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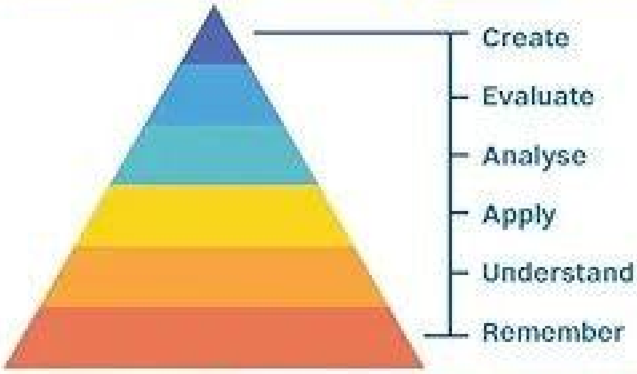
10 SAMPLE QUESTION PAPERS

CLASS 12

MATHEMATICS

Strictly as per the Latest Syllabus & Circular issued by CISCE for 2025 Exam


Create
Evaluate
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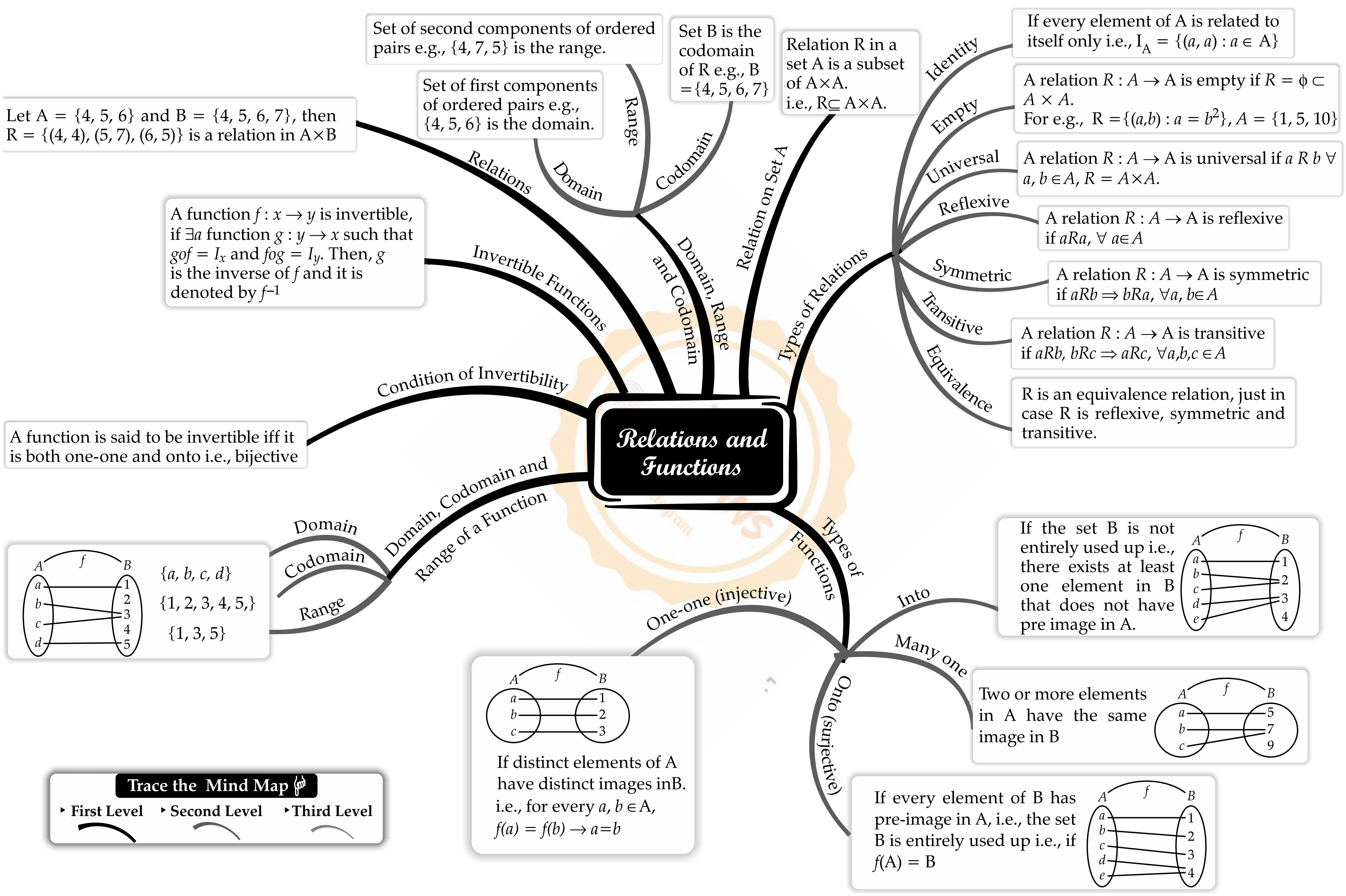
✓ ***Competency Focused Questions** Based on **Bloom's Taxonomy***

✓ ***Official Marking Scheme** Decoded for Students*



Relations and Functions

Relations

Let A = {4, 5, 6} and B = {4, 5, 6, 7}, then R = {(4, 4), (5, 7), (6, 5)} is a relation in A×B

Domain, Range and Codomain

- Range: Set of second components of ordered pairs e.g., {4, 7, 5} is the range.
- Domain: Set of first components of ordered pairs e.g., {4, 5, 6} is the domain.
- Codomain: Set B is the codomain of R e.g., B = {4, 5, 6, 7}

Relation on Set A

Relation R in a set A is a subset of A×A. i.e., R⊆ A×A.

Types of Relations

- Identity: If every element of A is related to itself only i.e., $I_A = \{(a, a) : a \in A\}$
- Empty: A relation $R : A \to A$ is empty if $R = \phi \subset A \times A$. For e.g., $R = \{(a,b) : a = b^2\}$, $A = \{1, 5, 10\}$
- Universal: A relation $R : A \to A$ is universal if $a\,R\,b\ \forall\ a, b \in A$, $R = A \times A$.
- Reflexive: A relation $R : A \to A$ is reflexive if aRa, $\forall\ a \in A$
- Symmetric: A relation $R : A \to A$ is symmetric if $aRb \Rightarrow bRa$, $\forall a, b \in A$
- Transitive: A relation $R : A \to A$ is transitive if aRb, $bRc \Rightarrow aRc$, $\forall a,b,c \in A$
- Equivalence: R is an equivalence relation, just in case R is reflexive, symmetric and transitive.

Invertible Functions

A function $f : x \to y$ is invertible, if $\exists a$ function $g : y \to x$ such that $gof = I_x$ and $fog = I_y$. Then, g is the inverse of f and it is denoted by f^{-1}

Condition of Invertibility

A function is said to be invertible iff it is both one-one and onto i.e., bijective

Domain, Codomain and Range of a Function

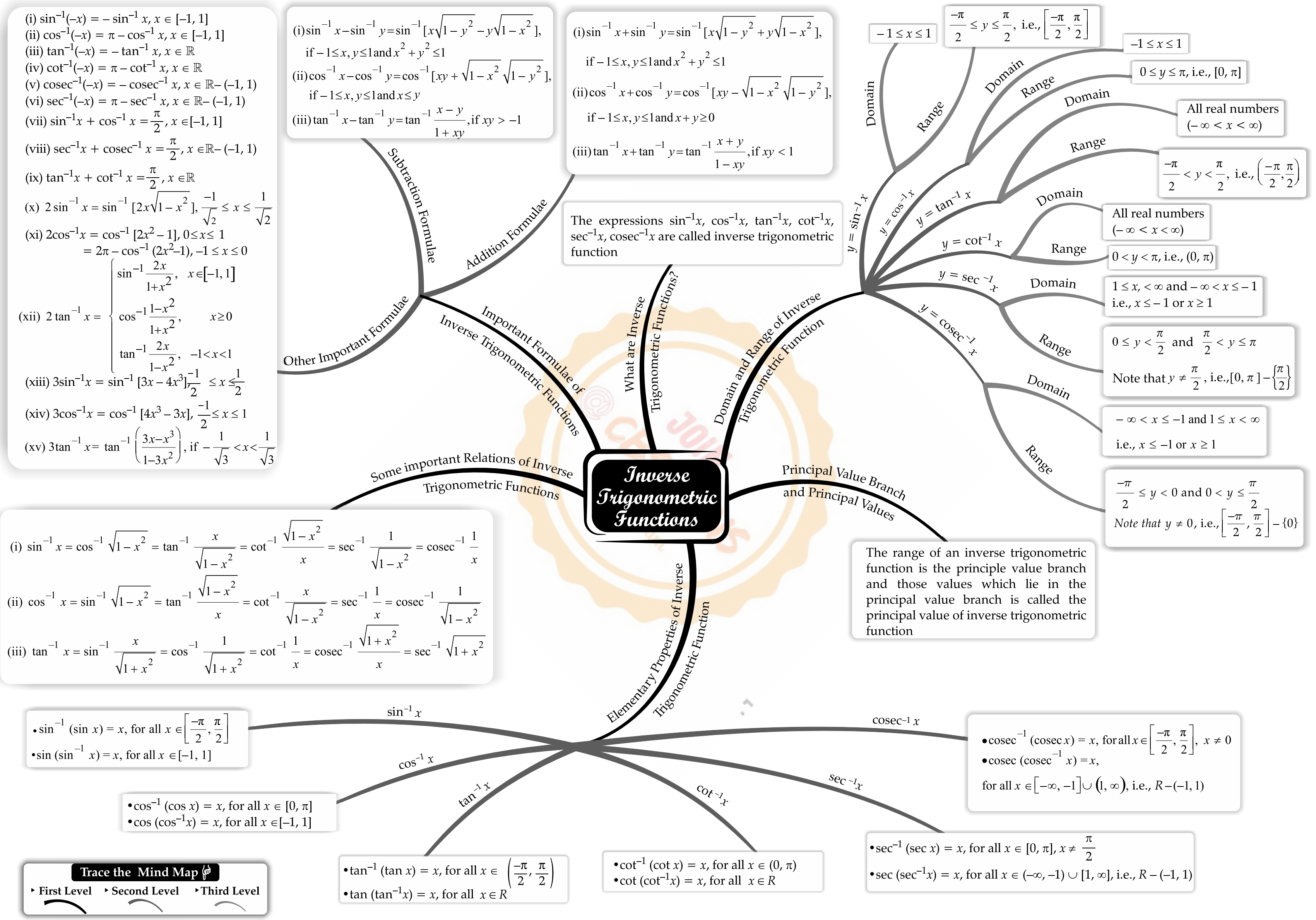
- Domain: $\{a, b, c, d\}$
- Codomain: $\{1, 2, 3, 4, 5,\}$
- Range: $\{1, 3, 5\}$

Types of Functions

- Into: If the set B is not entirely used up i.e., there exists at least one element in B that does not have pre image in A.
- Many one: Two or more elements in A have the same image in B
- Onto (surjective): If every element of B has pre-image in A, i.e., the set B is entirely used up i.e., if f(A) = B
- One-one (injective): If distinct elements of A have distinct images inB. i.e., for every $a, b \in A$, $f(a) = f(b) \to a=b$

Trace the Mind Map

- First Level
- Second Level
- Third Level



Inverse Trigonometric Functions

What are Inverse Trigonometric Functions?

The expressions $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, $\cot^{-1}x$, $\sec^{-1}x$, $\mathrm{cosec}^{-1}x$ are called inverse trigonometric function

Domain and Range of Inverse Trigonometric Function

$y = \sin^{-1} x$
- Domain: $-1 \le x \le 1$
- Range: $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$, i.e., $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$y = \cos^{-1} x$
- Domain: $-1 \le x \le 1$
- Range: $0 \le y \le \pi$, i.e., $[0, \pi]$

$y = \tan^{-1} x$
- Domain: All real numbers $(-\infty < x < \infty)$
- Range: $\frac{-\pi}{2} < y < \frac{\pi}{2}$, i.e., $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$y = \cot^{-1} x$
- Domain: All real numbers $(-\infty < x < \infty)$
- Range: $0 < y < \pi$, i.e., $(0, \pi)$

$y = \sec^{-1} x$
- Domain: $1 \le x, < \infty$ and $-\infty < x \le -1$ i.e., $x \le -1$ or $x \ge 1$
- Range: $0 \le y < \frac{\pi}{2}$ and $\frac{\pi}{2} < y \le \pi$ Note that $y \ne \frac{\pi}{2}$, i.e., $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

$y = \mathrm{cosec}^{-1} x$
- Domain: $-\infty < x \le -1$ and $1 \le x < \infty$ i.e., $x \le -1$ or $x \ge 1$
- Range: $\frac{-\pi}{2} \le y < 0$ and $0 < y \le \frac{\pi}{2}$ *Note that* $y \ne 0$, i.e., $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Principal Value Branch and Principal Values

The range of an inverse trigonometric function is the principle value branch and those values which lie in the principal value branch is called the principal value of inverse trigonometric function

Important Formulae of Inverse Trigonometric Functions

Addition Formulae

(i) $\sin^{-1} x + \sin^{-1} y = \sin^{-1}[x\sqrt{1-y^2} + y\sqrt{1-x^2}]$, if $-1 \le x, y \le 1$ and $x^2 + y^2 \le 1$

(ii) $\cos^{-1} x + \cos^{-1} y = \cos^{-1}[xy - \sqrt{1-x^2}\sqrt{1-y^2}]$, if $-1 \le x, y \le 1$ and $x + y \ge 0$

(iii) $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$, if $xy < 1$

Subtraction Formulae

(i) $\sin^{-1} x - \sin^{-1} y = \sin^{-1}[x\sqrt{1-y^2} - y\sqrt{1-x^2}]$, if $-1 \le x, y \le 1$ and $x^2 + y^2 \le 1$

(ii) $\cos^{-1} x - \cos^{-1} y = \cos^{-1}[xy + \sqrt{1-x^2}\sqrt{1-y^2}]$, if $-1 \le x, y \le 1$ and $x \le y$

(iii) $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$, if $xy > -1$

Other Important Formulae

(i) $\sin^{-1}(-x) = -\sin^{-1} x$, $x \in [-1, 1]$
(ii) $\cos^{-1}(-x) = \pi - \cos^{-1} x$, $x \in [-1, 1]$
(iii) $\tan^{-1}(-x) = -\tan^{-1} x$, $x \in \mathbb{R}$
(iv) $\cot^{-1}(-x) = \pi - \cot^{-1} x$, $x \in \mathbb{R}$
(v) $\mathrm{cosec}^{-1}(-x) = -\mathrm{cosec}^{-1} x$, $x \in \mathbb{R} - (-1, 1)$
(vi) $\sec^{-1}(-x) = \pi - \sec^{-1} x$, $x \in \mathbb{R} - (-1, 1)$
(vii) $\sin^{-1}x + \cos^{-1} x = \frac{\pi}{2}$, $x \in [-1, 1]$
(viii) $\sec^{-1}x + \mathrm{cosec}^{-1} x = \frac{\pi}{2}$, $x \in \mathbb{R} - (-1, 1)$
(ix) $\tan^{-1}x + \cot^{-1} x = \frac{\pi}{2}$, $x \in \mathbb{R}$
(x) $2 \sin^{-1} x = \sin^{-1}[2x\sqrt{1-x^2}]$, $\frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$
(xi) $2\cos^{-1}x = \cos^{-1}[2x^2 - 1]$, $0 \le x \le 1$
$= 2\pi - \cos^{-1}(2x^2 - 1)$, $-1 \le x \le 0$

(xii) $$2\tan^{-1} x = \begin{cases} \sin^{-1} \frac{2x}{1+x^2}, & x \in [-1, 1] \\ \cos^{-1} \frac{1-x^2}{1+x^2}, & x \ge 0 \\ \tan^{-1} \frac{2x}{1-x^2}, & -1 < x < 1 \end{cases}$$

(xiii) $3\sin^{-1}x = \sin^{-1}[3x - 4x^3]$, $\frac{-1}{2} \le x \le \frac{1}{2}$
(xiv) $3\cos^{-1}x = \cos^{-1}[4x^3 - 3x]$, $\frac{-1}{2} \le x \le 1$
(xv) $3\tan^{-1} x = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$, if $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

Some important Relations of Inverse Trigonometric Functions

(i) $\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x} = \sec^{-1}\frac{1}{\sqrt{1-x^2}} = \mathrm{cosec}^{-1}\frac{1}{x}$

(ii) $\cos^{-1} x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}} = \sec^{-1}\frac{1}{x} = \mathrm{cosec}^{-1}\frac{1}{\sqrt{1-x^2}}$

(iii) $\tan^{-1} x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}} = \cot^{-1}\frac{1}{x} = \mathrm{cosec}^{-1}\frac{\sqrt{1+x^2}}{x} = \sec^{-1}\sqrt{1+x^2}$

Elementary Properties of Inverse Trigonometric Function

$\sin^{-1} x$
- $\sin^{-1}(\sin x) = x$, for all $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
- $\sin(\sin^{-1} x) = x$, for all $x \in [-1, 1]$

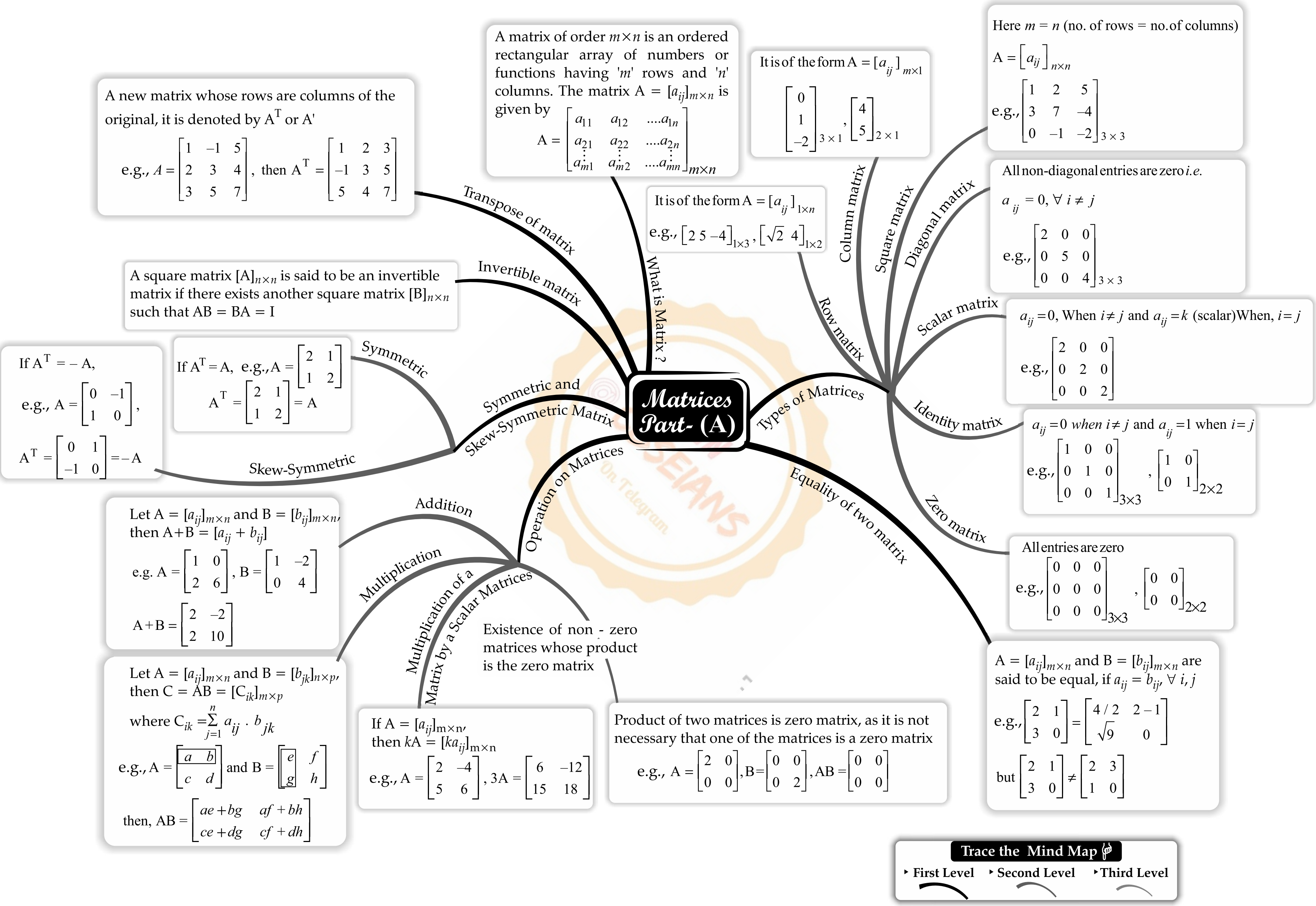
$\cos^{-1} x$
- $\cos^{-1}(\cos x) = x$, for all $x \in [0, \pi]$
- $\cos(\cos^{-1}x) = x$, for all $x \in [-1, 1]$

$\tan^{-1} x$
- $\tan^{-1}(\tan x) = x$, for all $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
- $\tan(\tan^{-1}x) = x$, for all $x \in R$

$\cot^{-1} x$
- $\cot^{-1}(\cot x) = x$, for all $x \in (0, \pi)$
- $\cot(\cot^{-1}x) = x$, for all $x \in R$

$\sec^{-1} x$
- $\sec^{-1}(\sec x) = x$, for all $x \in [0, \pi]$, $x \ne \frac{\pi}{2}$
- $\sec(\sec^{-1}x) = x$, for all $x \in (-\infty, -1) \cup [1, \infty]$, i.e., $R - (-1, 1)$

$\mathrm{cosec}^{-1} x$
- $\mathrm{cosec}^{-1}(\mathrm{cosec}\, x) = x$, for all $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$, $x \ne 0$
- $\mathrm{cosec}(\mathrm{cosec}^{-1} x) = x$, for all $x \in [-\infty, -1] \cup (1, \infty)$, i.e., $R - (-1, 1)$



Matrices Part- (A)

What is Matrix ?

A matrix of order $m\times n$ is an ordered rectangular array of numbers or functions having 'm' rows and 'n' columns. The matrix $A = [a_{ij}]_{m\times n}$ is given by

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots a_{1n} \\ a_{21} & a_{22} & \dots a_{2n} \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots a_{mn} \end{bmatrix}_{m\times n}$$

Types of Matrices

Row matrix

It is of the form $A = [a_{ij}]_{1\times n}$

e.g., $[2\ 5\ -4]_{1\times 3}, [\sqrt{2}\ 4]_{1\times 2}$

Column matrix

It is of the form $A = [a_{ij}]_{m\times 1}$

$$\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}_{3\times 1}, \begin{bmatrix} 4 \\ 5 \end{bmatrix}_{2\times 1}$$

Square matrix

Here $m = n$ (no. of rows = no.of columns)

$A = [a_{ij}]_{n\times n}$

e.g., $\begin{bmatrix} 1 & 2 & 5 \\ 3 & 7 & -4 \\ 0 & -1 & -2 \end{bmatrix}_{3\times 3}$

Diagonal matrix

All non-diagonal entries are zero *i.e.*

$a_{ij} = 0, \forall\ i \neq j$

e.g., $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}_{3\times 3}$

Scalar matrix

$a_{ij} = 0$, When $i \neq j$ and $a_{ij} = k$ (scalar)When, $i = j$

e.g., $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Identity matrix

$a_{ij} = 0$ *when* $i \neq j$ and $a_{ij} = 1$ when $i = j$

e.g., $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

Zero matrix

All entries are zero

e.g., $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$

Equality of two matrix

$A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ are said to be equal, if $a_{ij} = b_{ij}, \forall\ i, j$

e.g., $\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 4/2 & 2-1 \\ \sqrt{9} & 0 \end{bmatrix}$

but $\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} \neq \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$

Operation on Matrices

Existence of non - zero matrices whose product is the zero matrix

Product of two matrices is zero matrix, as it is not necessary that one of the matrices is a zero matrix

e.g., $A = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}, AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Multiplication of a Matrix by a Scalar Matrices

If $A = [a_{ij}]_{m\times n}$, then $kA = [ka_{ij}]_{m\times n}$

e.g., $A = \begin{bmatrix} 2 & -4 \\ 5 & 6 \end{bmatrix}, 3A = \begin{bmatrix} 6 & -12 \\ 15 & 18 \end{bmatrix}$

Multiplication

Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$, then $C = AB = [C_{ik}]_{m\times p}$

where $C_{ik} = \sum_{j=1}^{n} a_{ij} \cdot b_{jk}$

e.g., $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

then, $AB = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$

Addition

Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$, then $A+B = [a_{ij} + b_{ij}]$

e.g. $A = \begin{bmatrix} 1 & 0 \\ 2 & 6 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ 0 & 4 \end{bmatrix}$

$A+B = \begin{bmatrix} 2 & -2 \\ 2 & 10 \end{bmatrix}$

Symmetric and Skew-Symmetric Matrix

Symmetric

If $A^T = A$, e.g., $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$A^T = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = A$

Skew-Symmetric

If $A^T = -A$,

e.g., $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$,

$A^T = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -A$

Invertible matrix

A square matrix $[A]_{n\times n}$ is said to be an invertible matrix if there exists another square matrix $[B]_{n\times n}$ such that $AB = BA = I$

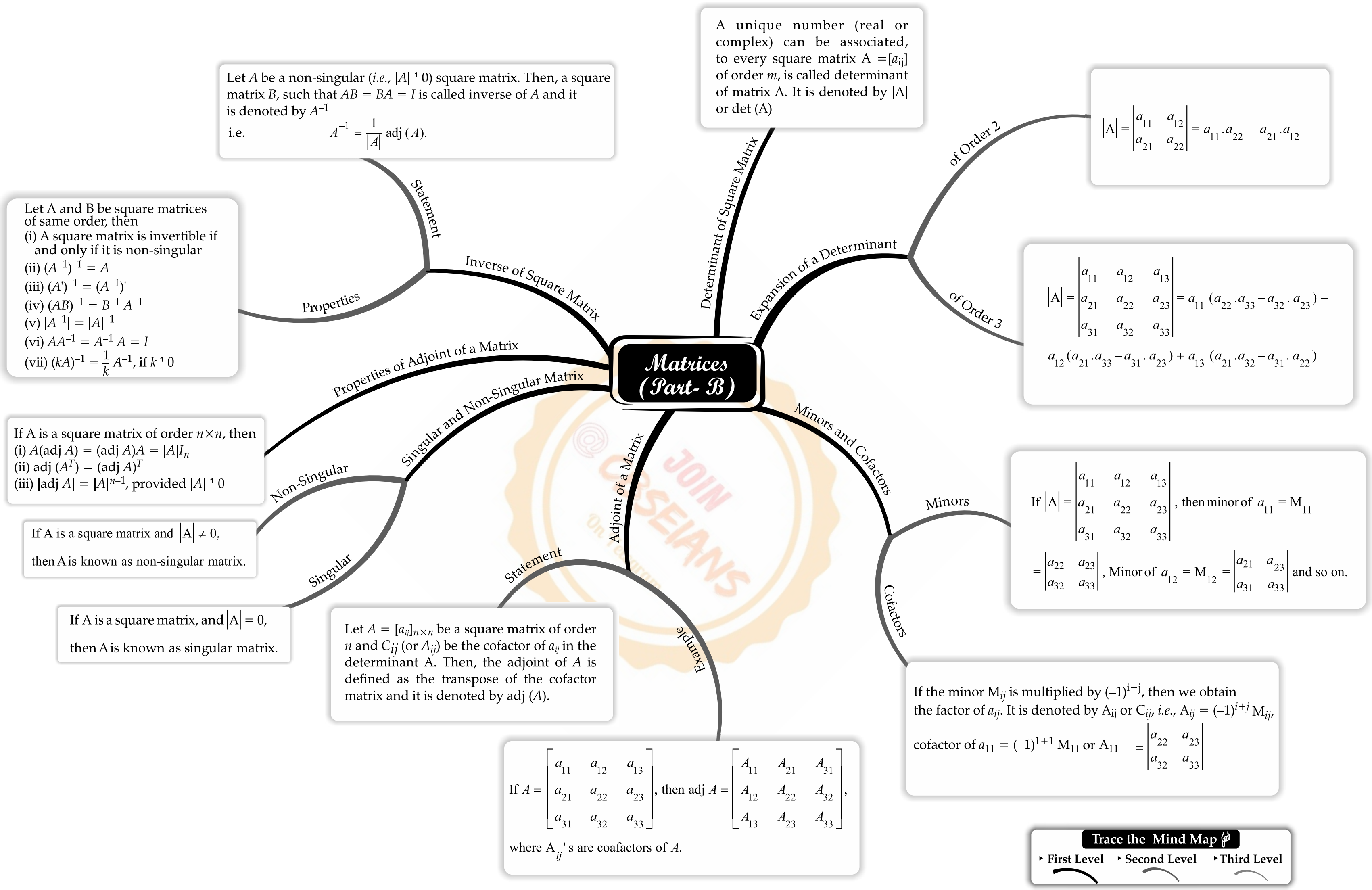
Transpose of matrix

A new matrix whose rows are columns of the original, it is denoted by A^T or A'

e.g., $A = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$, then $A^T = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 5 \\ 5 & 4 & 7 \end{bmatrix}$

Trace the Mind Map

- First Level
- Second Level
- Third Level



Matrices (Part- B)

Determinant of Square Matrix

A unique number (real or complex) can be associated, to every square matrix A $=[a_{ij}]$ of order m, is called determinant of matrix A. It is denoted by $|A|$ or det (A)

Expansion of a Determinant

of Order 2

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}.a_{22} - a_{21}.a_{12}$$

of Order 3

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\,(a_{22}.a_{33} - a_{32}.a_{23}) - a_{12}\,(a_{21}.a_{33} - a_{31}.a_{23}) + a_{13}\,(a_{21}.a_{32} - a_{31}.a_{22})$$

Minors and Cofactors

Minors

If $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, then minor of $a_{11} = M_{11}$ $= \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$, Minor of $a_{12} = M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ and so on.

Cofactors

If the minor M_{ij} is multiplied by $(-1)^{i+j}$, then we obtain the factor of a_{ij}. It is denoted by A_{ij} or C_{ij}, *i.e.*, $A_{ij} = (-1)^{i+j} M_{ij}$,

cofactor of $a_{11} = (-1)^{1+1} M_{11}$ or $A_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

Adjoint of a Matrix

Statement

Let $A = [a_{ij}]_{n\times n}$ be a square matrix of order n and C_{ij} (or A_{ij}) be the cofactor of a_{ij} in the determinant A. Then, the adjoint of A is defined as the transpose of the cofactor matrix and it is denoted by adj (A).

Example

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then adj $A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$,

where A_{ij}' s are coafactors of A.

Singular and Non-Singular Matrix

Non-Singular

If A is a square matrix and $|A| \neq 0$,

then A is known as non-singular matrix.

Singular

If A is a square matrix, and $|A| = 0$,

then A is known as singular matrix.

Properties of Adjoint of a Matrix

If A is a square matrix of order $n\times n$, then
(i) $A(\text{adj } A) = (\text{adj } A)A = |A|I_n$
(ii) adj $(A^T) = (\text{adj } A)^T$
(iii) $|\text{adj } A| = |A|^{n-1}$, provided $|A| \neq 0$

Inverse of Square Matrix

Statement

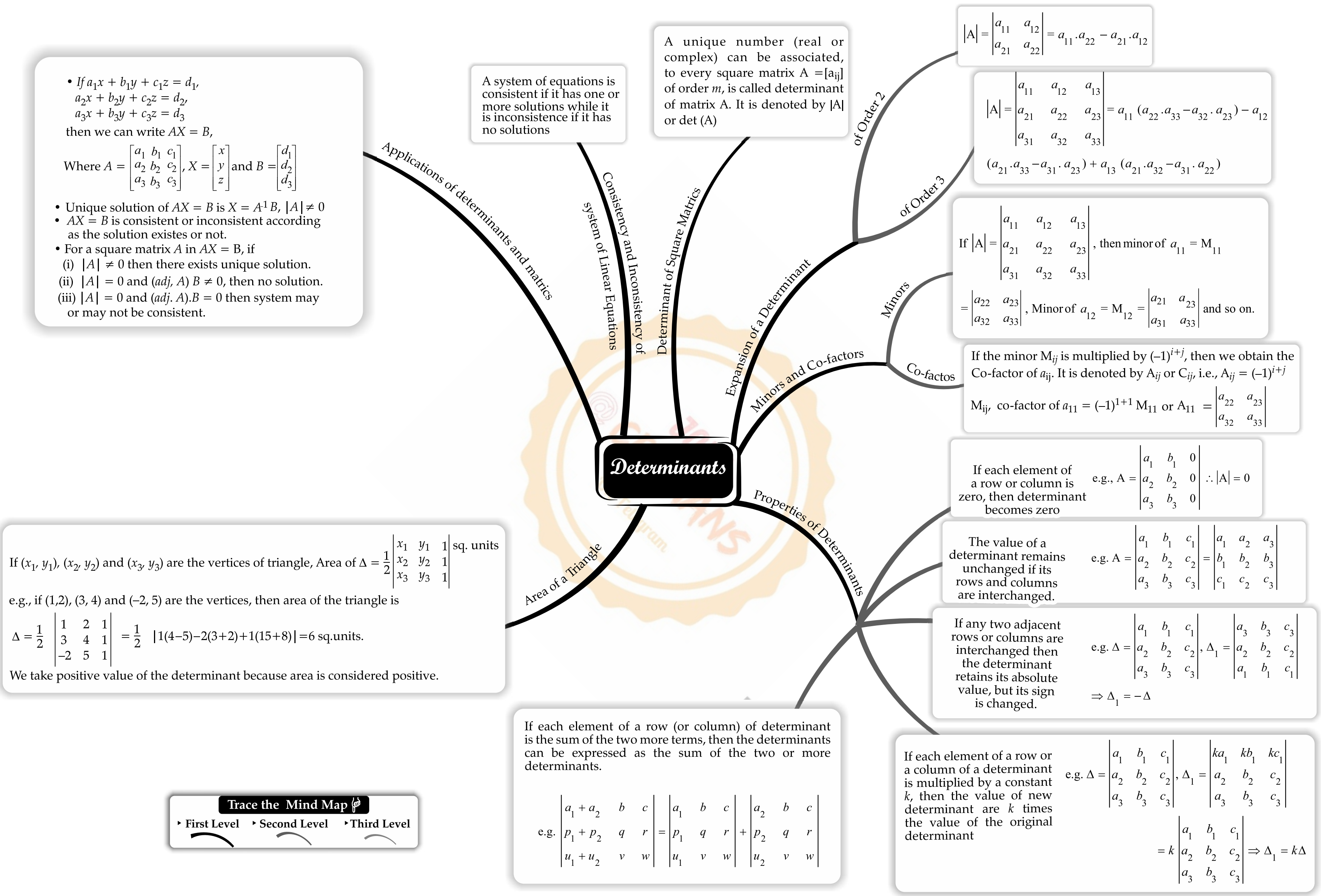
Let A be a non-singular (*i.e.*, $|A| \neq 0$) square matrix. Then, a square matrix B, such that $AB = BA = I$ is called inverse of A and it is denoted by A^{-1}

i.e. $A^{-1} = \frac{1}{|A|}$ adj (A).

Properties

Let A and B be square matrices of same order, then
(i) A square matrix is invertible if and only if it is non-singular
(ii) $(A^{-1})^{-1} = A$
(iii) $(A')^{-1} = (A^{-1})'$
(iv) $(AB)^{-1} = B^{-1}A^{-1}$
(v) $|A^{-1}| = |A|^{-1}$
(vi) $AA^{-1} = A^{-1}A = I$
(vii) $(kA)^{-1} = \frac{1}{k}A^{-1}$, if $k \neq 0$

Trace the Mind Map
- First Level
- Second Level
- Third Level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Properties of Determinants
Area of a Triangle
Consistency and Inconsistency of system of Linear Equations
Applications of determinants and matrics
Trace the Mind Map
First Level
Second Level
Third Level

A unique number (real or complex) can be associated, to every square matrix A =[a$_{ij}$] of order *m*, is called determinant of matrix A. It is denoted by |A| or det (A)

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}.a_{22} - a_{21}.a_{12}$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\,(a_{22}.a_{33} - a_{32}.a_{23}) - a_{12}$$
$$(a_{21}.a_{33} - a_{31}.a_{23}) + a_{13}\,(a_{21}.a_{32} - a_{31}.a_{22})$$

If $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, then minor of $a_{11} = M_{11}$

$= \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$, Minor of $a_{12} = M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ and so on.

If the minor M_{ij} is multiplied by $(-1)^{i+j}$, then we obtain the Co-factor of a_{ij}. It is denoted by A_{ij} or C_{ij}, i.e., $A_{ij} = (-1)^{i+j}$

M_{ij}, co-factor of $a_{11} = (-1)^{1+1} M_{11}$ or $A_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

If each element of a row or column is zero, then determinant becomes zero

e.g., $A = \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} \therefore |A| = 0$

The value of a determinant remains unchanged if its rows and columns are interchanged.

e.g. $A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

If any two adjacent rows or columns are interchanged then the determinant retains its absolute value, but its sign is changed.

e.g. $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta_1 = \begin{vmatrix} a_3 & b_3 & c_3 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix}$

$\Rightarrow \Delta_1 = -\Delta$

If each element of a row or a column of a determinant is multiplied by a constant *k*, then the value of new determinant are *k* times the value of the original determinant

e.g. $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta_1 = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$= k \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \Rightarrow \Delta_1 = k\Delta$

If each element of a row (or column) of determinant is the sum of the two more terms, then the determinants can be expressed as the sum of the two or more determinants.

e.g. $\begin{vmatrix} a_1 + a_2 & b & c \\ p_1 + p_2 & q & r \\ u_1 + u_2 & v & w \end{vmatrix} = \begin{vmatrix} a_1 & b & c \\ p_1 & q & r \\ u_1 & v & w \end{vmatrix} + \begin{vmatrix} a_2 & b & c \\ p_2 & q & r \\ u_2 & v & w \end{vmatrix}$

If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the vertices of triangle, Area of $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ sq. units

e.g., if (1,2), (3, 4) and (–2, 5) are the vertices, then area of the triangle is

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 4 & 1 \\ -2 & 5 & 1 \end{vmatrix} = \frac{1}{2}\ |1(4-5)-2(3+2)+1(15+8)| = 6 \text{ sq.units.}$$

We take positive value of the determinant because area is considered positive.

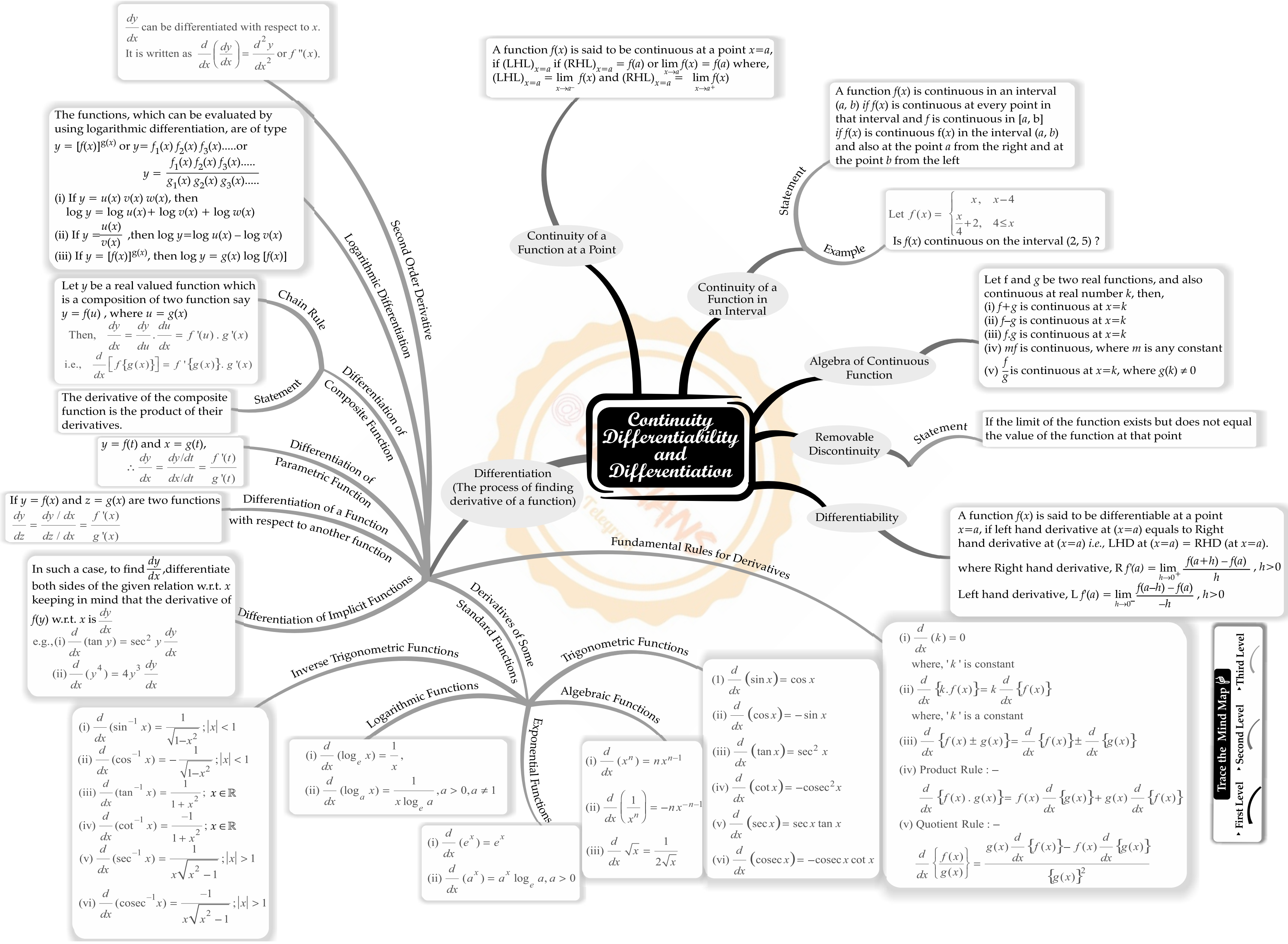
A system of equations is consistent if it has one or more solutions while it is inconsistence if it has no solutions

- If $a_1x + b_1y + c_1z = d_1$,
  $a_2x + b_2y + c_2z = d_2$,
  $a_3x + b_3y + c_3z = d_3$

then we can write $AX = B$,

Where $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

- Unique solution of $AX = B$ is $X = A^{-1}B$, $|A| \neq 0$
- $AX = B$ is consistent or inconsistent according as the solution existes or not.
- For a square matrix A in $AX =$ B, if
  - (i) $|A| \neq 0$ then there exists unique solution.
  - (ii) $|A| = 0$ and $(adj, A)\, B \neq 0$, then no solution.
  - (iii) $|A| = 0$ and $(adj.\ A).B = 0$ then system may or may not be consistent.



Continuity Differentiability and Differentiation

Continuity of a Function at a Point

A function $f(x)$ is said to be continuous at a point $x=a$, if $(\text{LHL})_{x=a}$ if $(\text{RHL})_{x=a} = f(a)$ or $\lim_{x\to a} f(x) = f(a)$ where, $(\text{LHL})_{x=a} = \lim_{x\to a^-} f(x)$ and $(\text{RHL})_{x=a} = \lim_{x\to a^+} f(x)$

Continuity of a Function in an Interval

Statement

A function $f(x)$ is continuous in an interval (a, b) if $f(x)$ is continuous at every point in that interval and f is continuous in $[a, b]$ if $f(x)$ is continuous $f(x)$ in the interval (a, b) and also at the point a from the right and at the point b from the left

Example

Let $f(x) = \begin{cases} x, & x-4 \\ \frac{x}{4}+2, & 4 \le x \end{cases}$

Is $f(x)$ continuous on the interval (2, 5) ?

Algebra of Continuous Function

Let f and g be two real functions, and also continuous at real number k, then,

(i) $f+g$ is continuous at $x=k$

(ii) $f-g$ is continuous at $x=k$

(iii) $f.g$ is continuous at $x=k$

(iv) mf is continuous, where m is any constant

(v) $\frac{f}{g}$ is continuous at $x=k$, where $g(k) \neq 0$

Removable Discontinuity

Statement

If the limit of the function exists but does not equal the value of the function at that point

Differentiability

A function $f(x)$ is said to be differentiable at a point $x=a$, if left hand derivative at $(x=a)$ equals to Right hand derivative at $(x=a)$ i.e., LHD at $(x=a)$ = RHD (at $x=a$).

where Right hand derivative, $\text{R}f'(a) = \lim_{h\to 0^+} \frac{f(a+h)-f(a)}{h}$, $h>0$

Left hand derivative, $\text{L}f'(a) = \lim_{h\to 0^-} \frac{f(a-h)-f(a)}{-h}$, $h>0$

Differentiation (The process of finding derivative of a function)

Fundamental Rules for Derivatives

(i) $\frac{d}{dx}(k) = 0$

where, 'k' is constant

(ii) $\frac{d}{dx}\{k.f(x)\} = k\frac{d}{dx}\{f(x)\}$

where, 'k' is a constant

(iii) $\frac{d}{dx}\{f(x) \pm g(x)\} = \frac{d}{dx}\{f(x)\} \pm \frac{d}{dx}\{g(x)\}$

(iv) Product Rule : –

$$\frac{d}{dx}\{f(x).g(x)\} = f(x)\frac{d}{dx}\{g(x)\} + g(x)\frac{d}{dx}\{f(x)\}$$

(v) Quotient Rule : –

$$\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x)\frac{d}{dx}\{f(x)\} - f(x)\frac{d}{dx}\{g(x)\}}{\{g(x)\}^2}$$

Derivatives of Some Standard Functions

Trigonometric Functions

(i) $\frac{d}{dx}(\sin x) = \cos x$

(ii) $\frac{d}{dx}(\cos x) = -\sin x$

(iii) $\frac{d}{dx}(\tan x) = \sec^2 x$

(iv) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

(v) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(vi) $\frac{d}{dx}(\text{cosec}\, x) = -\text{cosec}\, x \cot x$

Algebraic Functions

(i) $\frac{d}{dx}(x^n) = nx^{n-1}$

(ii) $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -nx^{-n-1}$

(iii) $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

Exponential Functions

(i) $\frac{d}{dx}(e^x) = e^x$

(ii) $\frac{d}{dx}(a^x) = a^x \log_e a, a > 0$

Logarithmic Functions

(i) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$,

(ii) $\frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a}, a > 0, a \neq 1$

Inverse Trigonometric Functions

(i) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}; |x| < 1$

(ii) $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}; |x| < 1$

(iii) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}; x \in \mathbb{R}$

(iv) $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}; x \in \mathbb{R}$

(v) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}; |x| > 1$

(vi) $\frac{d}{dx}(\text{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}; |x| > 1$

Differentiation of Implicit Functions

In such a case, to find $\frac{dy}{dx}$, differentiate both sides of the given relation w.r.t. x keeping in mind that the derivative of $f(y)$ w.r.t. x is $\frac{dy}{dx}$

e.g., (i) $\frac{d}{dx}(\tan y) = \sec^2 y \frac{dy}{dx}$

(ii) $\frac{d}{dx}(y^4) = 4y^3 \frac{dy}{dx}$

Differentiation of a Function with respect to another function

If $y = f(x)$ and $z = g(x)$ are two functions

$\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{f'(x)}{g'(x)}$

Differentiation of Parametric Function

$y = f(t)$ and $x = g(t)$,

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{f'(t)}{g'(t)}$

Differentiation of Composite Function

Chain Rule

Let y be a real valued function which is a composition of two function say $y = f(u)$, where $u = g(x)$

Then, $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = f'(u).g'(x)$

i.e., $\frac{d}{dx}[f\{g(x)\}] = f'\{g(x)\}.g'(x)$

Statement

The derivative of the composite function is the product of their derivatives.

Logarithmic Differentiation

The functions, which can be evaluated by using logarithmic differentiation, are of type $y = [f(x)]^{g(x)}$ or $y = f_1(x) f_2(x) f_3(x)$.....or

$$y = \frac{f_1(x) f_2(x) f_3(x).....}{g_1(x) g_2(x) g_3(x).....}$$

(i) If $y = u(x)\, v(x)\, w(x)$, then $\log y = \log u(x) + \log v(x) + \log w(x)$

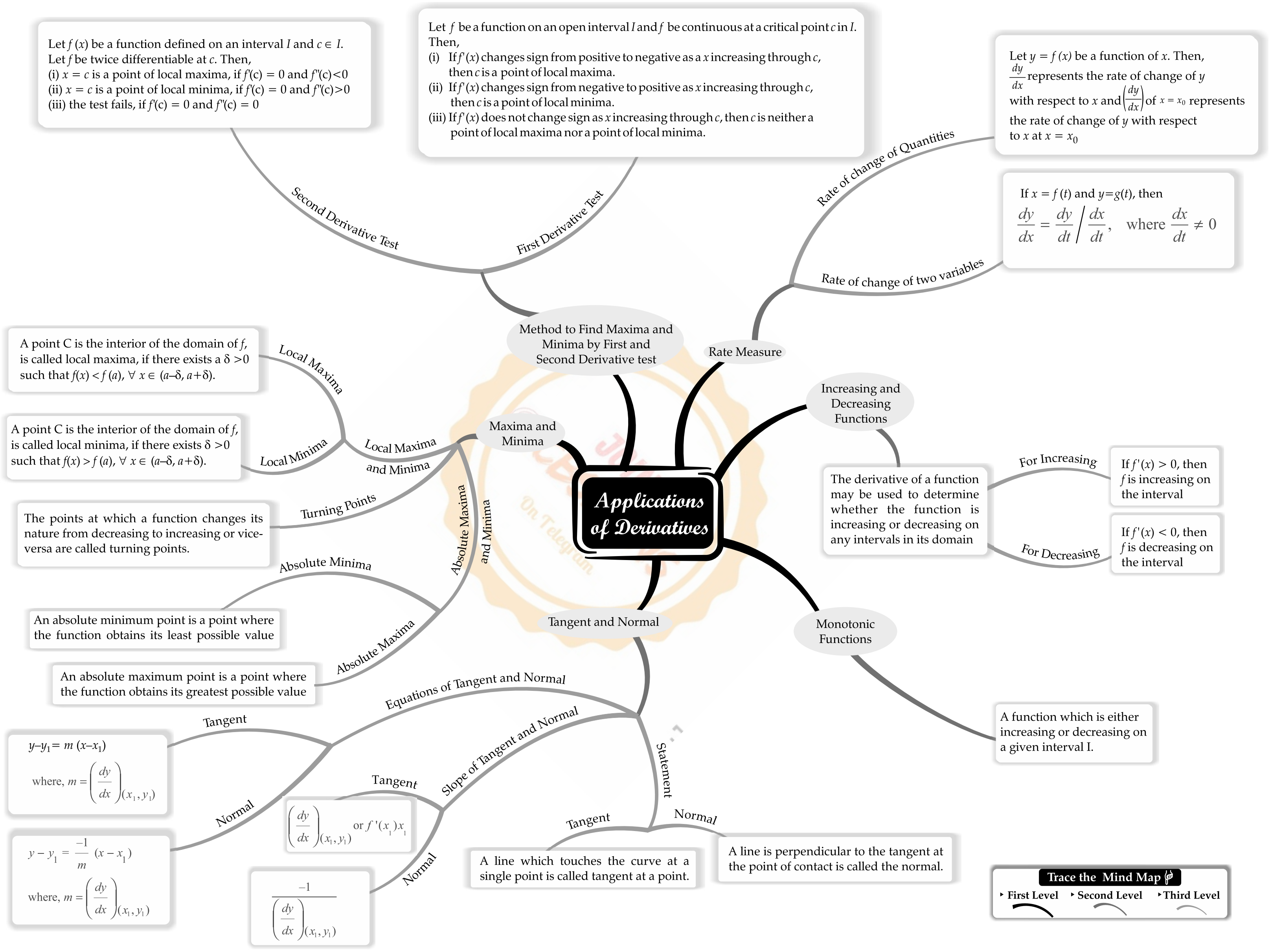
(ii) If $y = \frac{u(x)}{v(x)}$, then $\log y = \log u(x) - \log v(x)$

(iii) If $y = [f(x)]^{g(x)}$, then $\log y = g(x) \log [f(x)]$

Second Order Derivative

$\frac{dy}{dx}$ can be differentiated with respect to x.

It is written as $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$ or $f''(x)$.



Applications of Derivatives

Rate Measure

Rate of change of Quantities

Let $y = f(x)$ be a function of x. Then, $\frac{dy}{dx}$ represents the rate of change of y with respect to x and $\left(\frac{dy}{dx}\right)$ of $x = x_0$ represents the rate of change of y with respect to x at $x = x_0$

Rate of change of two variables

If $x = f(t)$ and $y = g(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$, where $\frac{dx}{dt} \neq 0$

Increasing and Decreasing Functions

The derivative of a function may be used to determine whether the function is increasing or decreasing on any intervals in its domain

For Increasing

If $f'(x) > 0$, then f is increasing on the interval

For Decreasing

If $f'(x) < 0$, then f is decreasing on the interval

Monotonic Functions

A function which is either increasing or decreasing on a given interval I.

Tangent and Normal

Statement

Tangent

A line which touches the curve at a single point is called tangent at a point.

Normal

A line is perpendicular to the tangent at the point of contact is called the normal.

Slope of Tangent and Normal

Tangent

$\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$ or $f'(x_1)x_1$

Normal

$\frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}}$

Equations of Tangent and Normal

Tangent

$y - y_1 = m(x - x_1)$

where, $m = \left(\frac{dy}{dx}\right)_{(x_1, y_1)}$

Normal

$y - y_1 = \frac{-1}{m}(x - x_1)$

where, $m = \left(\frac{dy}{dx}\right)_{(x_1, y_1)}$

Maxima and Minima

Local Maxima and Minima

Local Maxima

A point C is the interior of the domain of f, is called local maxima, if there exists a $\delta > 0$ such that $f(x) < f(a)$, $\forall\ x \in (a-\delta, a+\delta)$.

Local Minima

A point C is the interior of the domain of f, is called local minima, if there exists $\delta > 0$ such that $f(x) > f(a)$, $\forall\ x \in (a-\delta, a+\delta)$.

Turning Points

The points at which a function changes its nature from decreasing to increasing or vice-versa are called turning points.

Absolute Maxima and Minima

Absolute Minima

An absolute minimum point is a point where the function obtains its least possible value

Absolute Maxima

An absolute maximum point is a point where the function obtains its greatest possible value

Method to Find Maxima and Minima by First and Second Derivative test

First Derivative Test

Let f be a function on an open interval I and f be continuous at a critical point c in I. Then,

(i) If $f'(x)$ changes sign from positive to negative as a x increasing through c, then c is a point of local maxima.

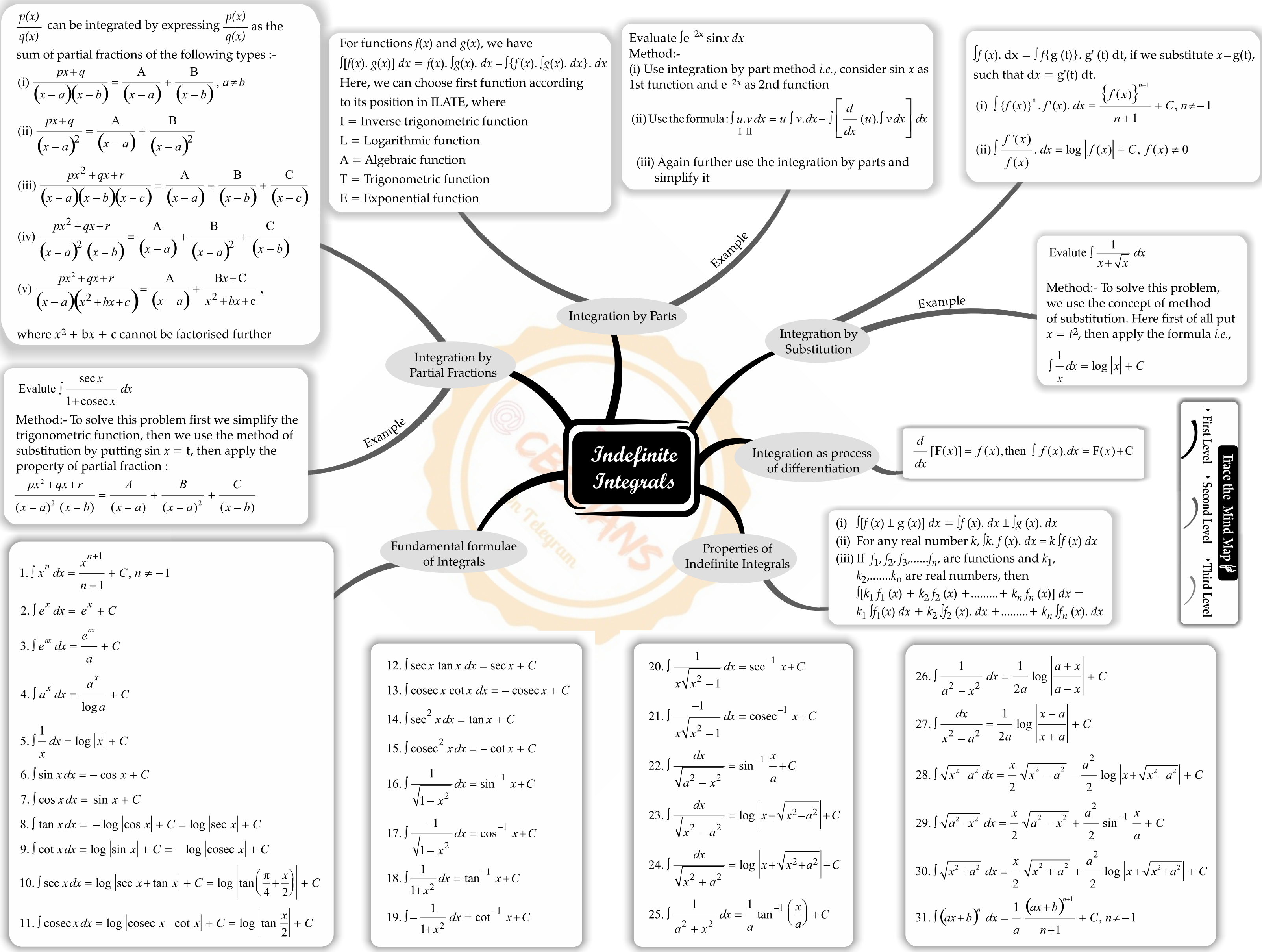
(ii) If $f'(x)$ changes sign from negative to positive as x increasing through c, then c is a point of local minima.

(iii) If $f'(x)$ does not change sign as x increasing through c, then c is neither a point of local maxima nor a point of local minima.

Second Derivative Test

Let $f(x)$ be a function defined on an interval I and $c \in I$.
Let f be twice differentiable at c. Then,
(i) $x = c$ is a point of local maxima, if $f'(c) = 0$ and $f''(c) < 0$
(ii) $x = c$ is a point of local minima, if $f'(c) = 0$ and $f''(c) > 0$
(iii) the test fails, if $f'(c) = 0$ and $f''(c) = 0$

Trace the Mind Map
- First Level
- Second Level
- Third Level



Trace the Mind Map

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Indefinite Integrals

Integration by Partial Fractions

$\frac{p(x)}{q(x)}$ can be integrated by expressing $\frac{p(x)}{q(x)}$ as the sum of partial fractions of the following types :-

(i) $\frac{px+q}{(x-a)(x-b)} = \frac{A}{(x-a)} + \frac{B}{(x-b)}, a \neq b$

(ii) $\frac{px+q}{(x-a)^2} = \frac{A}{(x-a)} + \frac{B}{(x-a)^2}$

(iii) $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$

(iv) $\frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$

(v) $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{(x-a)} + \frac{Bx+C}{x^2+bx+c}$,

where $x^2 + bx + c$ cannot be factorised further

Example

Evalute $\int \frac{\sec x}{1+\operatorname{cosec} x} dx$

Method:- To solve this problem first we simplify the trigonometric function, then we use the method of substitution by putting sin x = t, then apply the property of partial fraction :

$$\frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$$

Integration by Parts

For functions $f(x)$ and $g(x)$, we have

$\int [f(x).\,g(x)]\,dx = f(x).\int g(x).\,dx - \int \{f'(x).\int g(x).\,dx\}.\,dx$

Here, we can choose first function according to its position in ILATE, where

I = Inverse trigonometric function
L = Logarithmic function
A = Algebraic function
T = Trigonometric function
E = Exponential function

Example

Evaluate $\int e^{-2x} \sin x\, dx$

Method:-

(i) Use integration by part method *i.e.*, consider sin x as 1st function and e^{-2x} as 2nd function

(ii) Use the formula : $\int \underset{\text{I}}{u}.\underset{\text{II}}{v}\, dx = u \int v.dx - \int \left[\frac{d}{dx}(u).\int v\,dx\right] dx$

(iii) Again further use the integration by parts and simplify it

Integration by Substitution

$\int f(x).\,dx = \int f\{g(t)\}.\,g'(t)\,dt$, if we substitute x=g(t), such that dx = g'(t) dt.

(i) $\int \{f(x)\}^n . f'(x).\,dx = \frac{\{f(x)\}^{n+1}}{n+1} + C, n \neq -1$

(ii) $\int \frac{f'(x)}{f(x)}.\,dx = \log|f(x)| + C, f(x) \neq 0$

Example

Evalute $\int \frac{1}{x+\sqrt{x}} dx$

Method:- To solve this problem, we use the concept of method of substitution. Here first of all put $x = t^2$, then apply the formula *i.e.*,

$\int \frac{1}{x} dx = \log|x| + C$

Integration as process of differentiation

$\frac{d}{dx}[F(x)] = f(x)$, then $\int f(x).dx = F(x)+C$

Properties of Indefinite Integrals

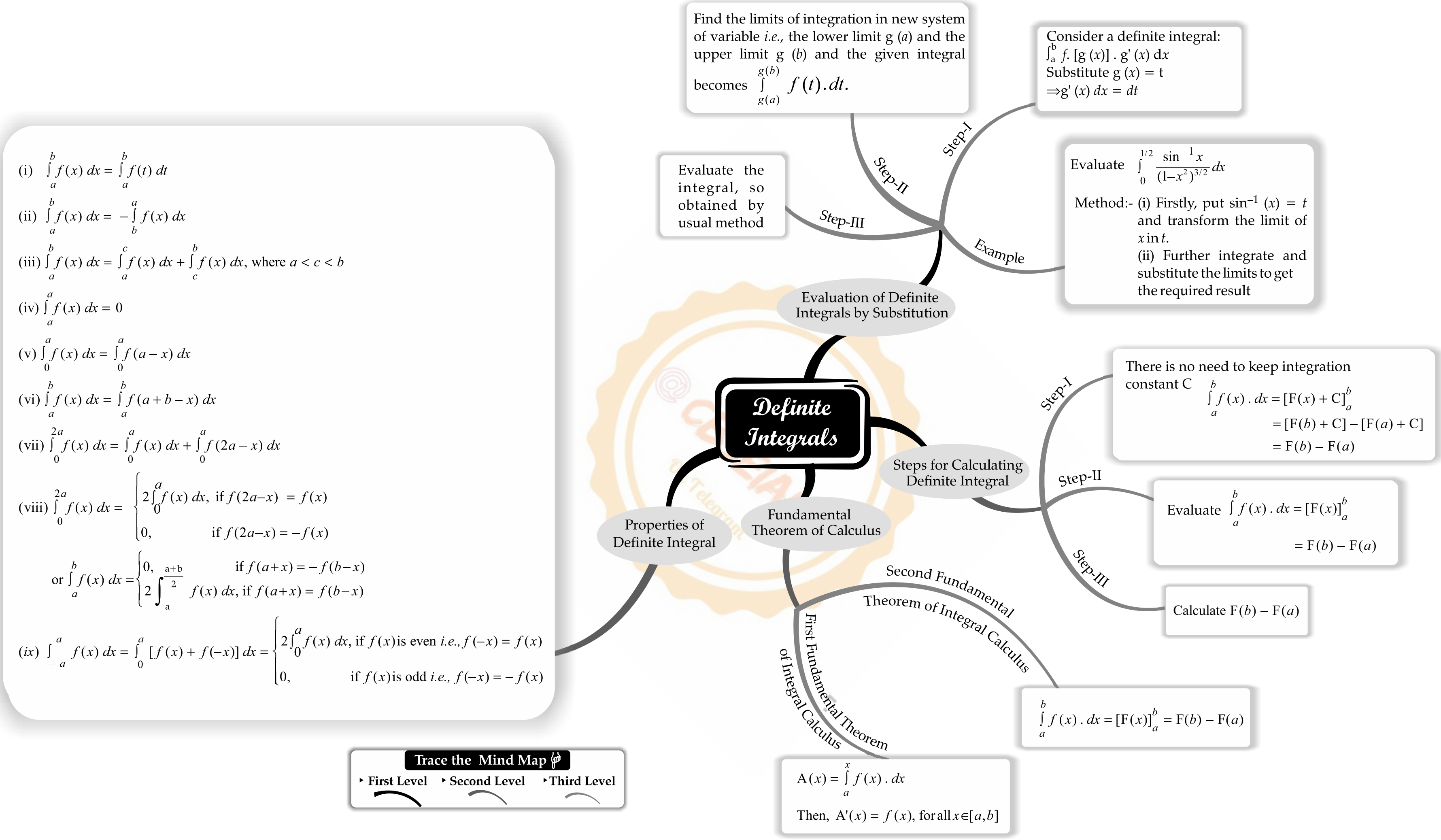
(i) $\int [f(x) \pm g(x)]\,dx = \int f(x).\,dx \pm \int g(x).\,dx$

(ii) For any real number k, $\int k.\,f(x).\,dx = k\int f(x)\,dx$

(iii) If $f_1, f_2, f_3, ...f_n$, are functions and $k_1, k_2, ...k_n$ are real numbers, then $\int [k_1 f_1(x) + k_2 f_2(x) + ... + k_n f_n(x)]\,dx = k_1 \int f_1(x)\,dx + k_2 \int f_2(x).\,dx + ... + k_n \int f_n(x).\,dx$

Fundamental formulae of Integrals

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
2. $\int e^x dx = e^x + C$
3. $\int e^{ax} dx = \frac{e^{ax}}{a} + C$
4. $\int a^x dx = \frac{a^x}{\log a} + C$
5. $\int \frac{1}{x} dx = \log|x| + C$
6. $\int \sin x\, dx = -\cos x + C$
7. $\int \cos x\, dx = \sin x + C$
8. $\int \tan x\, dx = -\log|\cos x| + C = \log|\sec x| + C$
9. $\int \cot x\, dx = \log|\sin x| + C = -\log|\operatorname{cosec} x| + C$
10. $\int \sec x\, dx = \log|\sec x + \tan x| + C = \log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + C$
11. $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| + C = \log\left|\tan\frac{x}{2}\right| + C$
12. $\int \sec x \tan x\, dx = \sec x + C$
13. $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$
14. $\int \sec^2 x\, dx = \tan x + C$
15. $\int \operatorname{cosec}^2 x\, dx = -\cot x + C$
16. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
17. $\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C$
18. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
19. $\int -\frac{1}{1+x^2} dx = \cot^{-1} x + C$
20. $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C$
21. $\int \frac{-1}{x\sqrt{x^2-1}} dx = \operatorname{cosec}^{-1} x + C$
22. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$
23. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + C$
24. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right| + C$
25. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
26. $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$
27. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$
28. $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$
29. $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$
30. $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$
31. $\int (ax+b)^n dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + C, n \neq -1$



Definite Integrals

Evaluation of Definite Integrals by Substitution

Step-I: Consider a definite integral: $\int_a^b f.[g(x)].g'(x)\,dx$
Substitute $g(x) = t$
$\Rightarrow g'(x)\,dx = dt$

Step-II: Find the limits of integration in new system of variable *i.e.,* the lower limit $g(a)$ and the upper limit $g(b)$ and the given integral becomes $\int_{g(a)}^{g(b)} f(t).dt$.

Step-III: Evaluate the integral, so obtained by usual method

Example: Evaluate $\int_0^{1/2} \frac{\sin^{-1} x}{(1-x^2)^{3/2}}\,dx$

Method:- (i) Firstly, put $\sin^{-1}(x) = t$ and transform the limit of x in t.
(ii) Further integrate and substitute the limits to get the required result

Steps for Calculating Definite Integral

Step-I: There is no need to keep integration constant C

$$\int_a^b f(x).dx = [F(x)+C]_a^b$$
$$= [F(b)+C]-[F(a)+C]$$
$$= F(b) - F(a)$$

Step-II: Evaluate $\int_a^b f(x).dx = [F(x)]_a^b$
$$= F(b) - F(a)$$

Step-III: Calculate $F(b) - F(a)$

Fundamental Theorem of Calculus

Second Fundamental Theorem of Integral Calculus

$$\int_a^b f(x).dx = [F(x)]_a^b = F(b) - F(a)$$

First Fundamental Theorem of Integral Calculus

$$A(x) = \int_a^x f(x).dx$$

Then, $A'(x) = f(x)$, for all $x \in [a,b]$

Properties of Definite Integral

(i) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

(ii) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(iii) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, where $a < c < b$

(iv) $\int_a^a f(x)\,dx = 0$

(v) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

(vi) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(vii) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

(viii) $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$

or $\int_a^b f(x)\,dx = \begin{cases} 0, & \text{if } f(a+x) = -f(b-x) \\ 2\int_a^{\frac{a+b}{2}} f(x)\,dx, & \text{if } f(a+x) = f(b-x) \end{cases}$

(ix) $\int_{-a}^{a} f(x)\,dx = \int_0^a [f(x)+f(-x)]\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is even } i.e., f(-x) = f(x) \\ 0, & \text{if } f(x) \text{ is odd } i.e., f(-x) = -f(x) \end{cases}$

Trace the Mind Map
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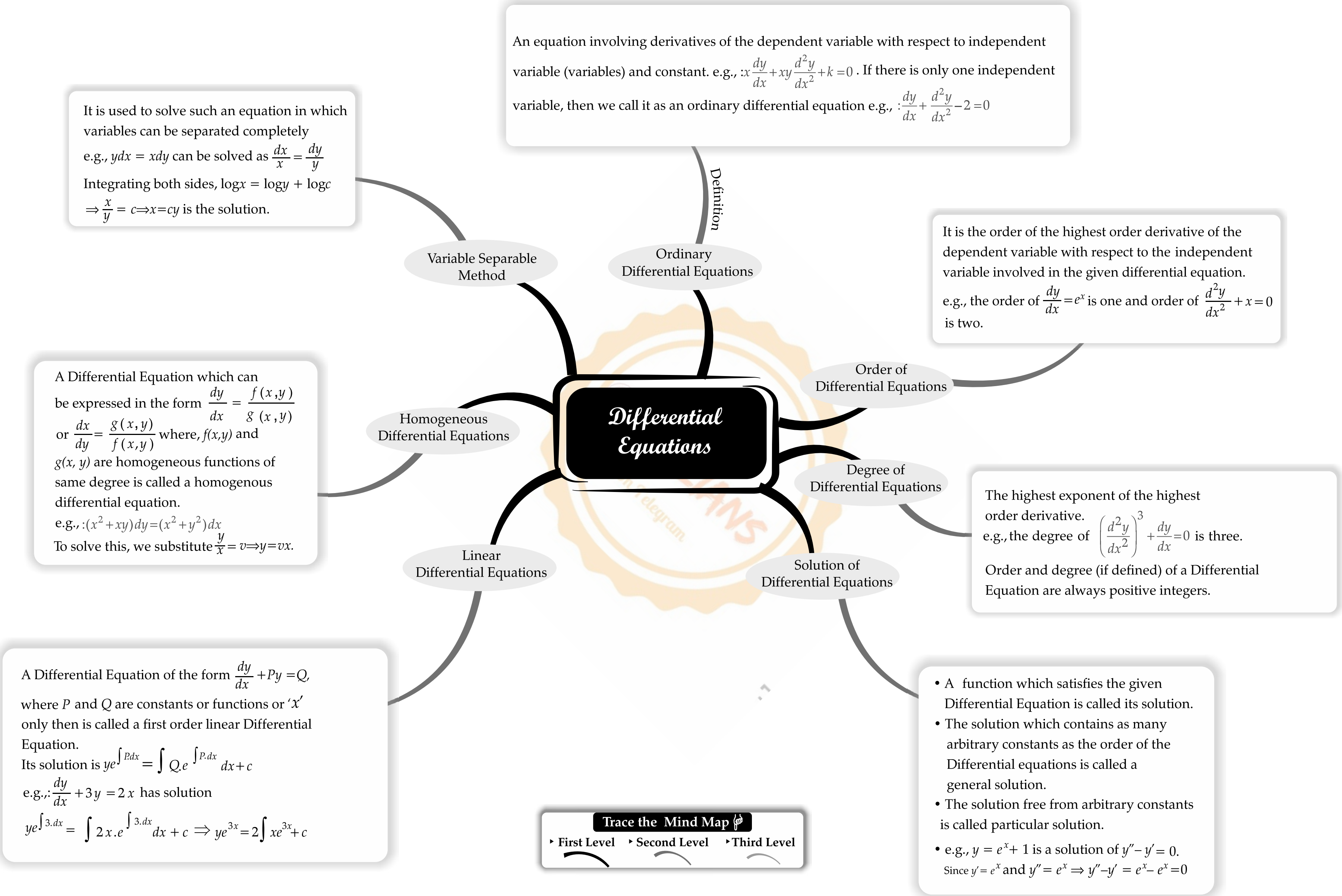

Differential Equations
Ordinary Differential Equations
Definition
Variable Separable Method
Homogeneous Differential Equations
Linear Differential Equations
Order of Differential Equations
Degree of Differential Equations
Solution of Differential Equations
Trace the Mind Map
First Level
Second Level
Third Level

An equation involving derivatives of the dependent variable with respect to independent variable (variables) and constant. e.g., :$x\frac{dy}{dx}+xy\frac{d^2y}{dx^2}+k=0$. If there is only one independent variable, then we call it as an ordinary differential equation e.g., :$\frac{dy}{dx}+\frac{d^2y}{dx^2}-2=0$

It is used to solve such an equation in which variables can be separated completely

e.g., $ydx = xdy$ can be solved as $\frac{dx}{x}=\frac{dy}{y}$

Integrating both sides, $\log x = \log y + \log c$

$\Rightarrow \frac{x}{y} = c \Rightarrow x = cy$ is the solution.

A Differential Equation which can be expressed in the form $\frac{dy}{dx}=\frac{f(x,y)}{g(x,y)}$ or $\frac{dx}{dy}=\frac{g(x,y)}{f(x,y)}$ where, $f(x,y)$ and $g(x, y)$ are homogeneous functions of same degree is called a homogenous differential equation.

e.g., :$(x^2+xy)dy=(x^2+y^2)dx$

To solve this, we substitute $\frac{y}{x}=v \Rightarrow y=vx$.

A Differential Equation of the form $\frac{dy}{dx}+Py=Q$, where P and Q are constants or functions or 'x' only then is called a first order linear Differential Equation.

Its solution is $ye^{\int P.dx}=\int Q.e^{\int P.dx}dx+c$

e.g.,:$\frac{dy}{dx}+3y=2x$ has solution

$ye^{\int 3.dx}=\int 2x.e^{\int 3.dx}dx+c \Rightarrow ye^{3x}=2\int xe^{3x}+c$

It is the order of the highest order derivative of the dependent variable with respect to the independent variable involved in the given differential equation.

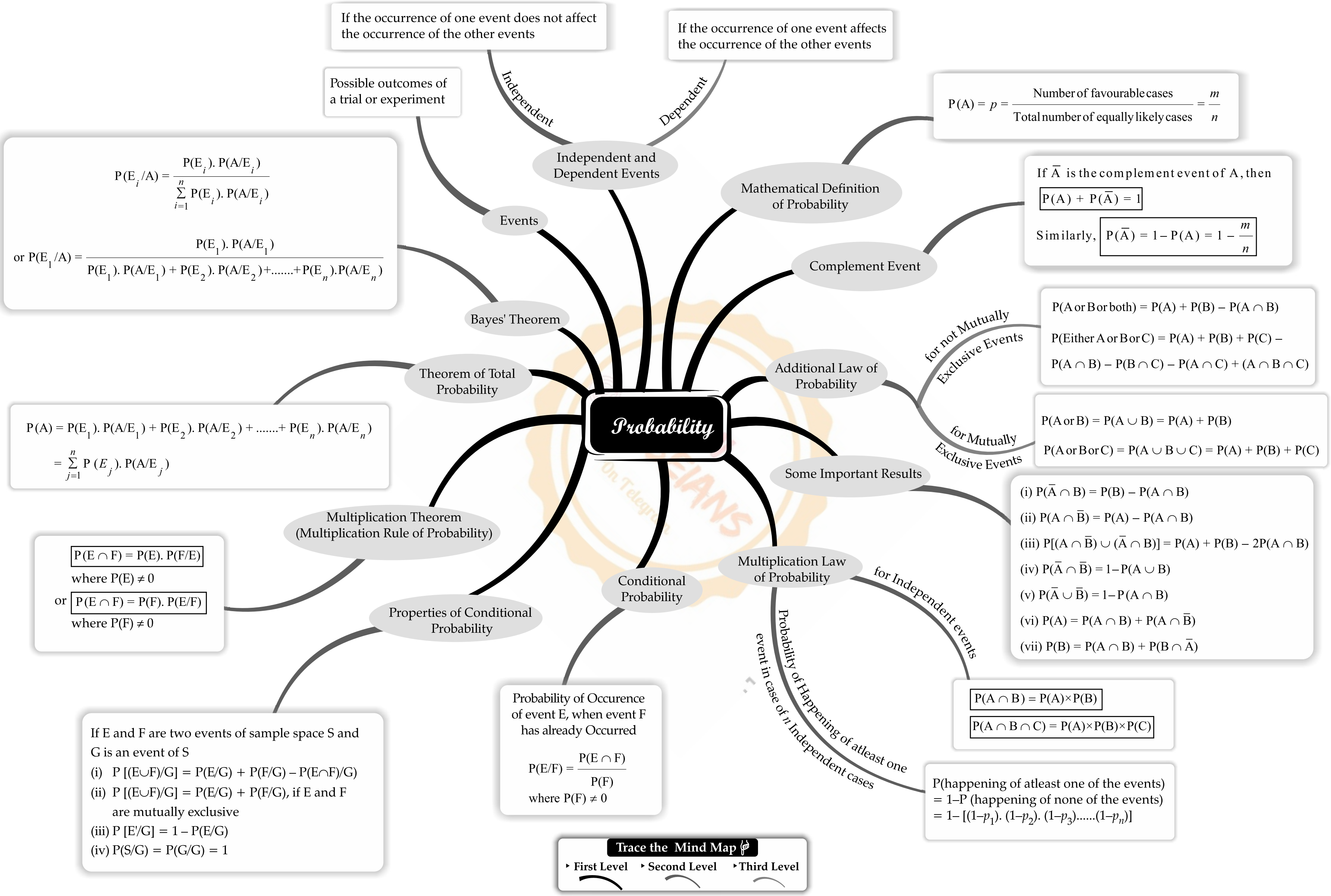
e.g., the order of $\frac{dy}{dx}=e^x$ is one and order of $\frac{d^2y}{dx^2}+x=0$ is two.

The highest exponent of the highest order derivative.

e.g., the degree of $\left(\frac{d^2y}{dx^2}\right)^3+\frac{dy}{dx}=0$ is three.

Order and degree (if defined) of a Differential Equation are always positive integers.

- A function which satisfies the given Differential Equation is called its solution.
- The solution which contains as many arbitrary constants as the order of the Differential equations is called a general solution.
- The solution free from arbitrary constants is called particular solution.
- e.g., $y = e^x + 1$ is a solution of $y'' - y' = 0$.
  Since $y' = e^x$ and $y'' = e^x \Rightarrow y'' - y' = e^x - e^x = 0$


Probability
Independent and Dependent Events
Independent
Dependent
Events
Mathematical Definition of Probability
Complement Event
Additional Law of Probability
for not Mutually Exclusive Events
for Mutually Exclusive Events
Some Important Results
Multiplication Law of Probability
for Independent events
Probability of Happening of atleast one event in case of n Independent cases
Conditional Probability
Properties of Conditional Probability
Multiplication Theorem (Multiplication Rule of Probability)
Theorem of Total Probability
Bayes' Theorem
Trace the Mind Map
First Level
Second Level
Third Level

If the occurrence of one event does not affect the occurrence of the other events

If the occurrence of one event affects the occurrence of the other events

Possible outcomes of a trial or experiment

$$P(A) = p = \frac{\text{Number of favourable cases}}{\text{Total number of equally likely cases}} = \frac{m}{n}$$

If $\bar{A}$ is the complement event of A, then $\boxed{P(A) + P(\bar{A}) = 1}$

Similarly, $\boxed{P(\bar{A}) = 1 - P(A) = 1 - \frac{m}{n}}$

P(A or B or both) = $P(A) + P(B) - P(A \cap B)$

P(Either A or B or C) = $P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + (A \cap B \cap C)$

P(A or B) = $P(A \cup B) = P(A) + P(B)$

P(A or B or C) = $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

(i) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

(ii) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

(iii) $P[(A \cap \bar{B}) \cup (\bar{A} \cap B)] = P(A) + P(B) - 2P(A \cap B)$

(iv) $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

(v) $P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$

(vi) $P(A) = P(A \cap B) + P(A \cap \bar{B})$

(vii) $P(B) = P(A \cap B) + P(B \cap \bar{A})$

$\boxed{P(A \cap B) = P(A) \times P(B)}$

$\boxed{P(A \cap B \cap C) = P(A) \times P(B) \times P(C)}$

P(happening of atleast one of the events)
= 1–P (happening of none of the events)
= $1 - [(1-p_1).\ (1-p_2).\ (1-p_3) \ldots\ldots (1-p_n)]$

Probability of Occurence of event E, when event F has already Occurred

$$P(E/F) = \frac{P(E \cap F)}{P(F)}$$

where $P(F) \neq 0$

If E and F are two events of sample space S and G is an event of S

(i) $P[(E \cup F)/G] = P(E/G) + P(F/G) - P(E \cap F)/G)$

(ii) $P[(E \cup F)/G] = P(E/G) + P(F/G)$, if E and F are mutually exclusive

(iii) $P[E'/G] = 1 - P(E/G)$

(iv) $P(S/G) = P(G/G) = 1$

$\boxed{P(E \cap F) = P(E).\ P(F/E)}$

where $P(E) \neq 0$

or $\boxed{P(E \cap F) = P(F).\ P(E/F)}$

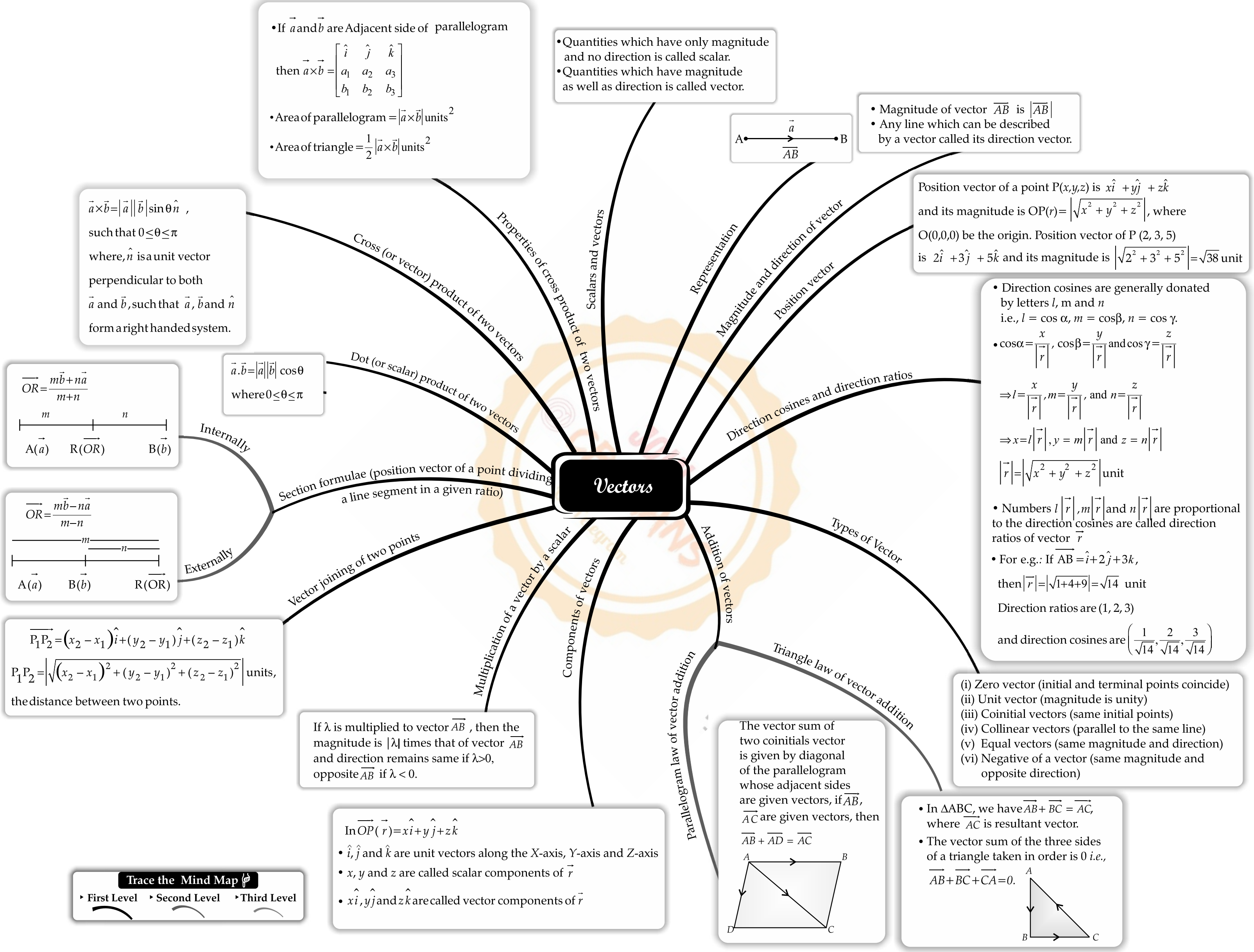
where $P(F) \neq 0$

$$P(A) = P(E_1).\ P(A/E_1) + P(E_2).\ P(A/E_2) + \ldots\ldots + P(E_n).\ P(A/E_n)$$

$$= \sum_{j=1}^{n} P(E_j).\ P(A/E_j)$$

$$P(E_i/A) = \frac{P(E_i).\ P(A/E_i)}{\sum_{i=1}^{n} P(E_i).\ P(A/E_i)}$$

$$\text{or } P(E_1/A) = \frac{P(E_1).\ P(A/E_1)}{P(E_1).\ P(A/E_1) + P(E_2).\ P(A/E_2) + \ldots\ldots + P(E_n).P(A/E_n)}$$



Vectors

Scalars and vectors

- Quantities which have only magnitude and no direction is called scalar.
- Quantities which have magnitude as well as direction is called vector.

Representation

Magnitude and direction of vector

- Magnitude of vector $\overline{AB}$ is $|\overline{AB}|$
- Any line which can be described by a vector called its direction vector.

Position vector

Position vector of a point P(*x*,*y*,*z*) is $x\hat{i}+y\hat{j}+z\hat{k}$ and its magnitude is $OP(r)=\left|\sqrt{x^2+y^2+z^2}\right|$, where O(0,0,0) be the origin. Position vector of P (2, 3, 5) is $2\hat{i}+3\hat{j}+5\hat{k}$ and its magnitude is $\left|\sqrt{2^2+3^2+5^2}\right|=\sqrt{38}$ unit

Direction cosines and direction ratios

- Direction cosines are generally donated by letters *l*, m and *n* i.e., $l=\cos\alpha$, $m=\cos\beta$, $n=\cos\gamma$.
- $\cos\alpha=\frac{x}{|\vec{r}|}$, $\cos\beta=\frac{y}{|\vec{r}|}$ and $\cos\gamma=\frac{z}{|\vec{r}|}$

$\Rightarrow l=\frac{x}{|\vec{r}|}, m=\frac{y}{|\vec{r}|}$, and $n=\frac{z}{|\vec{r}|}$

$\Rightarrow x=l|\vec{r}|, y=m|\vec{r}|$ and $z=n|\vec{r}|$

$|\vec{r}|=\left|\sqrt{x^2+y^2+z^2}\right|$ unit

- Numbers $l|\vec{r}|, m|\vec{r}|$ and $n|\vec{r}|$ are proportional to the direction cosines are called direction ratios of vector $\vec{r}$
- For e.g.: If $\overrightarrow{AB}=\hat{i}+2\hat{j}+3k$,

then $|\vec{r}|=|\sqrt{1+4+9}|=\sqrt{14}$ unit

Direction ratios are (1, 2, 3)

and direction cosines are $\left(\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}\right)$

Types of Vector

(i) Zero vector (initial and terminal points coincide)
(ii) Unit vector (magnitude is unity)
(iii) Coinitial vectors (same initial points)
(iv) Collinear vectors (parallel to the same line)
(v) Equal vectors (same magnitude and direction)
(vi) Negative of a vector (same magnitude and opposite direction)

Addition of vectors

Triangle law of vector addition

- In ΔABC, we have $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$, where $\overrightarrow{AC}$ is resultant vector.
- The vector sum of the three sides of a triangle taken in order is 0 *i.e.*, $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=0.$

Parallelogram law of vector addition

The vector sum of two coinitials vector is given by diagonal of the parallelogram whose adjacent sides are given vectors, if $\overrightarrow{AB}$, $\overrightarrow{AC}$ are given vectors, then $\overline{AB}+\overline{AD}=\overline{AC}$

Components of vectors

In $\overrightarrow{OP}(\vec{r})=x\hat{i}+y\hat{j}+z\hat{k}$

- $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along the *X*-axis, *Y*-axis and *Z*-axis
- *x*, *y* and *z* are called scalar components of $\vec{r}$
- $x\hat{i}, y\hat{j}$ and $z\hat{k}$ are called vector components of $\vec{r}$

Multiplication of a vector by a scalar

If λ is multiplied to vector $\overrightarrow{AB}$, then the magnitude is |λ| times that of vector $\overrightarrow{AB}$ and direction remains same if λ>0, opposite $\overrightarrow{AB}$ if λ < 0.

Vector joining of two points

$\overrightarrow{P_1P_2}=(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}$

$P_1P_2=\left|\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}\right|$ units, the distance between two points.

Section formulae (position vector of a point dividing a line segment in a given ratio)

Internally

$\overrightarrow{OR}=\frac{m\vec{b}+n\vec{a}}{m+n}$

Externally

$\overrightarrow{OR}=\frac{m\vec{b}-n\vec{a}}{m-n}$

Dot (or scalar) product of two vectors

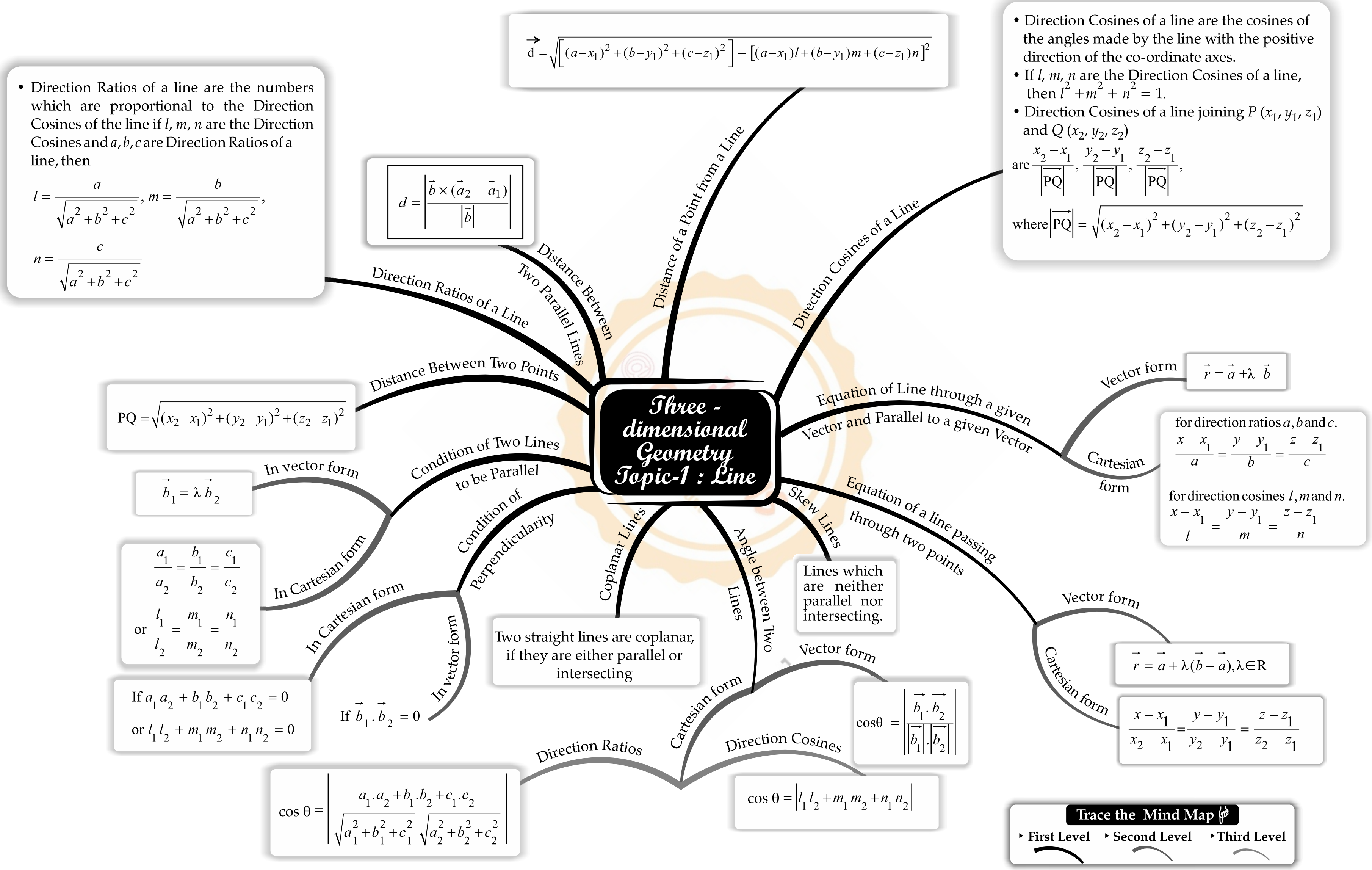
$\vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta$ where $0\le\theta\le\pi$

Cross (or vector) product of two vectors

$\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\hat{n}$, such that $0\le\theta\le\pi$ where, $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, such that $\vec{a}, \vec{b}$ and $\hat{n}$ form a right handed system.

Properties of cross product of two vectors

- If $\vec{a}$ and $\vec{b}$ are Adjacent side of parallelogram then $\vec{a}\times\vec{b}=\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{bmatrix}$
- Area of parallelogram $=|\vec{a}\times\vec{b}|$ units^2
- Area of triangle $=\frac{1}{2}|\vec{a}\times\vec{b}|$ units^2



Three - dimensional Geometry Topic-1 : Line

Direction Cosines of a Line

- Direction Cosines of a line are the cosines of the angles made by the line with the positive direction of the co-ordinate axes.
- If l, m, n are the Direction Cosines of a line, then $l^2 + m^2 + n^2 = 1$.
- Direction Cosines of a line joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$

are $\frac{x_2-x_1}{|\overrightarrow{PQ}|}, \frac{y_2-y_1}{|\overrightarrow{PQ}|}, \frac{z_2-z_1}{|\overrightarrow{PQ}|}$,

where $|\overrightarrow{PQ}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Distance of a Point from a Line

$\vec{d} = \sqrt{\left[(a-x_1)^2+(b-y_1)^2+(c-z_1)^2\right] - \left[(a-x_1)l+(b-y_1)m+(c-z_1)n\right]^2}$

Distance Between Two Parallel Lines

$$\boxed{d = \left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|}$$

Direction Ratios of a Line

- Direction Ratios of a line are the numbers which are proportional to the Direction Cosines of the line if l, m, n are the Direction Cosines and a, b, c are Direction Ratios of a line, then

$l = \frac{a}{\sqrt{a^2+b^2+c^2}}, m = \frac{b}{\sqrt{a^2+b^2+c^2}},$

$n = \frac{c}{\sqrt{a^2+b^2+c^2}}$

Distance Between Two Points

$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Condition of Two Lines to be Parallel

In vector form: $\vec{b}_1 = \lambda\,\vec{b}_2$

In Cartesian form: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ or $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

Condition of Perpendicularity

In vector form: If $\vec{b}_1 . \vec{b}_2 = 0$

In Cartesian form: If $a_1a_2 + b_1b_2 + c_1c_2 = 0$ or $l_1l_2 + m_1m_2 + n_1n_2 = 0$

Coplanar Lines

Two straight lines are coplanar, if they are either parallel or intersecting

Angle between Two Lines

Vector form: $\cos\theta = \left|\frac{\vec{b}_1 . \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

Cartesian form

Direction Ratios: $\cos\theta = \left|\frac{a_1.a_2+b_1.b_2+c_1.c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

Direction Cosines: $\cos\theta = |l_1l_2+m_1m_2+n_1n_2|$

Skew Lines

Lines which are neither parallel nor intersecting.

Equation of a line passing through two points

Vector form: $\vec{r} = \vec{a} + \lambda(\vec{b}-\vec{a}), \lambda \in R$

Cartesian form: $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

Equation of Line through a given Vector and Parallel to a given Vector

Vector form: $\vec{r} = \vec{a} + \lambda\,\vec{b}$

Cartesian form

for direction ratios a, b and c.

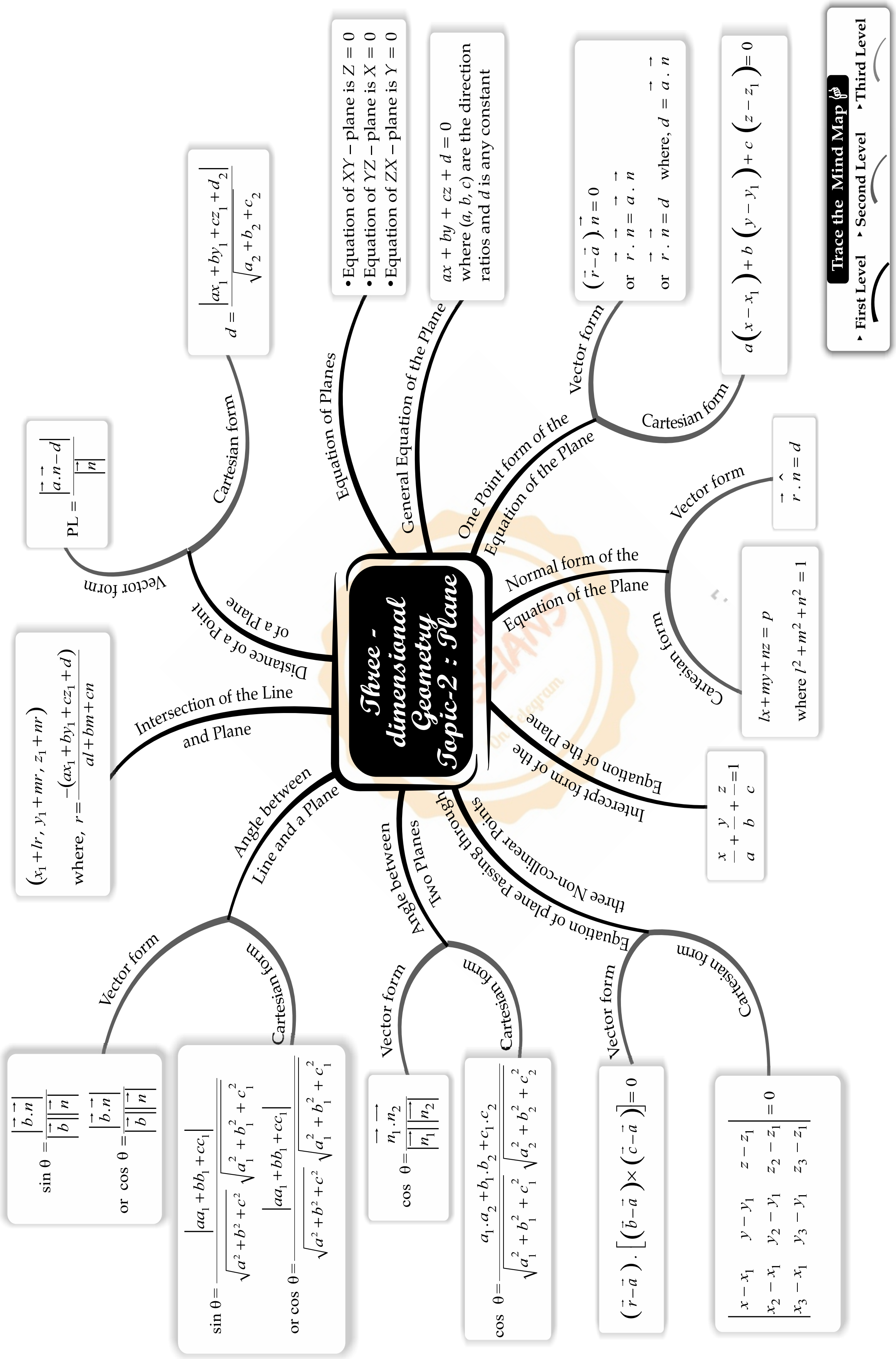
$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

for direction cosines l, m and n.

$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$

Trace the Mind Map

- First Level
- Second Level
- Third Level



Three - dimensional Geometry Topic-2 : Plane

Distance of a Point of a Plane

Vector form

$PL = \frac{|\vec{a}.\vec{n} - d|}{|\vec{n}|}$

Cartesian form

$d = \frac{|ax_1 + by_1 + cz_1 + d_2|}{\sqrt{a_2 + b_2 + c_2}}$

Equation of Planes

- Equation of XY – plane is Z = 0
- Equation of YZ – plane is X = 0
- Equation of ZX – plane is Y = 0

General Equation of the Plane

$ax + by + cz + d = 0$

where (a, b, c) are the direction ratios and d is any constant

One Point form of the Equation of the Plane

Vector form

$(\vec{r} - \vec{a}).\vec{n} = 0$

or $\vec{r}.\vec{n} = \vec{a}.\vec{n}$

or $\vec{r}.\vec{n} = d$ where, $d = \vec{a}.\vec{n}$

Cartesian form

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Normal form of the Equation of the Plane

Vector form

$\vec{r}.\hat{n} = d$

Cartesian form

$lx + my + nz = p$

where $l^2 + m^2 + n^2 = 1$

Equation of the Intercept form of the Plane

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Equation of plane Passing through three Non-collinear Points

Vector form

$(\vec{r} - \vec{a}).[(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$

Cartesian form

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Angle between Two Planes

Vector form

$\cos\theta = \frac{\vec{n_1}.\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$

Cartesian form

$\cos\theta = \frac{a_1.a_2 + b_1.b_2 + c_1.c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

Angle between Line and a Plane

Vector form

$\sin\theta = \frac{|\vec{b}.\vec{n}|}{|\vec{b}||\vec{n}|}$

or $\cos\theta = \frac{|\vec{b}.\vec{n}|}{|\vec{b}||\vec{n}|}$

Cartesian form

$\sin\theta = \frac{|aa_1 + bb_1 + cc_1|}{\sqrt{a^2 + b^2 + c^2}\sqrt{a_1^2 + b_1^2 + c_1^2}}$

or $\cos\theta = \frac{|aa_1 + bb_1 + cc_1|}{\sqrt{a^2 + b^2 + c^2}\sqrt{a_1^2 + b_1^2 + c_1^2}}$

Intersection of the Line and Plane

$(x_1 + lr, y_1 + mr, z_1 + nr)$

where, $r = \frac{-(ax_1 + by_1 + cz_1 + d)}{al + bm + cn}$

Trace the Mind Map

- First Level
- Second Level
- Third Level

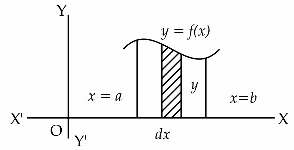
Application of Integrals

Area under Simple Curves and Coordinate Axis

Case I

The area of the region bounded by the curve $y = f(x)$ above X – axis and between the lines $x=a$ and $x=b$ $(b > a)$ is given by

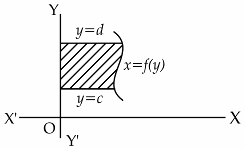
$$A=\int_a^b y\,dx \text{ or } \int_a^b f(x)\,dx.$$



Case II

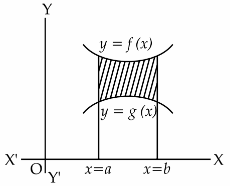
The area of the region bounded by the curve $x = f(y)$ right to the Y – axis and between the lines $y=c$ and $y=d$ $(d > c)$ is given by

$$A=\int_c^d x\,dy \text{ or } \int_c^d f(y)\,dy.$$



Area enclosed between two curves

Case I



The areas of the region enclosed between two curves $y = f(x)$, $y = g(x)$ and the lines $x = a$, $x = b$ is given by

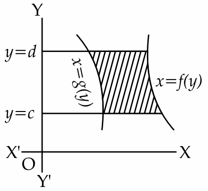
$$A=\int_a^b [f(x)-g(x)]\,dx, \text{ where } f(x) \geq g(x) \text{ in } [a,b]$$

Case II

The area of the region enclosed between curves $x = f(y)$, $x = g(y)$ and the lines $y = c$, $y = d$ is given by

$$A=\int_c^d [f(y) - g(y)]\,dy$$

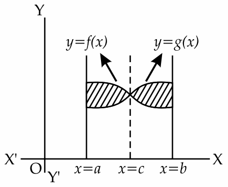
where $f(y) \geq g(y)$ in $[d, c]$



Case III

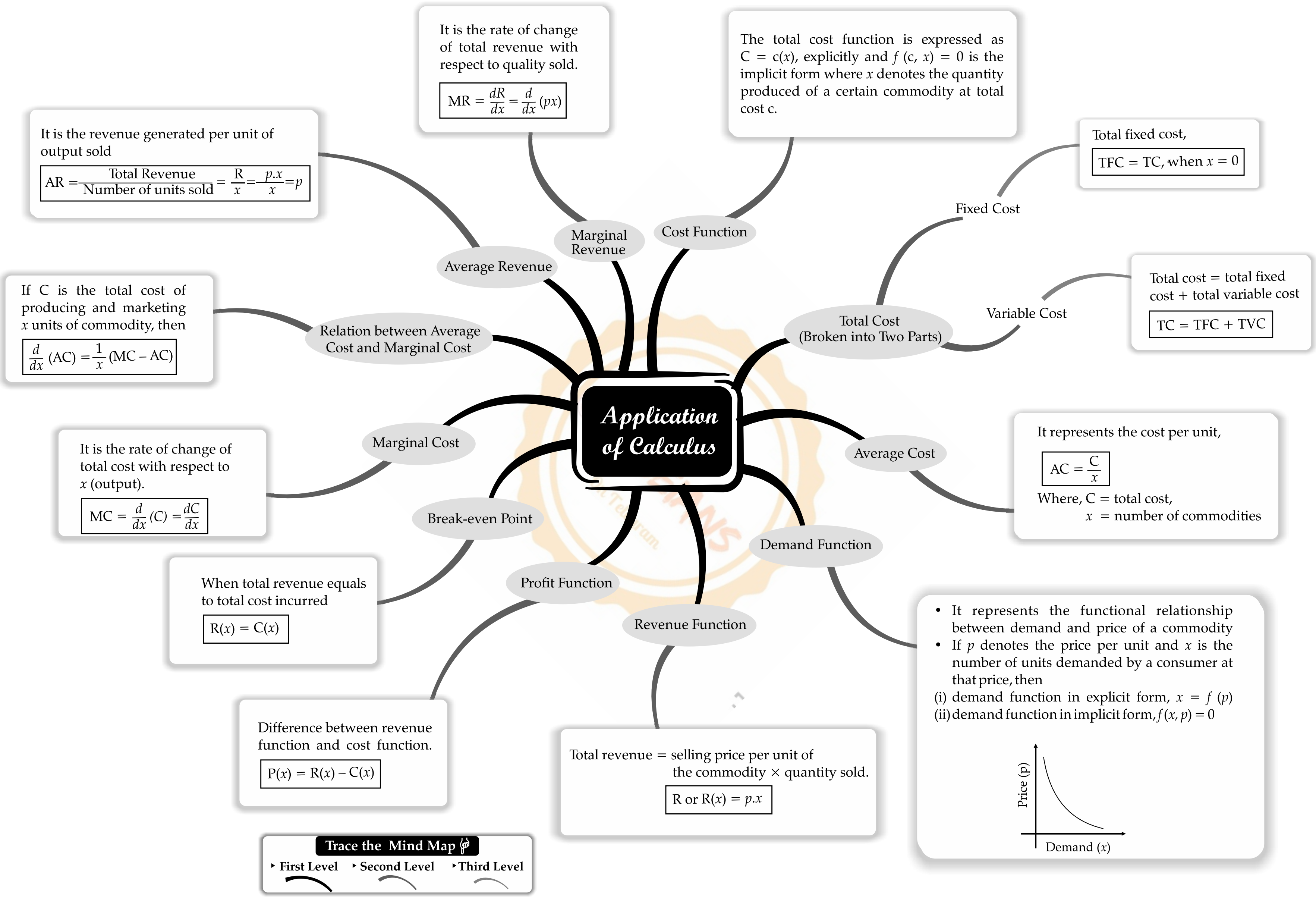
If $f(x) \geq g(x)$ in $[a, c]$ and $f(x) \leq g(x)$ in $[c, b]$ where $a < c < b$, then the area of the region bounded by the curves is

$$A=\int_a^c [f(x) - g(x)]\,dx + \int_c^b [g(x) - f(x)]\,dx$$



Trace the Mind Map

- First Level
- Second Level
- Third Level



Application of Calculus

Cost Function

The total cost function is expressed as C = c(x), explicitly and f (c, x) = 0 is the implicit form where x denotes the quantity produced of a certain commodity at total cost c.

Total Cost (Broken into Two Parts)

Fixed Cost

Total fixed cost,

$\boxed{\text{TFC} = \text{TC, when } x = 0}$

Variable Cost

Total cost = total fixed cost + total variable cost

$\boxed{\text{TC} = \text{TFC} + \text{TVC}}$

Average Cost

It represents the cost per unit,

$\boxed{\text{AC} = \frac{\text{C}}{x}}$

Where, C = total cost,
x = number of commodities

Demand Function

- It represents the functional relationship between demand and price of a commodity
- If p denotes the price per unit and x is the number of units demanded by a consumer at that price, then

(i) demand function in explicit form, $x = f(p)$
(ii) demand function in implicit form, $f(x, p) = 0$

Revenue Function

Total revenue = selling price per unit of the commodity × quantity sold.

$\boxed{\text{R or R}(x) = p.x}$

Profit Function

Difference between revenue function and cost function.

$\boxed{\text{P}(x) = \text{R}(x) - \text{C}(x)}$

Break-even Point

When total revenue equals to total cost incurred

$\boxed{\text{R}(x) = \text{C}(x)}$

Marginal Cost

It is the rate of change of total cost with respect to x (output).

$\boxed{\text{MC} = \frac{d}{dx}(\text{C}) = \frac{dC}{dx}}$

Relation between Average Cost and Marginal Cost

If C is the total cost of producing and marketing x units of commodity, then

$\boxed{\frac{d}{dx}(\text{AC}) = \frac{1}{x}(\text{MC} - \text{AC})}$

Average Revenue

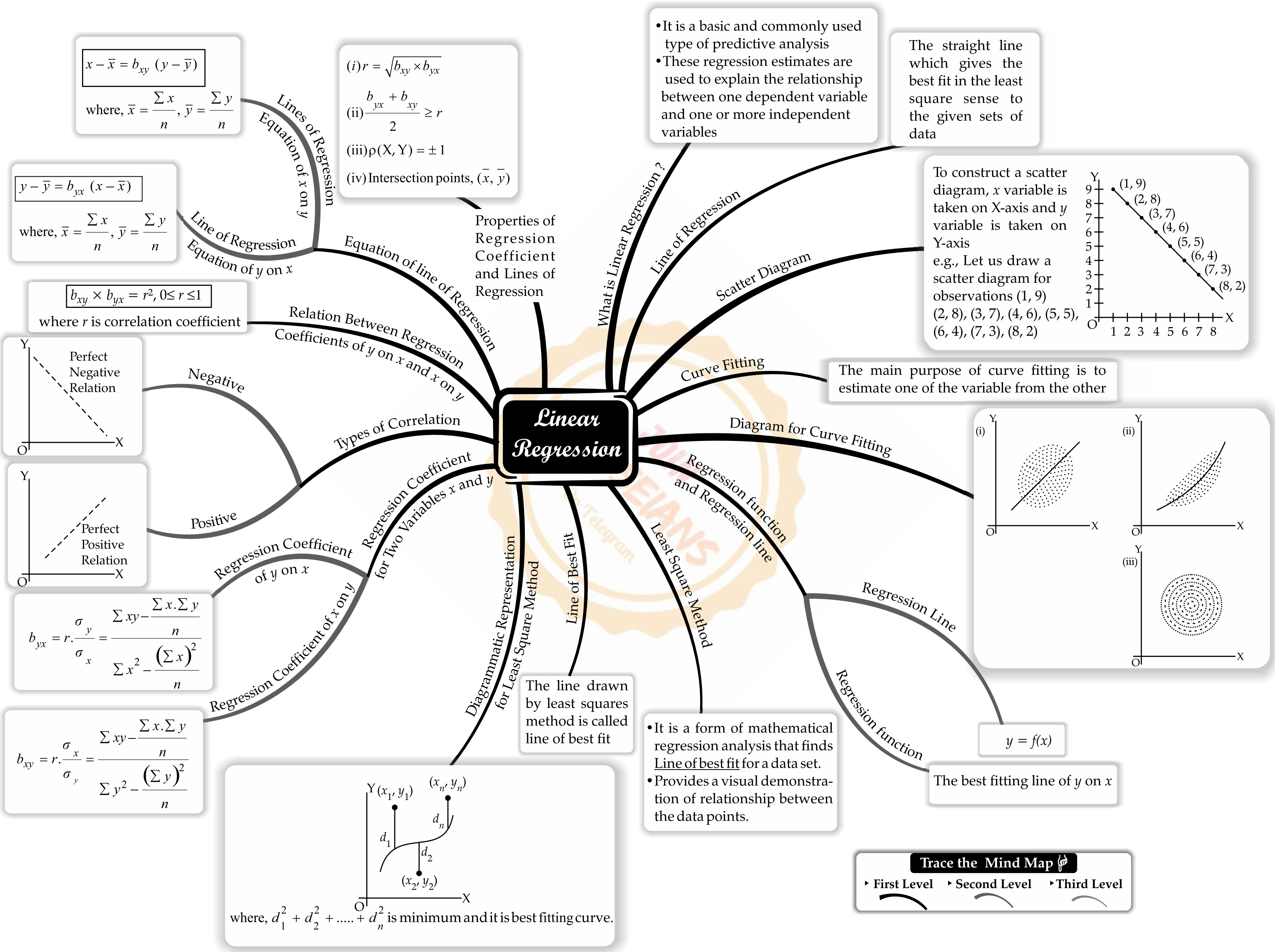
It is the revenue generated per unit of output sold

$\boxed{\text{AR} = \frac{\text{Total Revenue}}{\text{Number of units sold}} = \frac{\text{R}}{x} = \frac{p.x}{x} = p}$

Marginal Revenue

It is the rate of change of total revenue with respect to quality sold.

$\boxed{\text{MR} = \frac{dR}{dx} = \frac{d}{dx}(px)}$

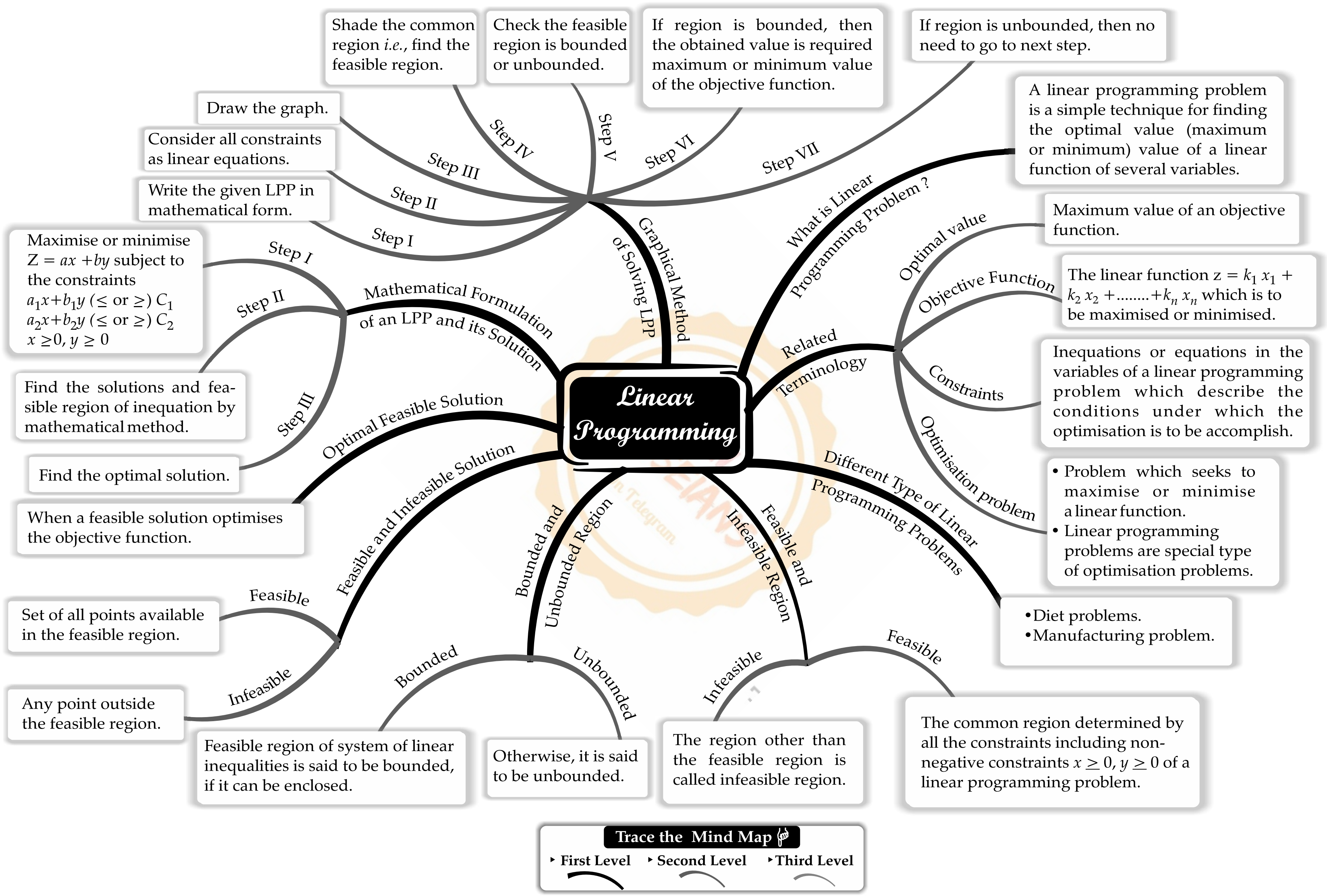


Linear Regression

- What is Linear Regression ?
  - It is a basic and commonly used type of predictive analysis
  - These regression estimates are used to explain the relationship between one dependent variable and one or more independent variables
- Line of Regression
  - The straight line which gives the best fit in the least square sense to the given sets of data
- Scatter Diagram
  - To construct a scatter diagram, x variable is taken on X-axis and y variable is taken on Y-axis
  - e.g., Let us draw a scatter diagram for observations (1, 9) (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2)
- Curve Fitting
  - The main purpose of curve fitting is to estimate one of the variable from the other
- Diagram for Curve Fitting
  - (i)
  - (ii)
  - (iii)
- Regression function and Regression line
  - Regression Line
    - $y = f(x)$
  - Regression function
    - The best fitting line of y on x
- Least Square Method
  - It is a form of mathematical regression analysis that finds Line of best fit for a data set.
  - Provides a visual demonstration of relationship between the data points.
- Line of Best Fit
  - The line drawn by least squares method is called line of best fit
- Diagrammatic Representation for Least Square Method
  - where, $d_1^2 + d_2^2 + ... + d_n^2$ is minimum and it is best fitting curve.
- Regression Coefficient for Two Variables x and y
  - Regression Coefficient of y on x
    - $$b_{yx} = r.\frac{\sigma_y}{\sigma_x} = \frac{\Sigma xy - \frac{\Sigma x.\Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$$
  - Regression Coefficient of x on y
    - $$b_{xy} = r.\frac{\sigma_x}{\sigma_y} = \frac{\Sigma xy - \frac{\Sigma x.\Sigma y}{n}}{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}$$
- Types of Correlation
  - Negative
    - Perfect Negative Relation
  - Positive
    - Perfect Positive Relation
- Relation Between Regression Coefficients of y on x and x on y
  - $\boxed{b_{xy} \times b_{yx} = r^2, 0 \le r \le 1}$
  - where r is correlation coefficient
- Equation of line of Regression
  - Lines of Regression Equation of x on y
    - $\boxed{x - \bar{x} = b_{xy}\ (y - \bar{y})}$
    - where, $\bar{x} = \frac{\Sigma x}{n}, \bar{y} = \frac{\Sigma y}{n}$
  - Line of Regression Equation of y on x
    - $\boxed{y - \bar{y} = b_{yx}\ (x - \bar{x})}$
    - where, $\bar{x} = \frac{\Sigma x}{n}, \bar{y} = \frac{\Sigma y}{n}$
- Properties of Regression Coefficient and Lines of Regression
  - (i) $r = \sqrt{b_{xy} \times b_{yx}}$
  - (ii) $\frac{b_{yx} + b_{xy}}{2} \ge r$
  - (iii) $\rho(X, Y) = \pm 1$
  - (iv) Intersection points, $(\bar{x}, \bar{y})$

Trace the Mind Map

- First Level
- Second Level
- Third Level



Linear Programming

- What is Linear Programming Problem ?
  - A linear programming problem is a simple technique for finding the optimal value (maximum or minimum) value of a linear function of several variables.
- Related Terminology
  - Optimal value
    - Maximum value of an objective function.
  - Objective Function
    - The linear function z = $k_1\,x_1$ + $k_2\,x_2$ +........+$k_n\,x_n$ which is to be maximised or minimised.
  - Constraints
    - Inequations or equations in the variables of a linear programming problem which describe the conditions under which the optimisation is to be accomplish.
  - Optimisation problem
    - Problem which seeks to maximise or minimise a linear function.
    - Linear programming problems are special type of optimisation problems.
- Different Type of Linear Programming Problems
  - Diet problems.
  - Manufacturing problem.
- Feasible and Infeasible Region
  - Feasible
    - The common region determined by all the constraints including non-negative constraints $x \geq 0, y \geq 0$ of a linear programming problem.
  - Infeasible
    - The region other than the feasible region is called infeasible region.
- Bounded and Unbounded Region
  - Unbounded
    - Otherwise, it is said to be unbounded.
  - Bounded
    - Feasible region of system of linear inequalities is said to be bounded, if it can be enclosed.
- Feasible and Infeasible Solution
  - Feasible
    - Set of all points available in the feasible region.
  - Infeasible
    - Any point outside the feasible region.
- Optimal Feasible Solution
  - When a feasible solution optimises the objective function.
- Mathematical Formulation of an LPP and its Solution
  - Step I
    - Maximise or minimise Z = $ax + by$ subject to the constraints
      a_1x+b_1y ($\leq$ or $\geq$) C_1
      a_2x+b_2y ($\leq$ or $\geq$) C_2
      $x \geq 0, y \geq 0$
  - Step II
    - Find the solutions and feasible region of inequation by mathematical method.
  - Step III
    - Find the optimal solution.
- Graphical Method of Solving LPP
  - Step I
    - Write the given LPP in mathematical form.
  - Step II
    - Consider all constraints as linear equations.
  - Step III
    - Draw the graph.
  - Step IV
    - Shade the common region *i.e.*, find the feasible region.
  - Step V
    - Check the feasible region is bounded or unbounded.
  - Step VI
    - If region is bounded, then the obtained value is required maximum or minimum value of the objective function.
  - Step VII
    - If region is unbounded, then no need to go to next step.

OSWAAL EXAM TOOLS

ON TIPS

NOTES

Note making is a skill that we use in many walks of life: at school, university and in the world of work. However, accurate note making requires a thorough understanding of concepts. We, at Oswaal, have tried to encapsulate all the chapters from the given syllabus into the following **ON TIPS NOTES**. These notes will not only facilitate better understanding of concepts, but will also ensure that each and every concept is taken up and every chapter is covered in totality. So, go ahead and use these to your advantage… go get the **OSWAAL ADVANTAGE!!**

Chapter 1 - Relations and Functions

Relation Let A be a non-empty set and $R \in A \times A$. Then, R is called a relation on A. If $(a, b) \in R$, then say that 'a' is related to 'b' and write $\boldsymbol{aRb}$. If $(a, b) \notin R$, then we write $a \not{R} b$.

Types of Relation

- **Empty Relation:** A relation R on a set A is called an empty or null or void relation, if no element of A is related to any element of A i.e., $R = \phi \subset A \times A$.
- **Identity Relation:** A relation I on a set A is called an identity relation, if every element of a set A is related to itself only. The identity relation on set A is defined as $I_A = \{(x, y) \in A \times A \mid x = y\}$
- **Universal Relation:** A relation R on a set A is called universal relation, if each element of A is related to every element of A i.e., $R = A \times A$.
- **Reflexive Relation:** A relation R defined on set A is said to be reflexive, if $(x, x) \in R, \forall\, x \in A$ i.e., $x\,R\,x, \forall\, x \in A$.
- **Symmetric Relation:** A relation R defined on set A is said to be symmetric, if $(x, y) \in R \Rightarrow (y, x) \in R,\ \forall\, x, y \in A$ i.e., $x\,R\,y \Rightarrow y\,R\,x\ \forall\ x, y \in A$.
- **Transitive Relation:** A relation R defined on set A is said to be transitive, if $(x, y) \in R$ and $(y, z) \in R$ $\Rightarrow (x, z) \in R,\ \forall\ x, y, z \in A$ i.e., $x\,R\,y$ and $y\,R\,z \Rightarrow x\,R\,z\ \forall\ x, y, z \in A$.
- **Equivalence Relation:** A relation R on a set A is called an equivalence relation, if it is reflexive, symmetric and transitive.
- **Equivalence Classes:** Consider, an arbitrary equivalence relation on R on an arbitrary set X, R divides X into mutually disjoint subsets A_i called partitions or subdivisions of X, satisfying:
  - **(i)** all elements of A_i are related to each other for all i
  - **(ii)** no element of A_i is related to any element of A_j; $i \neq j$.
  - **(iii)** $A_i \cup A_j = X$ and $A_i \cap A_j = \phi$, $i \neq j$. Then, subset A_i are called equivalence classes.

Function

Let A and B be two non-empty sets. Then, a relation f from A to B which associates each element $x \in A$, to a unique element of $f(x) \in B$ is called a function from A to B and we write $f : A \rightarrow B$. Here, A is called the **domain** of f. i.e., dom (f) = A and B is called the **codomain** of f. Also, $\{f(x) : x \in A\} \subseteq B$ is called the **range** of f.

Note: *Every function is a relation but every relation is not a function.*

Types of Function

1. **One-one (Injective) Function:** A function $f : A \rightarrow B$ is said to be one-one, if distinct element of A have distinct images in B, i.e., if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ or $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)\ \forall,\ x_1, x_2 \in A$.

2. **Onto (Surjective) Function:** A function $f : A \to B$ is said to be onto, if every element in B has its pre-image in A, i.e., if for each $y \in B$, there exists an element $x \in A$, such that $f(x) = y$.
3. **One-one and Onto (Bijective) Function:** A function $f : X \to Y$ is said to be one-one and onto, if f is both one-one and onto.
4. **Invertible Function:** A function $f : X \to Y$ is defined to be invertible if there exists a function $g : Y \to X$ such that $gof = I_X$ and $fog = I_Y$. The function g is called the inverse of f and is denoted by f^{-1}.
   An important note is that, if f is invertible, then f must be one-one and onto and conversely if f is one-one and onto, then f must be invertible.

Chapter 2 - Inverse Trigonometric Function

Inverse of a Function: We know that corresponding to every bijection (one-one onto function) $f : A \to B$ there exists a bijection $g : B \to A$ defined by

$$g(y) = x \text{ if and only } f(x) = y.$$

The function $g : B \to A$ is called the inverse of function $f : A \to B$ and is denoted by f^{-1}.

Thus, we have

$$f(x) = y \leftrightarrow f^{-1}(y) = x.$$

Trigonometric functions are not one-one and onto on their natural domains and ranges, so their inverse does not exist in all values but their inverse may exist in some interval of their restricted domains and ranges. Thus, we can say that, inverse of trigonometric functions are defined within restricted domains of corresponding trigonometric functions. Inverse of f is denoted by f^{-1}.

Note: **(i)** $\sin^{-1} x \neq (\sin x)^{-1}$

(ii) $\sin^{-1} x \neq \sin^{-1}\left(\dfrac{1}{x}\right)$

(iii) $\sin^{-1} x \neq \left(\dfrac{1}{\sin x}\right)$

Domain and Principal Value branch (Range) of Inverse Trigonometric Functions

Function	Domain	Principal value branch (Range)
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\text{cosec}^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$\sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\cot^{-1} x$	R	$(0, \pi)$

T-ratios of Some Standard Angles

Angle (θ) / Ratio	$0° = 0$	$30° = \frac{\pi}{6}$	$45° = \frac{\pi}{4}$	$60° = \frac{\pi}{3}$	$90° = \frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined

Properties of Inverse Trigonometric Functions

Property I

(i) $\sin^{-1}(\sin\theta) = \theta$, for all $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(ii) $\cos^{-1}(\cos\theta) = \theta$, for all $\theta \in [0, \pi]$

(iii) $\tan^{-1}(\tan\theta) = \theta$, for all $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iv) $\cot^{-1}(\cot\theta) = \theta$, for all $\theta \in (0, \pi)$

(v) $\text{cosec}^{-1}(\text{cosec}\,\theta) = \theta$, for all $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

(vi) $\sec^{-1}(\sec\theta) = \theta$, for all $\theta \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

(vii) $\sin(\sin^{-1} x) = x$, for all $x \in [-1, 1]$

(viii) $\cos(\cos^{-1} x) = x$, for all $x \in [-1, 1]$

(ix) $\tan(\tan^{-1} x) = x$, for all $x \in R$

(x) $\cot(\cot^{-1} x) = x$, for all $x \in R$

(xi) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

(xii) $\sec(\sec^{-1} x) = x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

Property II

(i) $\sin^{-1}(-x) = -\sin^{-1} x$, for all $x \in [-1, 1]$

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1} x$, for all $x \in [-1, 1]$

(iii) $\tan^{-1}(-x) = -\tan^{-1} x$, for all $x \in R$

(iv) $\cot^{-1}(-x) = \pi - \cot^{-1} x$, for all $x \in R$

(v) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

(vi) $\sec^{-1}(-x) = \pi - \sec^{-1} x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

Property III

(i) $\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1} x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

(ii) $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x$, for all $x \in (-\infty, -1] \cup [1, \infty)$

(iii) $\tan^{-1}\frac{1}{x} = \begin{cases} \cot^{-1} x, & x > 0 \\ -\pi + \cot^{-1} x, & x < 0 \end{cases}$

Property IV

(i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$, $x \in [-1, 1]$

(ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, $x \in R$

(iii) $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$, $x \in (-\infty, -1] \cup [1, \infty)$

Property V

(i) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x + y}{1 - xy}\right)$, $xy < 1$

(ii) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x - y}{1 + xy}\right)$, $xy > -1$

(iii) $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x + y}{1 - xy}\right)$, $xy > 1; x, y > 0$

Property VI

(i) $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1 - x^2})$, $-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

(ii) $2\cos^{-1} x = \cos^{-1}(2x^2 - 1)$, $0 \le x \le 1$

(iii) $2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1 - x^2}\right)$, $-1 < x < 1$

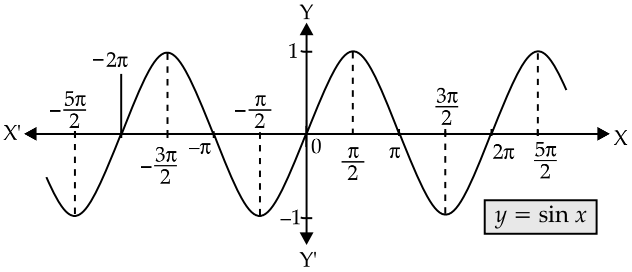
Property VII

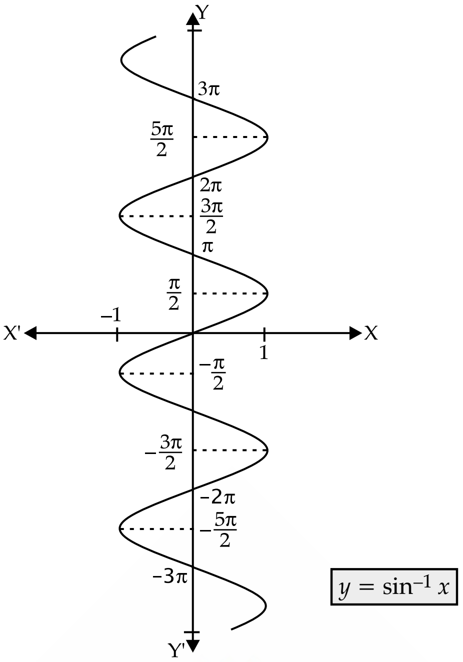
(i)
$$\sin^{-1} x + \sin^{-1} y = \begin{cases} \sin^{-1}\{x\sqrt{1 - y^2} + y\sqrt{1 - x^2}\} & \text{, if } -1 \le x, y \le 1 \text{ and } x^2 + y^2 \le 1 \\ & \text{or } xy < 0 \text{ and } x^2 + y^2 > 1 \\ \pi - \sin^{-1}\{x\sqrt{1 - y^2} + y\sqrt{1 - x^2}\} & \text{, if } 0 < x, y \le 1 \text{ and } x^2 + y^2 > 1 \\ -\pi - \sin^{-1}\{x\sqrt{1 - y^2} + y\sqrt{1 - x^2}\} & \text{, if } -1 \le x, y < 0 \text{ and } x^2 + y^2 > 1 \end{cases}$$

(ii)
$$\sin^{-1} x - \sin^{-1} y = \begin{cases} \sin^{-1}\{x\sqrt{1 - y^2} - y\sqrt{1 - x^2}\} & \text{, if } -1 \le x, y \le 1 \text{ and } x^2 + y^2 \le 1 \\ & \text{or } xy > 0 \text{ and } x^2 + y^2 > 1 \\ \pi - \sin^{-1}\{x\sqrt{1 - y^2} - y\sqrt{1 - x^2}\} & \text{, if } 0 < x \le 1, -1 \le y \le 0 \text{ and } x^2 + y^2 > 1 \\ -\pi - \sin^{-1}\{x\sqrt{1 - y^2} - y\sqrt{1 - x^2}\} & \text{, if } -1 \le x < 0, 0 < y \le 1 \text{ and } x^2 + y^2 > 1 \end{cases}$$

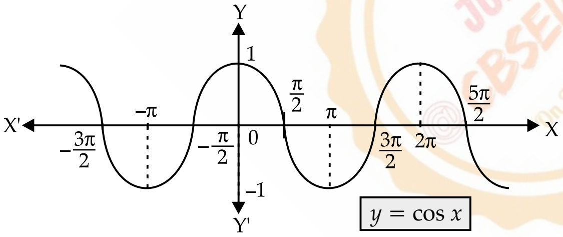
Graph of Functions and their Inverses

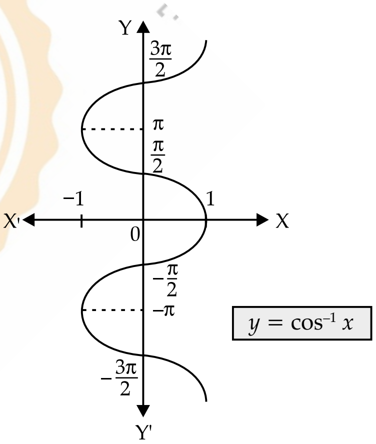
➢ **Principal value of branch function $\sin^{-1}$:** It is a function with domain $[-1, 1]$ and range $\left[\frac{-3\pi}{2}, \frac{-\pi}{2}\right]$, $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ or $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ and so on corresponding to each interval, we get a branch of the function $\sin^{-1} x$. The branch with range $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is called the principal value branch. Thus, $\sin^{-1}[-1, 1] \rightarrow \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.





- **Principal value branch of function $\cos^{-1}$:** The graph of the function $\cos^{-1}$ is as shown in figure. Domain of the function $\cos^{-1}$ is $[-1, 1]$. Its range in one of the intervals $[-\pi, 0]$, $[0, \pi]$, $[\pi, 2\pi]$, etc. is one-one and onto with the range $[-1, 1]$. The branch with range $[0, \pi]$ is called the principal value branch of the function $\cos^{-1}$.
  Thus, $\cos^{-1}[-1,1] \rightarrow [0,\pi]$

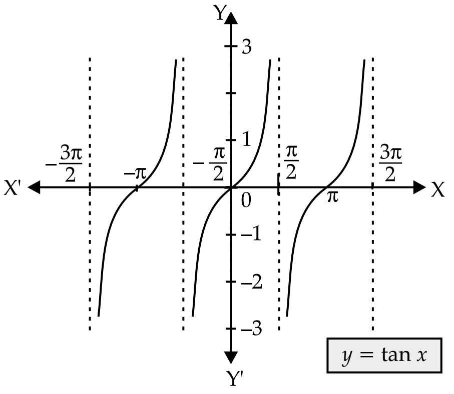


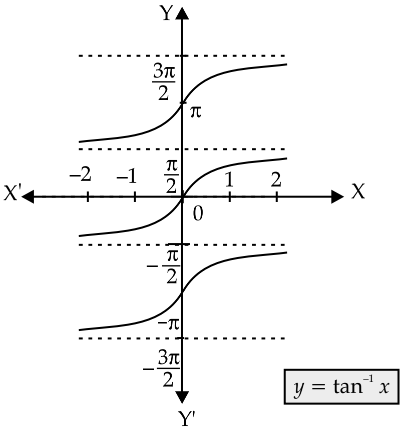


- **Principal value branch function of $\tan^{-1}$:** The function $\tan^{-1}$ is defined whose domain is set of real numbers and range is one of the intervals

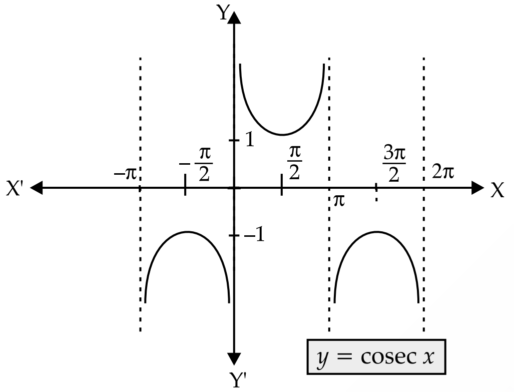
$$\left(\frac{-3\pi}{2}, \frac{-\pi}{2}\right), \left(\frac{-\pi}{2}, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{3\pi}{2}\right), \ldots$$

- **Graph of the function is as shown in the figure:**



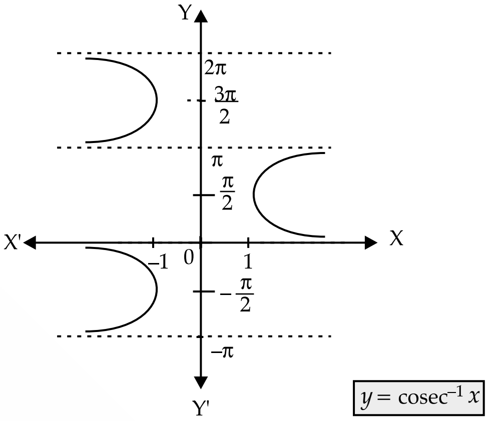


The branch with range $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ is called the principal value branch of function $\tan^{-1}$. Thus, $\tan^{-1}: R \to \left(\frac{-\pi}{2},\frac{\pi}{2}\right)$



- **Principal value branch of function cosec^{-1} :** The graph of function cosec^{-1} is shown in figure. The cosec^{-1} is defined on a function whose domain is $R - (-1, 1)$ and the range is any one of the interval,

$$\left[\frac{-3\pi}{2},\frac{-\pi}{2}\right]-\{\pi\},\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\},\left[\frac{\pi}{2},\frac{3\pi}{2}\right]-\{\pi\},\ldots$$

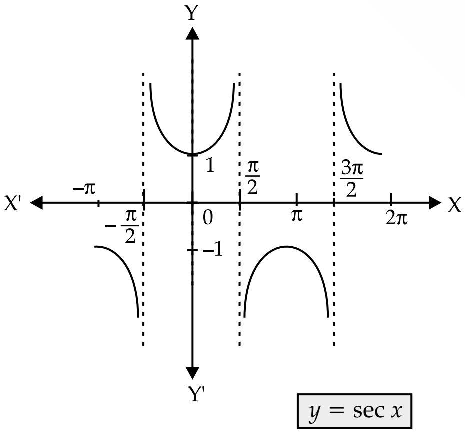


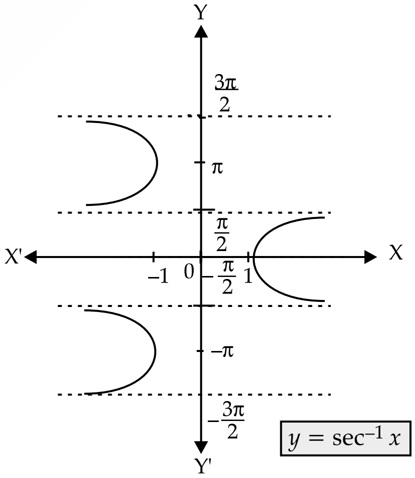
The function corresponding to the range $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\}$ is called the principal value branch of cosec^{-1}.

Thus, $\text{cosec}^{-1}: R-(-1,1) \to \left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\}$

- **Principal value branch of function $\sec^{-1}$:** The graph of function $\sec^{-1}$ is shown in figure. The $\sec^{-1}$ is defined as a function whose domain $R-(-1,1)$ and range is $[-\pi,0]-\left[\frac{-\pi}{2}\right],[0-\pi]-\left\{\frac{\pi}{2}\right\},[\pi-2\pi]-\left\{\frac{3\pi}{2}\right\}$, etc. Function corresponding to range $[0-\pi]-\left\{\frac{\pi}{2}\right\}$ is known as the principal value branch of $\sec^{-1}$.

Thus, $\sec^{-1}: R-(-1,1) \to [0,\pi]-\left\{\frac{\pi}{2}\right\}$





- **The principal value branch of function $\cot^{-1}$:** The graph of function $\cot^{-1}$ is shown below. The $\cot^{-1}$ function is defined on function whose domain is R and the range is any of the intervals, $(-\pi, 0)$, $(0, \pi)$, $(\pi, 2\pi)$, …. The function corresponding to $(0, \pi)$ is called the principal value branch of the function $\cot^{-1}$. Then, $\cot^{-1}: R \to (0, \pi)$

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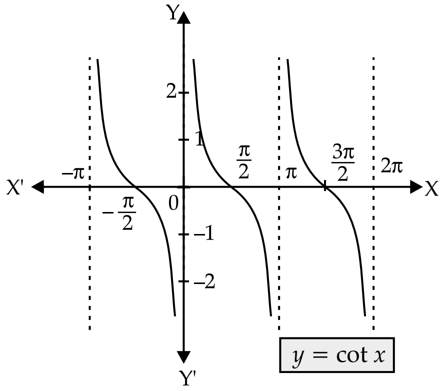

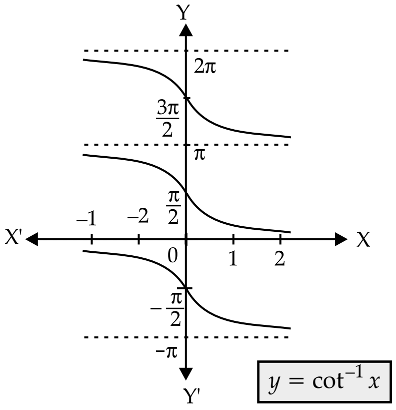

Some Useful Trigonometric Formulae

(i) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(ii) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(iii) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

(iv) $\cot(A \pm B) = \dfrac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

(v) $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

(vi) $2 \cos A \sin B = \sin(A+B) - \sin(A-B)$

(vii) $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$

(viii) $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

(ix) $\sin C + \sin D = 2 \sin\left(\dfrac{C+D}{2}\right) \cos\left(\dfrac{C-D}{2}\right)$

(x) $\sin C - \sin D = 2 \cos\left(\dfrac{C+D}{2}\right) \sin\left(\dfrac{C-D}{2}\right)$

(xi) $\cos C + \cos D = 2 \cos\left(\dfrac{C+D}{2}\right) \cos\left(\dfrac{C-D}{2}\right)$

(xii) $\cos C - \cos D = -2 \sin\left(\dfrac{C+D}{2}\right) \sin\left(\dfrac{C-D}{2}\right)$

or $2 \sin\left(\dfrac{C+D}{2}\right) \sin\left(\dfrac{D-C}{2}\right)$

(xiii) $\sin 2x = 2 \sin x \cos x$

(xiv) $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

(xv) $\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$

(xvi) $1 + \cos 2x = 2\cos^2 x;\ 1 - \cos 2x = 2 \sin^2 x$

(xvii) $\sin 2x = \dfrac{2 \tan x}{1 + \tan^2 x};\ \cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$

(xviii) $\sin 3x = 3 \sin x - 4 \sin^3 x$

(xix) $\cos 3x = 4 \cos^3 x - 3 \cos x$

(xx) $\tan 3x = \dfrac{3 \tan x - \tan^3 x}{1 - 3\tan^2 x}$

(xxi) $\sin^2 \theta + \cos^2 \theta = 1$

(xxii) $1 + \tan^2 \theta = \sec^2 \theta$

(xxiii) $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

- **To simplify inverse trigonometrical expressions, following substitutions can be considered:**

Expression	Substitution
$a^2 + x^2$ or $\sqrt{a^2 + x^2}$	$x = a \tan \theta$ or $x = a \cot \theta$
$a^2 - x^2$ or $\sqrt{a^2 - x^2}$	$x = a \sin \theta$ or $x = a \cos \theta$
$x^2 - a^2$ or $\sqrt{x^2 - a^2}$	$x = a \sec \theta$ or $x = a \operatorname{cosec} \theta$
$\sqrt{\dfrac{a-x}{a+x}}$ or $\sqrt{\dfrac{a+x}{a-x}}$	$x = a \cos 2\theta$
$\sqrt{\dfrac{a^2-x^2}{a^2+x^2}}$ or $\sqrt{\dfrac{a^2+x^2}{a^2-x^2}}$	$x^2 = a^2 \cos 2\theta$
$\sqrt{\dfrac{x}{a-x}}$ or $\sqrt{\dfrac{a-x}{x}}$	$x = a \sin^2\theta$ or $x = a \cos^2\theta$
$\sqrt{\dfrac{x}{a+x}}$ or $\sqrt{\dfrac{a+x}{x}}$	$x = a \tan^2\theta$ or $x = a \cot^2\theta$

Chapter 3 – Matrices

Matrix

A matrix is a rectangular array of numbers or functions. The number or functions are called the elements or the entries of the matrix.

Order of Matrix

A matrix of order $m \times n$ is of the form

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix},$$

where m represents number of rows and n represents number of columns.

In notation form, it can be rewritten as

$$A = [a_{ij}]_{m \times n}$$

where, $1 \le i \le m$, $1 \le j \le n$ and $i, j \in N$. Here, a_{ij} is an element lying in the i^{th} row and j^{th} column.

Types of Matrices

(i) **Row matrix:** A matrix having only one row and many columns, is called a row matrix.

(ii) **Column matrix:** A matrix having only one column and many rows, is called a column matrix.

(iii) **Zero matrix or null matrix:** If all the elements of a matrix are zero, then it is called a zero matrix or null matrix. It is denoted by symbol O.

(iv) **Square matrix:** A matrix in which number of rows and number of columns are equal, is called a square matrix.

(v) **Diagonal matrix:** A square matrix is said to be a diagonal matrix, if all the elements lying outside the principal diagonal elements are zero.

(vi) **Scalar matrix:** A diagonal matrix in which all principal diagonal elements are equal, is called a scalar matrix.

(vii) **Unit matrix or identity matrix:** A square matrix having 1 (one) on its principal diagonal and 0 (zero) elsewhere, is called an identity matrix. It is denoted by symbol I.

(viii) **Equality of matrix:** Two matrices are said to be equal, if their order is same and their corresponding elements are also equal.

Addition of Matrices

Let A and B be two matrices each of order $m \times n$. Then, the sum of matrices $A+B$ is a matrix whose elements are obtained by adding the corresponding elements of A and B.

i.e., if $\quad A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{m \times n}$

Then, $\quad A + B = [a_{ij} + b_{ij}]_{m \times n}$

Difference of Matrices

If $A = [a_{ij}]$ and $B = [b_{ij}]$ are two matrices of the same order $m \times n$, then difference $A - B$ is defined as a matrix $D = [d_{ij}]$, where $d_{ij} = a_{ij} - b_{ij}$, $\forall i, j$.

Note: *If A and B are not of same order, then A + B is not defined.*

Multiplication of a Matrix by a Scalar

Let $A = [a_{ij}]_{m \times n}$ be a matrix and k be any scalar. Then, kA is another matrix which is obtained by multiplying each element of A by k i.e., $kA = [ka_{ij}]_{m \times n}$.

(i) $k(A+B) = kA+kB$ **(ii)** $(k_1+k_2)A = k_1A+k_2A$

Multiplication of Matrices

Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{n \times p}$ be two matrices such that the number of columns of A is equal to the number of rows of B, then multiplication of A and B is denoted by AB, is given by $C_{ij} = \sum_{k=1}^{n} a_{ij} b_{kj}$ where, C_{ij} is the element of matrix C and $C = AB$.

Note: *Generally, multiplication of matrices is not commutative* i.e., $AB \neq BA$.

Transpose of a Matrix

The matrix obtained by interchanging the rows and columns of a given matrix A, is called transpose of a matrix. It is denoted by A' or A^T.

Properties of Transpose of Matrices

(i) $(A \pm B)' = A' \pm B'$

(ii) $(kA)' = kA'$, where k is any constant.

(iii) $(AB)' = B'A'$

(iv) $(A')' = A$

Symmetric and Skew-Symmetric Matrix

A square matrix A is called symmetric, if $A' = A$ and a square matrix A is called skew-symmetric, if $A' = -A$.

Properties of Symmetric and Skew-Symmetric Matrix

(i) For any square matrix A with real number entries, $A + A'$ is a symmetric matrix and $A - A'$ is a skew-symmetric matrix.

(ii) Any square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrices.

i.e., $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

(iii) The principal diagonal elements of a skew-symmetric matrix are always zero.

Invertible Matrices

A is a square matrix of order m is said to be invertible, if there exists another square matrix B of the same order m, such that $AB = BA = I$, where I is a unit matrix of same order m. The matrix B is called the inverse matrix of A and it is denoted by A^{-1}.

Note: (i) *A rectangular matrix does not possess inverse matrix.*

(ii) *If B is an inverse of A, then A is also the inverse of B.*

Properties of Invertible Matrices

Let A and B be two non-zero invertible matrices of same order.

(i) Uniqueness of inverse: If inverse of a square matrix exists, then it is unique.

(ii) $AA^{-1} = A^{-1}A = I$

(iii) $(AB)^{-1} = B^{-1}A^{-1}$

(iv) $(A^{-1})^{-1} = A$

(v) $(A')^{-1} = (A^{-1})'$ or $(A^T)^{-1} = (A^{-1})^T$

where, A' or A^T is transpose of a matrix A.

Chapter 4 – Determinants

Determinant

Every square matrix A is associated to a number, which is called its determinant and it is denoted by det (A) or $|A|$.

Expansion of Determinant of Order (2×2)

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Expansion of Determinant of Order (3×3)

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

Note *Similarly, we can expand the above determinant corresponding to any row or column.*

If A is a $n \times n$ *matrix, then* $|kA| = k^n|A|$.

Properties of Determinants

(i) If the rows and columns of a determinant are interchanged, then the value of the determinant does not change.

(ii) If any two rows (columns) of a determinant are interchanged, then sign of determinant changes.

(iii) If any two rows (columns) of a determinant are identical, then the value of the determinant is zero.

(iv) If each element of a row (column) is multiplied by a non-zero constant k, then the value of the determinant is multiplied by k.

(v) If each element of any row (column) of a determinant is added k times the corresponding element of another row (column), then the value of the determinant remains unchanged.

(vi) If some or all elements of a row (column) of a determinant are expressed as sum of two (more) terms, then the determinant can be expressed as sum of two (more) determinants of the same order.

(vii) If all elements of any two rows (or column) of a determinant are proportional, then the value of such determinant becomes zero.

(viii) If all the elements of any row (or column) of a determinant are zero, then the value of such determinant becomes zero.

Minors and Cofactors

Minors: Minor of an element a_{ij} of a matrix is the determinant obtained by deleting i^{th} row and j^{th} column in which element a_{ij} lies. It is denoted by M_{ij}.

$$\text{If } A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \text{ then}$$

Minors of A are

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix},$$

$$M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix},$$

$$M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}, \text{ etc.}$$

Note: *The minor of an element of a determinant of order n* ($n \geq$ 2) *is a determinant of order* $n - 1$.

Cofactors: If M_{ij} is the minor of an element a_{ij}, then the cofactor of a_{ij} is denoted by C_{ij} or A_{ij} and defined as follows

$$A_{ij} \text{ or } C_{ij} = (-1)^{i+j} M_{ij}$$

Note: *If elements of a row (column) are multiplied with cofactors of any other row (column), then their sum is zero.*

Adjoint of a Matrix

The adjoint of a square matrix A is defined as the transpose of the matrix formed by cofactors of A. Let $A=[a_{ij}]$ be a square matrix of order n, then adjoint of A, i.e., *adj* $A = C^T$, where $C = [C_{ij}]$ is the cofactor matrix of A.

Properties of Adjoint of Square Matrix

If A and B are two non-singular matrices of order n, then

(i) $A(adj\,A) = |A|\, I_n = (adj\,A)\,A$

(ii) $adj(A^T) = (adj\,A)^T$

(iii) $|adj\,A| = |A|^{n-1}$, if $|A| \neq 0$

(iv) $|\text{adj}[\text{adj}(A)]| = |A|^{(n-1)^2}$, if $|A| \neq 0$

(v) $(adj\,AB) = (adj\,B)\,(adj\,A)$

Singular and Non-singular Matrix

A square matrix A is said to be a singular matrix, if $|A| =$ 0 and if $|A| \neq 0$, then matrix A is said to be non-singular matrix.

Inverse of a Matrix

A square matrix A has inverse, if A is a non-singular ($|A| \neq 0$) matrix. The inverse of A is denoted by A^{-1}

i.e., $A^{-1} = \frac{1}{|A|}(adj\,A), |A| \neq 0$

Properties of Inverse of a Square Matrix

(i) $(A^{-1})^{-1} = A$
(ii) $(AB)^{-1} = B^{-1}A^{-1}$
(iii) $(A^T)^{-1} = (A^{-1})^T$
(iv) $(kA)^{-1} = kA^{-1}$
(v) $adj\,(A^{-1}) = (adj\,A)^{-1}$

System of Linear Equations, Martin's Rule

Let the system of linear equations be

$$a_1x+b_1y+c_1z = d_1,$$
$$a_2x+b_2y+c_2z = d_2$$
$$\text{and } a_3x+b_3y+c_3z = d_3$$

Then, in matrix form this system of equations can be written as $AX = B$

$$\text{where, } A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

A system of equations is consistent or inconsistent accordingly as its solution exists or not.

(i) For a square matrix A in matrix equation $AX = B$. So, $X = A^{-1}B$

(a) If $|A| \neq 0$, then system of equations is **consistent** and has unique solution given by $X = A^{-1}B$.

(ii) When $B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ in such cases, we have

(a) $|A| \neq 0 \Rightarrow$ System has only trivial solutions i.e., $x = 0, y = 0$ and $z = 0$

(b) $|A| = 0 \Rightarrow$ System has infinitely many solutions.

Chapter 5 – Continuity, Differentiability and Differentiation

Limit and Continuity

Limit: For a function $f(x)$, $\lim_{x \to a} f(x)$ exists if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

➢ Continuity at a point

A function $f(x)$ is continuous at a point $x = a$, were $a \in$ domain of $f(x)$, if $\lim_{x \to a^-} f(x) = f(a) = \lim_{x \to a^+} f(x)$

Where, $\lim_{x \to a^-} f(x)$ is Left Hand Limit (LHL) of $f(x)$ at $x = a$

$\lim_{x \to a^+} f(x)$ is Right Hand Limit (RHL) of $f(x)$ at $x = a$ and $f(a)$ is the value of $f(x)$ at $x = a$.

➢ Continuity on an interval

- **Continuity on an open interval:** A function $f(x)$ is said to be continuous on an open interval (a, b) if it is continuous at every point on the interval (a, b).
- **Continuity on a closed interval:** A function $f(x)$ is said to be continuous on closed interval $[a, b]$ if it is continuous on $[a, b]$ and it is continuous at 'a' from the right side and at 'b' from the left. i.e., for $f(x)$ to be continuous on $[a, b]$

(i) f is continuous on (a, b)

(ii) $\lim_{x \to a^+} f(x) = f(a)$

(iii) $\lim_{x \to a^-} f(x) = f(b)$

➢ Algebra of continuity

Let f and g be two real and continuous function at $x = a$, then

(i) $f \pm g$ is continuous at $x = a$

(ii) $f\,g$ is continuous at $x = a$.

(iii) $\frac{f}{g}$ is continuous at $x = a$, Provided $g(a) \neq 0$

(iv) $f(x)$ is continuous at $x = a$, where a is real number.

➢ Differentiability

Let $f(x)$ be defined at any point c in the interval (a, b) Then $f(x)$ is said to be differentiate at $x = c$ if the function has a derivative at this point, i.e., if $f'(c)$ exists. Hence, if $\lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$ then the function $f(x)$ is called differentiable at point $x = c$.

For the existence of this limit it is necessary that when $h \to 0$, the *left–hand* and *right–hand* limits both must exist and they must be equal.

Important Facts

- If a function is differentiable at a point, it is continuous at that point as well.
- If a function is not differentiable at a point, it may or may not be continuous at that point.
- If a function is continuous at a point, it may or may not differentiable at that point.
- If a function is discontinuous at a point, it is not differentiable at that point.

Formulae For Limits

(a) $\lim_{x \to 0} \frac{\sin x}{x} = 1 = \lim_{x \to 0} \frac{x}{\sin x}$

(b) $\lim_{x \to 0} \frac{\tan x}{x} = 1 = \lim_{x \to 0} \frac{x}{\tan x}$

(c) $\lim\limits_{x\to 0}\dfrac{\sin^{-1}x}{x}=1=\lim\limits_{x\to 0}\dfrac{x}{\sin^{-1}x}$

(d) $\lim\limits_{x\to 0}\dfrac{\tan^{-1}x}{x}=1=\lim\limits_{x\to 0}\dfrac{x}{\tan^{-1}x}$

(e) $\lim\limits_{x\to 0}\dfrac{a^x-1}{x}=\log_e a,\ a>0$

(f) $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}=1$

(g) $\lim\limits_{x\to 0}\dfrac{\log_e(1+x)}{x}=1$

(h) $\lim\limits_{x\to 0}\dfrac{x^n-a^n}{x-a}=na^{n-1}$

(i) $\lim\limits_{x\to 0}(1+kx)^{\frac{1}{x}}=e^k$, where k is any constant

(j) $\lim\limits_{x\to\infty}\dfrac{\log_e(1+x)}{x}=\log_e e,\ a>0\neq 1$

(k) $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}=\lim\limits_{x\to\infty}\dfrac{\cos x}{x}=0$

(l) $\lim\limits_{x\to 0}\cos x=1$

Derivatives of Standard Functions

(i) $\dfrac{d}{dx}(x^n)=nx^{n-1}$

(ii) $\dfrac{d}{dx}(\text{constant})=0$

(iii) $\dfrac{d}{dx}(cx^n)=cn\,x^{n-1}$, where c is a constant.

(iv) $\dfrac{d}{dx}(\sin x)=\cos x$

(v) $\dfrac{d}{dx}(\cos x)=-\sin x$

(vi) $\dfrac{d}{dx}(\tan x)=\sec^2 x$

(vii) $\dfrac{d}{dx}(\text{cosec } x)=-\text{cosec}\,x\cot x$

(viii) $\dfrac{d}{dx}(\sec x)=\sec x\tan x$

(ix) $\dfrac{d}{dx}(\cot x)=-\text{cosec}^2 x$

(x) $\dfrac{d}{dx}(e^x)=e^x$

(xi) $\dfrac{d}{dx}(a^x)=a^x\log_e a,\ a>0$

(xii) $\dfrac{d}{dx}(\log_e x)=\dfrac{1}{x},\ x>0$

(xiii) $\dfrac{d}{dx}(\log_a x)=\dfrac{1}{x\log_e a},\ a>0,\ a\neq 1$

(xiv) $\dfrac{d}{dx}(\sin^{-1}x)=\dfrac{1}{\sqrt{1-x^2}},\ -1<x<1$

(xv) $\dfrac{d}{dx}(\cos^{-1}x)=\dfrac{-1}{\sqrt{1-x^2}},\ -1<x<1$

(xvi) $\dfrac{d}{dx}(\tan^{-1}x)=\dfrac{1}{1+x^2}$

(xvii) $\dfrac{d}{dx}(\cot^{-1}x)=\dfrac{-1}{1+x^2}$

(xviii) $\dfrac{d}{dx}(\sec^{-1}x)=\dfrac{1}{|x|\sqrt{x^2-1}},\ |x|>1$

(xix) $\dfrac{d}{dx}(\text{cosec}^{-1}x)=\dfrac{-1}{|x|\sqrt{x^2-1}},\ |x|>1$

Algebra of Derivatives

(i) $\dfrac{d}{dx}(u\pm v)=\dfrac{du}{dx}\pm\dfrac{dv}{dx}$ [sum and difference rule]

(ii) $\dfrac{d}{dx}(u\cdot v)=u\dfrac{d}{dx}(v)+v\dfrac{d}{dx}(u)$ [product rule]

(iii) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)=\dfrac{v\dfrac{d}{dx}(u)-u\dfrac{d}{dx}(v)}{v^2}$ [quotient rule]

Implicit functions

These are functions where a specific variable cannot be expressed as a function of the other variable. Even when the value of x is known, we cannot directly compute the value of y.
$x^2y^2+xy^2+e^{xy}=abc=$ constant is an implicit function. Here, we cannot express x in terms of y or y in terms of x.

Steps to compute the derivative of an implicit function

Given an implicit function with the dependent variable y and the independent variable x (or the other way around). Differentiate the entire equation with respect to the independent variable (it could be x or y).
After differentiating, we need to apply the chain rule of differentiation.

Solve the resultant equation for $\dfrac{dy}{dx}$ (or $\dfrac{dy}{dx}$ like wise) or differentiate again if the higher-order derivatives are needed.

Parametric functions:

A parametric function is a group of functions with one or more independent variables.

Steps to calculate derivatives of Parametric functions

Typically, the parametric equations are given in the form $x(t)$ and $y(t)$. we start by finding $x'(t)$ and $y'(t)$.

The derivative of a parametric equation, $\frac{dy}{dx}$ is given by the formula $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{y'(t)}{x'(t)}$. Therefore, we divide $y'(t)$ by $x'(t)$ and simplify to obtain $\frac{dy}{dx}$.

Derivative of Composite Function by Chain Rule

Let f be a real valued function which is a composite of two functions say $y = f(u)$ and $u = g(x)$.

Then, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = f'(u) \cdot g'(x)$

i.e., $\frac{d}{dx}[f\{g(x)\}] = f'[g(x)] \cdot g'(x)$

Derivative of a Function with respect to Another Function

Let $y = f(x)$ and $z = g(x)$, we firstly differentiate both functions $f(x)$ and $g(x)$ with respect to x separately and then put these values in the following formulae.

$$\frac{dy}{dz} = \frac{dy/dx}{dz/dx} \text{ or } \frac{dz}{dy} = \frac{dz/dx}{dy/dx}$$

Logarithmic Differentiation

Suppose, given function is of the form $[u(x)]^{v(x)}$ or $[u(x)]^m$ or $u(x) \cdot v(x)$. Then, we take logarithm and use properties of logarithm to simplify it and then differentiate it.

Basic Properties of Logarithms

$$\left.\begin{array}{ll} \textbf{(i)} & \log_a mn = \log_a m + \log_a n \\ \textbf{(ii)} & \log_a \frac{m}{n} = \log_a m - \log_a n \\ \textbf{(iii)} & \log_a m^n = n \log_a m \end{array}\right\} \text{For } m > 0, n > 0, a > 0 \text{ and } a \neq 1$$

(iv) $\log_a a = 1; a > 0$ and $a \neq 1$

(v) $\log_b a = \frac{\log_m a}{\log_m b}; a > 0, b > 0, b \neq 1, m > 0$ and $m \neq 1$

(vi) $a^{\log_a m} = m;\ a > 0, m > 0,\ a \neq 1$

Second Order Derivative

Let $y = f(x)$, then $\frac{dy}{dx} = f'(x)$ is called the first derivative of y or $f(x)$, $\frac{d}{dx}\left(\frac{dy}{dx}\right)$ is called the second order derivative of y w.r.t x and it is denoted by $\frac{d^2y}{dx^2}$ or y''.

L - Hospital's rule states that for functions f and g which are differentiable on an open interval I except possibly at a point c contained in I, its $\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = 0$ or $\neq \infty$; $g'(x) \neq 0$ for all x in I with $x \neq c$ and

$\lim_{x \to c} \frac{f'(x)}{g'(x)}$ exists, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(c)}{g'(c)}$$

The differentiation of numerator and denominator often simplifies the quotient or converts it to a limit that can be evaluated directly.

Chapter 6 – Application of Derivatives

Rate of Change of Quantities

If $y = f(x)$ is a function, where y is dependent variable and x is independent variable. Then, $\frac{dy}{dx}$ [or $f'(x)$] represents the rate of change of y w.r.t. x and $\left(\frac{dy}{dx}\right)_{x = x_0}$ [or $f'(x_0)$] represents the rate of change of y w.r.t. x at $x = x_0$.

(i) Average rate of change of y w.r.t. $x = \dfrac{\Delta y}{\Delta x}$.

(ii) Instantaneous rate of change of y w.r.t. $x = \dfrac{dy}{dx}$.

Note *Here,* $\dfrac{dy}{dx}$ *is positive, if y increases as x increases and* $\dfrac{dy}{dx}$ *is negative, if y decreases as x increases.*

Increasing and Decreasing Functions

(i) **Increasing functions:** Let I be an open interval contained in the domain of a real valued function f. Then, f is said to be

(a) Increasing on I, if $x_1 < x_2$

$$\Rightarrow \quad f(x_1) \le f(x_2),\ \forall\ x_1, x_2 \in I$$

(b) Strictly increasing on I, if $x_1 < x_2$

$$\Rightarrow \quad f(x_1) < f(x_2),\ \forall\ x_1, x_2 \in I$$

(ii) **Decreasing functions:** Let I be an open interval contained in the domain of a real valued function f. Then, f is said to be

(a) Decreasing on I, if $x_1 < x_2$

$$\Rightarrow \quad f(x_1) \ge f(x_2),\ \forall\ x_1, x_2 \in I$$

(b) Strictly decreasing on I, if $x_1 < x_2$

$$\Rightarrow \quad f(x_1) > f(x_2),\ \forall\ x_1, x_2 \in I$$

Theorem- Let f be continuous on $[a, b]$ and differentiable on (a, b). Then,

(a) If $f'(x) > 0$ for each $x \in (a, b,)$, then $f(x)$ is said to be increasing in $[a, b]$ and strictly increasing in (a, b).

(b) If $f'(x) < 0$ for each $x \in (a, b,)$, then $f(x)$ is said to be decreasing in $[a, b]$ and strictly decreasing in (a, b).

(c) If $f'(x) = 0$ for each $x \in (a, b,)$, then f is said to be a constant function in $[a, b]$.

Tangents and Normals

A **tangent** is a straight line, which touches the curve $y = f(x)$ at a point. $\dfrac{dy}{dx}$ represents the gradient or slope of a curve $y = f(x)$.

A **normal** is a straight line perpendicular to a tangent to the curve $y = f(x)$ intersecting at the point of contact.

Slope of tangent and normal

If a tangent line to the curve $y = f(x)$ makes an angle θ with X-axis in the positive direction, then $\dfrac{dy}{dx}$ = Slope of tangent $= \tan\theta$ and slope of normal $= \dfrac{-1}{\text{slop of tangent}} = \dfrac{-1}{dy/dx}$

Equation of Tangent and Normal

Let $y = f(x)$ be a curve and $P(x_1, y_1)$ be a point on it. Then,

(a) equation of tangent at $P(x_1, y_1)$ is,

$$(y - y_1) = \left[\frac{dy}{dx}\right]_{(x_1, y_1)} (x - x_1)$$

(b) equation of normal at $P(x_1, y_1)$ is,

$$(y - y_1) = \frac{-1}{\left[\dfrac{dy}{dx}\right]_{(x_1, y_1)}} (x - x_1)$$

Maxima and Minima

Let f be a real valued function and c be an interior point in the domain of f. Then,

(i) Point c is called a **local maxima**, if there is a $h > 0$ such that $f(c) \ge f(x)\, \forall x$ in $(c - h,\ c + h)$.
Here, value $f(c)$ is called the local maximum value of f.

(ii) Point c is called a point of **local minima**, if there is a $h > 0$ such that $f(c) \le f(x)\, \forall x$ in $(c - h,\ c + h)$.
Here, value $f(c)$ is called the local minimum value of f.

Critical Point

A point c in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable, is called a critical point of f.

First Derivative Test

Let f be a function defined on an open interval I and let f be continuous at a critical point c in I. Then,

(i) If $f'(x)$ change sign from positive to negative as x increases through c, then c is a point of local maxima.

(ii) If $f'(x)$ change sign from negative to positive as x increases through point c, then c is a point of local minima.

(iii) If $f'(x)$ does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. Infact, such a point is called **point of inflection**.

Second Derivative Test

Let f be a function defined on an interval I and $c \in I$. Let f be twice differentiable at c, then

(i) $x = c$ is a point of local maxima, if $f'(c) = 0$ and $f''(c) < 0$. The value $f(c)$ is local maximum value of f.

(ii) $x = c$ is a point of local minima, if $f'(c) = 0$ and $f''(c) > 0$. Then, $f(c)$ is local minimum value of f.

(iii) If $f'(c) = 0$ and $f''(c) = 0$, then the test fails.

Absolute Maxima and Absolute Minima

Let f be a differentiable function on $[a, b]$ and c be a point in $[a, b]$ such that $f'(c) = 0$. Then, find $f(a)$, $f(b)$ and $f(c)$. The maximum of these values gives a maxima or absolute maxima and minimum of these values gives a minima or absolute minima.

Chapter 7 – Integrals

Indefinite Integral

Let $F(x)$ and $f(x)$ be two functions connected together, such that $\frac{d}{dx}[F(x)] = f(x)$, then $F(x)$ is called **integral** of $f(x)$ or **indefinite integral** or **anti-derivative**.

Thus, $\int f(x)dx = F(x) + C$

where, C is an arbitrary constant.

Some Standard Formulae

(i) $\int x^n dx = \frac{x^{n+1}}{(n+1)} + C,\ n \neq -1$

(ii) $\int \frac{1}{x} dx = \log|x| + C,\ x \neq 0$

(iii) $\int e^x dx = e^x + C$

(iv) $\int a^x dx = \frac{a^x}{\log_e a} + C$

(v) $\int \sin x\, dx = -\cos x + C$

(vi) $\int \cos x\, dx = \sin x + C$

(vii) $\int \tan x\, dx = \log|\sec x| + C$

$= -\log|\cos x| + C$

(viii) $\int \cot x\, dx = \log|\sin x| + C$

(ix) $\int \sec x\, dx = \log|\sec x + \tan x| + C$

$= \log|\tan(\frac{\pi}{4} + \frac{x}{2})| + C$

(x) $\int \text{cosec}\, x\, dx = \log|\text{cosec}\, x - \cot x| + C$

$= \log|\tan(x/2)| + C$

(xi) $\int \sec^2 x\, dx = \tan x + C$

(xii) $\int \text{cosec}^2 x\, dx = -\cot x + C$

(xiii) $\int \sec x \cdot \tan x\, dx = \sec x + C$

(xiv) $\int \text{cosec}\, x \cdot \cot x\, dx = -\text{cosec}\, x + C$

(xv) $\int \frac{1}{\sqrt{1-x^2}} dx = -\cos^{-1} x + C$

(xvi) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$

(xvii) $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

(xviii) $\int \frac{1}{1+x^2} dx = -\cot^{-1} x + C$

(xix) $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$

(xx) $\int \frac{1}{x\sqrt{x^2-1}} dx = -\text{cosec}^{-1} x + C$

(xxi) $\int \sin^2 x\, dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$

(xxii) $\int \sin^3 x\, dx = \frac{-3}{4}\cos x + \frac{1}{12}\cos 3x + C$

(xxiii) $\int \sin^4 x\, dx = \frac{3x}{8} - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$

(xxiv) $\int \cos^2 x\, dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$

(xxv) $\int \cos^3 x\, dx = \frac{1}{4}\left[\frac{\sin 3x}{3} + 3\sin x\right] + C$

(xxvi) $\int \cos^4 x\, dx = \frac{3}{8x} + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$

Properties of Indefinite Integral

(i) The process of differentiation and integration are inverse of each other.

i.e., $\frac{d}{dx}\int f(x)dx = f(x)$ and $\int f'(x)dx = f(x) + C$

where, C is any arbitrary constant.

(ii) $\int \{f(x) \pm g(x)\}dx = \int f(x)dx \pm \int g(x)dx$

(iii) $\int K f(x)dx = K\int f(x)dx$

Integration by Substitution

The method of reducing a given integral into one or more standard integrals by changing the independent variable is called method of substitution.

Thus, $\int f(x)dx = \int f\{g(t)\} \cdot g'(t)dt$, if we substitute $x = g(t)$, such that $dx = g'(t)\, dt$.

Some Standard Substitutions

Expression	Substitution
$a^2 - x^2$	$x = a \sin\theta$ or $a \cos\theta$
$a^2 + x^2$	$x = a \tan\theta$ or $a \cot\theta$
$x^2 - a^2$	$x = a \sec\theta$ or $a\, \text{cosec}\, \theta$
$\sqrt{\frac{a+x}{a-x}}$ or $\sqrt{\frac{a-x}{a+x}}$	$x = a \cos 2\theta$
$\sqrt{\frac{x-\alpha}{\beta-x}}$ or $\sqrt{(x-\alpha)(x-\beta)}$	$x = \alpha\cos^2\theta + \beta\sin^2\theta$

Integration by Partial Fractions

Suppose, given integral is of the form $\int \frac{P(x)}{Q(x)}dx$, when $P(x)$ and $Q(x)$ are polynomials in x and $Q(x) \neq 0$. Then, to solve such integrals by partial fraction, we firstly take the given integrand $\frac{P(x)}{Q(x)}$ and decompose it into suitable partial fraction form and then integrate each term by using suitable method.

Form of the Rational Functions	Form of the Partial Fractions
$\frac{px \pm q}{(x \pm a)(x \pm b)}$, $a \neq b$	$\frac{A}{x \pm a} + \frac{B}{x \pm b}$
$\frac{px \pm q}{(x \pm a)^2}$	$\frac{A}{(x \pm a)} + \frac{B}{(x \pm a)^2}$
$\frac{px^2 \pm qx \pm r}{(x \pm a)(x \pm b)^2}$	$\frac{A}{(x \pm a)} + \frac{B}{(x \pm b)} + \frac{C}{(x \pm b)^2}$

Note: $\int \frac{x^2}{(x^2+a^2)(x^2+b^2)}dx$

Let $x^2 = t$ *and proceed for partial fraction of*

$$\frac{t}{(t+a^2)(t+b^2)}$$

Integration by Parts

Let u and v be two differentiable functions of a single variable x, then the integral of the product of two functions is

$$\int \underset{\text{I}}{u}\,\underset{\text{II}}{v}\;dx = u\int v\;dx - \int\left(\frac{d}{dx}u\int v\;dx\right)dx$$

If two functions are of different types, then consider the 1st function (i.e., u) which comes first in word ILATE, where

I : Inverse trigonometric function e.g., $\sin^{-1} x$

L : Logarithmic function e.g., $\log x$

A : Algebraic function e.g., $1, x, x^2$

T : Trigonometric function e.g., $\sin x$, $\cos x$

E : Exponential function e.g., e^x

Integrals of Some Particular Functions

(i) $\int \frac{dx}{(x^2-a^2)} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

(ii) $\int \frac{dx}{(a^2-x^2)} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

(iii) $\int \frac{dx}{\sqrt{(x^2-a^2)}} = \log\left|x+\sqrt{x^2-a^2}\right| + C$

(iv) $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right| + C$

(v) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

(vi) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

(vii) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

(viii) $\int e^x[f(x)+f'(x)]dx = e^x f(x) + C$

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Some Standard Integrands and Substitutions For Them

Integral	Substitution
$\int \frac{dx}{a \pm b\cos x}$	Put $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$, then put $\tan\frac{x}{2} = t$
$\int \frac{dx}{a \pm b\sin x}$	Put $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$, then put $\tan\frac{x}{2} = t$
$\int \frac{dx}{a\sin x + b\cos x}$	Put $a = r\cos\theta$ and $b = r\sin\theta$

Integral	Substitution
$\int \frac{dx}{a+b\sin^2 x}$, $\int \frac{dx}{a+b\cos^2 x}$ and $\int \frac{dx}{a\cos^2 x+b\sin^2 x}$	(a) Divide numerator and denominator by $\cos^2 x$ (b) Reduce $\sec^2 x$ in denominator as $1 + \tan^2 x$ (c) Put $\tan x = t$ and proceed for perfect square
$\int \frac{dx}{(ax+b)\sqrt{px+q}}$ *or* $\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$	Put $\sqrt{px+q}=t$
$\int \frac{dx}{(px+q)\left(\sqrt{ax^2+bx+c}\right)}$	Put $px+q=\frac{1}{t}$
$\int \frac{dx}{(px^2+q)\sqrt{ax^2+b}}$	Put $x=\frac{1}{t}$

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Definite Integral

An integral of the form $\int_a^b f(x)dx$ is known as definite integral and is given by

$$\int_a^b f(x)dx = g(b)-g(a)$$

where, $f(x)$ is derivative of $g(x)$, a and b are lower and upper limits of a definite integral.

Fundamental Theorem of Calculus

Theorem 1- Let f be a continuous function defined on the closed interval $[a, b]$ and $A(x)$ be the area of function.

i.e., $A(x)=\int_a^x f(x)dx$. Then, $A'(x)=f(x), \forall\, x\in[a,b]$.

Theorem 2- Let f be a continuous function defined on the closed interval $[a, b]$ and F be an anti-derivative of f. Then,

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b)-F(a)$$

Properties of Definite Integral

(i) $\int_a^b f(x)dx = \int_a^b f(t)dt$

(ii) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(iii) $\int_a^a f(x)dx = 0$

(iv) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where $a<c<b$.

(v) $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

(vi) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

(vii) $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

(viii) $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x)=f(x) \\ 0 \quad , & \text{if } f(2a-x)=-f(x) \end{cases}$

(ix) $\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(-x)=f(x) \text{ i.e., even} \\ 0 \quad , & \text{if } f(-x)=-f(x) \text{ i.e., odd} \end{cases}$

Chapter 8 – Differential Equations

Differential Equation

An equation containing an independent variable, dependent variable and derivative of dependent variable with respect to independent variable, is called a differential equation.

Order of a Differential Equation

The order of the highest order derivative occurring in the differential equation, is called the order of differential equation.

Degree of a Differential Equation

The highest power (positive integral index) of the highest order derivative involved in a differential equation, when it is a polynomial equation in derivative, i.e., all powers are integers, is called the degree of a differential equation.

Solution of a Differential Equation

A function in which variables involved in the differential equation will be a solution of differential equation, if this function and its derivatives satisfy the given differential equation.

General Solution of a Differential Equation

A solution of a differential equation which contains arbitrary constants, is called the general solution or primitive solution of the differential equation.

Particular Solution of a Differential Equation

The solution obtained by giving particular values to the arbitrary constants in general solution, is called a particular solution of the differential equation.
In other words, the solution free from arbitrary constants is called particular solution.

Formation of a Differential Equation

An equation with independent, dependent variables involving some arbitrary constants is given, then a differential equation is obtained as follows:

(i) Differentiate the given equation with respect to the independent variable (say x) as many times as the number of arbitrary constants in it.

(ii) Eliminate the arbitrary constants by these equations to get required differential equation.

Differential Equation in Variable Separable Form

Suppose, a first order and first degree differential equation is $\frac{dy}{dx}=f(x,y)$. …(i)

Then, expressed it as $\frac{dy}{dx}=h(g)\cdot g(x)$ …(ii)

If $h(g) \neq 0$, then separating the variables, eq. (ii) can be written as $\frac{1}{h(g)}dy=g(x)dx$.

On integrating both sides, we get the required solution of given differential equation.

Homogeneous Differential Equation

A function $F(x, y)$ is said to be homogeneous function of degree n, if $F(x,y)=x^n g\left(\frac{y}{x}\right)$ or $y^n h\left(\frac{x}{y}\right)$.

A differential equation of the form $\frac{dy}{dx}=F(x,y)$ is called a homogeneous differential equation.

Solution of Homogeneous Differential Equation

To solve homogeneous differential equation, firstly put $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$ in given differential equation and then use variable separable to get the required solution.

Linear Differential Equation

A first order and first degree differential equation in which the degree of dependent variable and its derivative is one and they do not get multiplied together, is called a linear differential equation.
There are two types of linear differential equations.

Type I If differential equation is of the form $\frac{dy}{dx}+Py=Q$, where P and Q are constants or functions of x. Then, its solution is $y \times (I.F.) = \int[Q\times(I\cdot F\cdot)]dx+C$, where $I\cdot F\cdot=e^{\int Pdx}$

Type II If differential equation is of the form $\frac{dx}{dy}+Px=Q$, where P and Q are constants or functions of y, then its solution is $x \times (I.F.) = \int[Q\times(I\cdot F\cdot)]dy+C$, where $I\cdot F\cdot=e^{\int Pdy}$.

Chapter 9 – Probability

Some Basic Definitions

(i) **Experiment:** An operation which can produce some well-defined outcomes, is called an experiment.

(ii) **Random Experiment:** An experiment in which total outcomes are known in advance but occurrence of specific outcome can be told only after completion of the experiment, is known as a random experiment.

(iii) **Trial:** When a random experiment is repeated under identical conditions and it does not give the same result each time but may result in one of the several possible outcomes, then such experiment is called a trial.

(iv) **Sample Space:** The set of all possible outcomes of a random experiment is called its sample space. It is usually denoted by S.

Event

A subset of the sample space associated with a random experiment is called an **event**.

Types of Event

Equally Likely Event

The given events are said to be equally likely, if none of them is expected to occur in preference to the other.

Mutually Exclusive Event

A set of events is said to be mutually exclusive, if the happening of one excludes the happening of the other i.e., if A and B are mutually exclusive, then $(A \cap B) = \phi$.

Exhaustive Event

A set of events is said to be exhaustive, if the performance of the experiment always results in the occurrence of atleast one of them.

If $E_1, E_2, \ldots, E_n$ are exhaustive events, then $E_1 \cup E_2 \cup \ldots \cup E_n = S$.

Complement of an Event

Let A be an event in a sample space S, then complement of A is the set of all sample points of the space other than the sample point in A and it is denoted by A' or $\bar{A}$.

i.e., $A' = \{n : n \in S, n \notin A\}$

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Some Basic Terms

Coin

A coin has two sides, head and tail. If an event consists of more than one coin, then coins are considered as distinct, if not otherwise stated.

(i) Sample space of one coin = {H, T}

(ii) Sample space of two coins = {(H, T), (T, H), (H, H), (T, T)}

(iii) Sample space of three coins = {(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, H, T), (T, T, H), (T, T, T)}

Die

A die has six faces marked 1, 2, 3, 4, 5 and 6. If we have more than one die, then all dice are considered as distinct, if not otherwise stated.

(i) Sample space of a die = {1, 2, 3, 4, 5, 6}

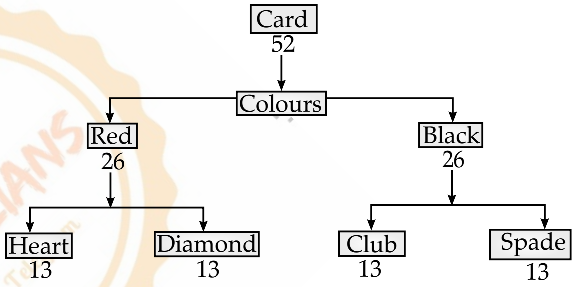
(ii) Sample space of two dice

$$= \begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{Bmatrix}$$

Playing Cards

A pack of playing cards has 52 cards. There are 4 suits (spade, heart, diamond and club), each having 13 cards. There are two colours, red (heart and diamond) and black (spade and club), each having 26 cards.

In 13 cards of each suit, there are 3 face cards namely king, queen and jack, so there are in all 12 face cards. Also, there are 16 honour cards, 4 of each suit namely ace, king, queen and jack.



Probability

In a random experiment, let S be the sample space and E be the event. Then,

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Number of all possible outcomes}} = \frac{n(E)}{n(S)}$$

(i) If E is an event and S is the sample space, then

(a) $0 \le P(E) \le 1$ **(b)** $P(\phi) = 0$ **(c)** $P(S) = 1$

(ii) $P(\bar{E}) = 1 - P(E)$

Addition Theorem of Probability

(a) For two events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B)$$

[for mutually exclusive, $P(A \cap B) = 0$]

(b) For three events A, B and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

If A, B and C are mutually exclusive events, then

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)$$

$$\left[\begin{array}{l}\text{for mutually exclusive events,}\\ P(A\cap B)=P(B\cap C)=P(C\cap A)\\ \qquad\qquad =P(A\cap B\cap C)=0\end{array}\right]$$

Important Results on Probability

(i) If A and B are two events associated to a random experiment, then

$$P(A\cap B)\le P(A)\le P(A\cup B)\le P(A)+P(B)$$

(ii) If A and B are two events associated with a random experiment, then

(a) $P(\bar{A}\cap B)=P(B)-P(A\cap B)$

(b) $P(A\cap\bar{B})=P(A)-P(A\cap B)$

(c) $P\left[(A\cap\bar{B})\cup(\bar{A}\cap B)\right]$

$$=P(A)+P(B)-2P(A\cap B)$$

(d) $P(\bar{A}\cap\bar{B})=1-P(A\cup B)$

(e) $P(\bar{A}\cup\bar{B})=1-P(A\cap B)$

(f) $P(A)=P(A\cap B)+P(A\cap\bar{B})$

(g) $P(B)=P(A\cap B)+P(B\cap\bar{A})$

(iii) **(a)** P (exactly one of A, B occurs)

$$=P(A)+P(B)-2P(A\cap B)$$
$$=P(A\cup B)-P(A\cap B)$$

(b) $P(\text{neither } A \text{ nor } B)$

$$P(\bar{A}\cap\bar{B})=1-P(A\cup B)$$

(iv) If, A, B and C are three events, then

P (exactly one of A, B, C occurs)

$$=P(A)+P(B)+P(C)-2P(A\cap B)-2P\ (B\cap C)$$
$$-2P(A\cap C)+3P(A\cap B\cap C)$$

(v) **(a)** $P(\bar{A})=1-P(A)$

(b) $P(A\cup\bar{A})=P(S),\ P(f)=0$

Conditional Probability

Let E and F be two events associated with a random experiment. Then, probability of occurrence of event E, when the event F has already occurred, is called conditional probability of event E over F and is denoted by $P(E/F)$.

$$P(E/F)=\frac{P(E\cap F)}{P(F)},\ \text{where } P(F)\neq 0.$$

Properties of Conditional Probability

Let A, B and C be the events of a sample space S. Then,

(i) $P(S/A)=P(A/A)=1$

(ii) $P\{(A\cup B)/C\}=P(A/C)+P(B/C)-P\{(A\cap B)/C\};$

$P(C)\neq 0$

(iii) $P(A'/B)=1-P(A/B)$,

where A' is complement of A.

Multiplication Theorem on Probability

Let A and B are two events associated with a random experiment, then

$$P(A\cap B)=\begin{cases}P(A)\cdot P(B/A), \text{ where } P(A)\neq 0\\ P(B)\cdot P(A/B), \text{ where } P(B)\neq 0\end{cases}$$

Multiplication Probability for more than Two Events

Let E, F and G be three events of sample space S, then

$$P(E\cap F\cap G)=P(E)\cdot P\left(\frac{F}{E}\right)\cdot P\left(\frac{G}{E\cap F}\right)$$

Independent Events

Two events A and B are said to be independent, if the occurrence or non-occurrence of one event does not affect the occurrence or non-occurrence of another event. Two events E and F are said to be independent, if $P(F/E)=P(F)$, $P(E)\neq 0$, and $P(E/F)=P(E)$, $P(F)\neq 0$.

Note *If E and F are independent vents, then* $P(E\cap F)=P(E)\cdot P(F)$.

Theorem of Total Probability

Let S be the sample space and $E_1, E_2, E_3, \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment.

If E is any event which occurs with $E_1, E_2, E_3, \ldots, E_n$.

Then,

$$P(E)=P(E_1)\cdot P(E/E_1)+P(E_2)\cdot P(E/E_2)$$
$$+P(E_3)\cdot P(E/E_3)+\ldots+P(E_n)\cdot P(E/E_n)$$

or $$P(E)=\sum_{i=1}^{n}P(E_i)\cdot P\left(\frac{E}{E_i}\right)$$

Bayes' Theorem

Let S be the sample space and $E_1, E_2, \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with $E_1, E_2, \ldots, E_n$, then probability of occurrence of E_i when A occurred,

$$P(E_i/A)=\frac{P(E_i)P(A/E_i)}{\sum\limits_{i=1}^{n}P(E_i)P(A/E_i)},\ i=1,2,\ldots,n$$

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Chapter 10 – Vectors

Vector

A quantity that has magnitude as well as direction, is called a vector.

Scalar

A quantity that has magnitude only, is called scalar.

Magnitude of a Vector

The length of the vector $\vec{AB}$ or $\vec{a}$ is called the magnitude of $\vec{AB}$ or $\vec{a}$ and it is represented by $|\vec{AB}|$ or $|\vec{a}|$ or a.

If $\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$, then $|\vec{a}| = \sqrt{a^2 + b^2 + c^2}$.

Note *Since, the length is never negative, so the notation*

$|\vec{a}| < 0$ *is meaningless.*

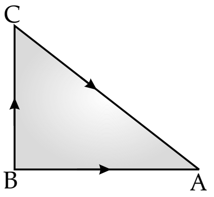
Types of Vector

1. **Zero or Null Vector:** A vector whose magnitude is zero i.e., whose initial and final points coincide, is called a null vector or zero vector.
2. **Unit Vector:** A vector whose magnitude is one unit. The unit vector in the direction of $\vec{a}$ is represented by $\hat{a}$. The unit vectors along X-axis, Y-axis and Z-axis are represented by $\hat{i}$, $\hat{j}$ and $\hat{k}$, respectively.
3. **Co-initial Vectors:** Two or more vectors having the same initial point are called co-initial vectors.
4. **Collinear Vectors:** The vectors which have same or parallel support are called collinear vectors.
5. **Like Vectors:** The vectors which have same direction are called like vectors.
6. **Unlike Vectors:** The vectors which have opposite directions are called unlike vectors.
7. **Equal Vectors:** Two vectors are equal, if they have same magnitude and direction.
8. **Negative of a Vector:** A vector whose magnitude is same as that of given vector but the direction is opposite is called negative vector of the given vector. e.g., Let $\vec{AB}$ be a vector, then $-\vec{AB}$ or $\vec{BA}$ is a negative vector.
9. **Coplanar Vectors:** A system of vectors is said to be coplanar, if their supports are parallel to same plane.

Addition of Vectors

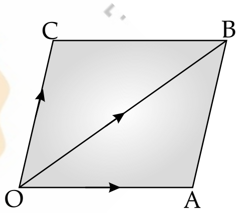
(i) **Triangle Law of Vector Addition:** If two vectors are represented along two sides of a triangle taken in order, then their resultant is represented by the third side taken in opposite direction i.e., in ΔABC, by triangle law of vector addition, we have

$$\vec{BC} + \vec{CA} = \vec{BA}$$



(ii) **Parallelogram Law of Vector Addition:** If two vectors are represented along the two adjacent sides of a parallelogram, then their resultant is represented by the diagonal of the parallelogram, which is co-initial with the given vector. If the sides OA and OC of a parallelogram $OABC$ represents $\vec{OA}$ and $\vec{OC}$, respectively, then we get

$$\vec{OA} + \vec{OC} = \vec{OB} \text{ or } \vec{OA} + \vec{AB} = \vec{OB} \quad [\because \vec{AB} = \vec{OC}]$$



Properties of Vector Addition

(i) For any two vectors $\vec{a}$ and $\vec{b}$,

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \quad \text{[commutative law]}$$

(ii) For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$,

$$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c} \quad \text{[associative law]}$$

(iii) For any vector $\vec{a}$, we have $\vec{a} + \vec{0} = \vec{0} + \vec{a} = \vec{a}$.

The zero vector $\vec{0}$ is called the additive identity for the vector addition.

(iv) For any vector $\vec{a}$, $\vec{a} + (-\vec{a}) = 0$

The vector $-\vec{a}$ is additive inverse of $\vec{a}$.

Difference of Vectors

If $\vec{a}$ and $\vec{b}$ are any two vectors, then their difference $\vec{a} - \vec{b}$ is defined as $\vec{a} + (-\vec{b})$.

Multiplication of a Vector by Scalar

Let $\vec{a}$ be a given vector and λ be a scalar. Then, the product of the vector $\vec{a}$ by the scalar λ, denoted by $\lambda\vec{a}$ is called the multiplication of vector $\vec{a}$ by the scalar λ.

Let $\vec{a}$ and $\vec{b}$ be any two vectors and k and m be any scalars.

Then,

(i) $k\vec{a}+m\vec{a}=(k+m)\vec{a}$

(ii) $k(m\vec{a})=(km)\vec{a}$

(iii) $k(\vec{a}+\vec{b})=k\vec{a}+k\vec{b}$

Section Formulae

Let A and B be two points with position vectors $\vec{a}$ and $\vec{b}$, respectively and P be a point which divides AB internally in the ratio m:n. Then, position vector of $\vec{P}$

$$=\frac{m\vec{b}+n\vec{a}}{m+n}.$$

If P divides AB externally in the ratio $m : n$. Then, position vector of $\vec{P}=\dfrac{m\vec{b}-n\vec{a}}{m-n}$.

If R is the mid-point of AB, then $\overrightarrow{OR}=\dfrac{\vec{a}+\vec{b}}{2}$.

Components of a Vector

If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, we say that the scalar components of $\vec{a}$ along X-axis, Y-axis and Z-axis are a_1, a_2 and a_3 respectively.

Important Results in Component Form

If $\vec{a}$ and $\vec{b}$ are any two vectors given in the component form such that $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$. Then, ($a_1$, a_2, a_3) and (b_1, b_2, b_3) are called direction ratios of $\vec{a}$ and $\vec{b}$, respectively.

(i) The sum (or resultant) of the vectors $\vec{a}$ and $\vec{b}$ is given by

$$\vec{a}+\vec{b}=(a_1+b_1)\hat{i}+(a_2+b_2)\hat{j}+(a_3+b_3)\hat{k}$$

(ii) The difference of the vectors $\vec{a}$ and $\vec{b}$ is given by

$$\vec{a}-\vec{b}=(a_1-b_1)\hat{i}+(a_2-b_2)\hat{j}+(a_3-b_3)\hat{k}$$

(iii) The vectors $\vec{a}$ and $\vec{b}$ are equal, iff

$$a_1=b_1,\ a_2=b_2$$

and $$a_3=b_3$$

(iv) The multiplication of vector $\vec{a}$ by any scalar λ is given by

$$\lambda\vec{a}=(\lambda a_1)\hat{i}+(\lambda a_2)\hat{j}+(\lambda a_3)\hat{k}$$

(v) If $\dfrac{b_1}{a_1}=\dfrac{b_2}{a_2}=\dfrac{b_3}{a_3}=k$ (constant)

Then, vectors $\vec{a}$ and $\vec{b}$ will be collinear.

(vi) If it is given that l, m and n are direction cosines of a vector, then $l\hat{i}+m\hat{j}+n\hat{k}=(\cos\alpha)\hat{i}+(\cos\beta)\hat{j}+(\cos\gamma)\hat{k}$ is the unit vector in the direction of that vector, where α, β and γ are the angles which the vector makes with X, Y and Z-axes, respectively.

Vector Joining Two Points

If $P_1(x_1,y_1,z_1)$ and $P_2(x_2,y_2,z_2)$ are any two points, then vector joining P_1 and P_2 is

$$\begin{aligned}\overrightarrow{P_1P_2}&=\overrightarrow{OP_2}-\overrightarrow{OP_1}\\&=(x_2\hat{i}+y_2\hat{j}+z_2\hat{k})-(x_1\hat{i}+y_1\hat{j}+z_1\hat{k})\\&=(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}\end{aligned}$$

Dot Product or Scalar Product

Let $\vec{a}$ and $\vec{b}$ be the two non-zero vectors inclined at an angle θ. Then, the scalar product or dot product of $\vec{a}$ and $\vec{b}$ is represented by $\vec{a}\cdot\vec{b}$, then $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$.

Important Results

(i) $\vec{a}\perp\vec{b}\Leftrightarrow\vec{a}\cdot\vec{b}=0$

(ii) $\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$ and $\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$

(iii) If $\theta=0$, then $\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|$

If $\theta=\pi$, then $\vec{a}\cdot\vec{b}=-|\vec{a}||\vec{b}|$

(iv) The angle between two non-zero vectors $\vec{a}$ and $\vec{b}$ is given by

$$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\text{ or }\theta=\cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)$$

(v) Properties of scalar product

(a) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ [commutative]

(b) $\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

(c) $\vec{a}\cdot\vec{a}=|\vec{a}|^2=a^2$

where, a represents magnitude of vector $\vec{a}$.

(d) $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=a^2-b^2$, where a and b represent the magnitude of vectors $\vec{a}$ and $\vec{b}$.

(e) $(\lambda\vec{a})\cdot\vec{b}=\lambda(\vec{a}\cdot\vec{b})$

(vi) Projection of $\vec{a}$ on $\vec{b}=\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)$

Note *If* α, β *and* γ *are the direction angles of vector*

$\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, then its DC's is given as,

$$\cos\alpha=\frac{a_1}{|\vec{a}|},\quad \cos\beta=\frac{a_2}{|\vec{a}|},\quad \cos\gamma=\frac{a_3}{|\vec{a}|}$$

Cross Product or Vector Product

Let θ be the angle between two non-zero vector $\vec{a}$ and $\vec{b}$, then the vector or cross product of $\vec{a}$ and $\vec{b}$ is represented by $\vec{a}\times\vec{b}$ and defined as $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{n}$, where, $\hat{n}$ is a unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$.

Important Results

(i) $\vec{a}\,||\,\vec{b}\Leftrightarrow\vec{a}\times\vec{b}=\vec{0}$

(ii) If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

(iii) $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$

(iv) $\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$

(v) $\vec{a}\times\vec{a}=\vec{0}$

(vi) $\hat{i}\times\hat{i}=\vec{0},\ \hat{j}\times\hat{j}=\vec{0},\ \hat{k}\times\hat{k}=\vec{0}$

(vii) $\hat{i}\times\hat{j}=\hat{k},\ \hat{j}\times\hat{k}=\hat{i},\ \hat{k}\times\hat{i}=\hat{j},$

$\hat{j}\times\hat{i}=-\hat{k},\ \hat{k}\times\hat{j}=-\hat{i},\ \hat{i}\times\hat{k}=-\hat{j}$

(viii) A unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ is given by $\hat{n}=\dfrac{(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$.

(ix) Area of a parallelogram with sides $\vec{a}$ and $\vec{b}$ $=|\vec{a}\times\vec{b}|$

(x) Area of a parallelogram with diagonals $\vec{d_1}$ and $\vec{d_2}=\dfrac{1}{2}|\vec{d_1}\times\vec{d_2}|$

(xi) Area of a $\Delta ABC=\dfrac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\dfrac{1}{2}|\overrightarrow{BC}\times\overrightarrow{BA}|=\dfrac{1}{2}|\overrightarrow{CB}\times\overrightarrow{CA}|$

Chapter 11 – Three-Dimensional Geometry

Direction Cosines and Direction Ratios

If a line makes angles α, β and γ with X-axis, Y-axis and Z-axis, respectively, then $l=\cos\alpha$, $m=\cos\beta$ and $n=\cos\gamma$ are called the **direction cosines** of the line.

The relation between direction cosines of a line is, $l^2+m^2+n^2=1$.

Numbers proportional to the direction cosines of a line are called the **direction ratios** of the line.

If a, b and c are the direction ratios of a line, then

$$l=\frac{\pm a}{\sqrt{a^2+b^2+c^2}},\quad m=\frac{\pm b}{\sqrt{a^2+b^2+c^2}}$$

and $n=\dfrac{\pm c}{\sqrt{a^2+b^2+c^2}}$

For any line, direction cosines are unique but direction ratios are not unique.

Direction Cosines and Direction Ratios of a Line

The direction cosines and direction ratios of the line segment joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are respectively given by

$$\frac{x_2-x_1}{PQ}, \frac{y_2-y_1}{PQ}, \frac{z_2-z_1}{PQ} \text{ and } (x_2-x_1, y_2-y_1, z_2-z_1)$$

where, $PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Line

A line (or straight line) is a curve such that all points on the line segment joining two points of it lies on it.

Equation of a Line

(i) Equation of a line through a given point and parallel to a given vector.

In vector form: Equation of a line through a given point A with position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is given by $\vec{r} = \vec{a}+\lambda\vec{b}$, where λ is a scalar.

In cartesian form:

The equation of a line passing through a point $A(x_1, y_1, z_1)$ and having direction ratios a, b and c is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

If l, m and n are the direction cosines of the line, the equation of the line is

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

(ii) Equation of a line passing through two given points

In vector form: The vector equation of a line passing through two points with position vectors $\vec{a}$ and $\vec{b}$ is given by $\vec{r} = \vec{a}+t(\vec{b}-\vec{a})$, where t is a scalar.

In cartesian form: The equation of a line passing through two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Angle between two lines

In vector form: If θ is the angle between two lines $\vec{r} = \vec{a}+t\vec{b}$ and $\vec{r} = \vec{c}+s\vec{d}$, then $\cos\theta = \dfrac{\vec{b}\cdot\vec{d}}{|\vec{b}||\vec{d}|}$.

If two lines are perpendicular, then $\vec{b}\cdot\vec{d} = 0$ and if two lines are parallel, then $\vec{b} = \lambda\vec{d}$.

In cartesian form: If a_1, b_1, c_1 and a_2, b_2, c_2 are direction ratios of two lines respectively, then the angle θ between the lines is given by $\cos\theta = \left|\dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

If θ is the angle between two lines with direction cosines l_1, m_1, n_1 and l_2, m_2, n_2, then $\cos\theta = l_1l_2+m_1m_2+n_1n_2$.

If lines are perpendicular, then $a_1a_2+b_1b_2+c_1c_2 = 0$,

If lines are parallel, then $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$,

Skew-lines

If two lines do not meet and not parallel, then they are known as skew-lines.

Shortest distance between two lines

The shortest distance between two skew-lines is the length of perpendicular to both the lines.

In vector form: Shortest distance between the skew-lines $\vec{r} = \vec{a}_1+\lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2+\lambda\vec{b}_2$ is given by

$$SD = \frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}$$

The shortest distance between the parallel lines $\vec{r} = \vec{a}_1+\lambda\vec{b}$ and $\vec{r} = \vec{a}_2+\mu\vec{b}$ is given by

$$SD = \frac{|\vec{b}\times(\vec{a}_2-\vec{a}_1)|}{|\vec{b}|}$$

In cartesian form: The shortest distance between the lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

and $\dfrac{x-x_2}{a_2} = \dfrac{y-y_2}{b_2} = \dfrac{z-z_2}{c_2}$ is

$$\left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}\right|$$

Plane

A plane is a surface such that a line segment joining any two points on it lies wholly on it. A straight line which

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perpendicular to every line lying on a plane is called a normal to the plane.

Equation of Plane

(i) Equation of a plane in Normal form

In vector form: Let O be the origin and $\hat{n}$ be a unit vector in the direction of the normal ON to the plane and let $ON = p$. Then, the equation of the plane is $\vec{r}\cdot\hat{n} = p$.

In cartesian form: The general equation of a plane is $ax + by + cz + d = 0$.

The direction ratios of the normal to this plane are a, b and c.

(ii) Intercept form of the equation of a plane: If a plane cuts (intercepts) a, b and c with the coordinate axes, then the equation of the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$

(iii) Equation of a plane perpendicular to a given vector and passing through a given point

In vector form: The equation of a plane passing through the point $\vec{a}$ and perpendicular to the given vector $\vec{n}$ is $(\vec{r}-\vec{a})\cdot\vec{n} = 0$

In cartesian form: The equation of a plane passing through a point (x_1, y_1, z_1) is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

where, a, b and c are direction ratios of perpendicular vector.

(iv) Equation of a plane passing through three non-collinear points

In vector form: The equation of plane passing through three non-collinear points $\vec{a}$, $\vec{b}$ and $\vec{c}$ is

$(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0.$

In cartesian form: The equation of a plane passing through three non-collinear points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C\,(x_3, y_3, z_3)$ is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

(v) Equation of a plane passing through the intersection of two planes

In vector form: The equation of plane passing through the intersection of two planes $\vec{r}\cdot\vec{n}_1 = d_1$ and $\vec{r}\cdot\vec{n}_2 = d_2$ is $\vec{r}\cdot(\vec{n}_1+\lambda\vec{n}_2)$ $= d_1+\lambda d_2$.

In cartesian form: The equation of a plane through the intersection of the planes

$$a_1x+b_1y+c_1z+d_1=0$$

and $$a_2x+b_2y+c_2z+d_2=0 \text{ is}$$

$$(a_1x+b_1y+c_1z+d_1)+\lambda(a_2x+b_2y+c_2z+d_2)=0$$

(vi) Equation of a plane parallel to a given plane

In vector form: The vector equation of a plane parallel to the given plane $\vec{r}\cdot\vec{n}=d_1$ is $\vec{r}\cdot\vec{n}=d_2$.

In cartesian form: The cartesian equation of a plane parallel to the given plane $ax + by + cz + d_1 = 0$ is $ax + by + cz + d_2 = 0$.

Angle between two planes

In vector form: The angle between two planes is the angle between their normals.

Angle between the planes $\vec{r}\cdot\vec{n}_1 = p_1$ and $\vec{r}\cdot\vec{n}_2 = p_2$ is given by $\cos\theta=\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$.

In cartesian form: If θ is the angle between the planes

$$a_1x + b_1y + c_1z + d_1 = 0$$

and $$a_2x + b_2y + c_2z + d_2 = 0, \text{ then}$$

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

If two planes are perpendicular, then

$$\vec{n}_1\cdot\vec{n}_2=0 \text{ or } a_1a_2+b_1b_2+c_1c_2=0$$

and if they are parallel, then $\vec{n}_1\times\vec{n}_2=0$ or $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$.

Coplanarity of Two Lines

In cartesian form: Suppose, vector equations of two lines are $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$. Then, these lines are coplanar, iff $(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2) = 0$.

Two lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$

are coplanar, if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

The equation of plane containing the above lines is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

In vector form: Two lines $\vec{r} = \vec{a}_1+\lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2+\lambda\vec{b}_2$ are coplanar, if $(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2) = 0$.

Distance of a point from a plane

In cartesian form: Distance of a point $P(x_1, y_1, z_1)$ from a plane $ax + by + cz + d = 0$ is given by

$$p = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

In vector form: The perpendicular distance of a point $\vec{a}$ from the plane $\vec{r}\cdot\vec{n} = d$, where $\vec{n}$ is normal to the plane, is

$$\frac{|\vec{a}\cdot\vec{n}-d|}{|\vec{n}|}$$

The length of the perpendicular from origin O to the plane $\vec{r}\cdot\vec{n} = d$ is $\frac{|d|}{|\vec{n}|}$.

Angle between a line and a plane

The angle ϕ between line $\vec{r} = \vec{a}+\lambda\vec{b}$ and plane $\vec{r}\cdot\vec{n} = d$ is $\cos\theta = \left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}\right|$ and $\sin\phi = \left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}\cdot|\vec{n}|}\right|$, where $\phi = 90° - \theta$.

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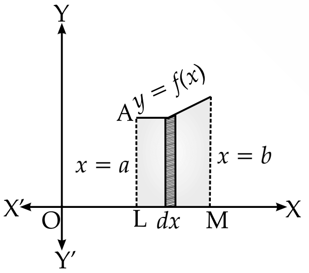
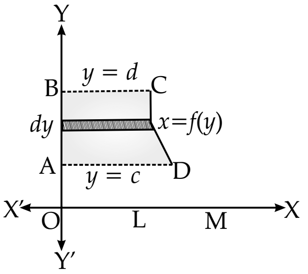
Chapter 12 – Application of Integrals

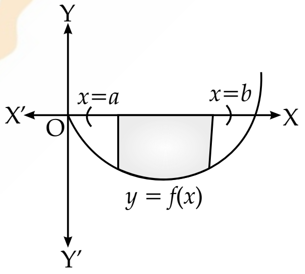
(i) The area enclosed by the curve $y = f(x)$, the X-axis and the lines at $x = a$ and $x = b$, is given by $\int_a^b y\,dx$.



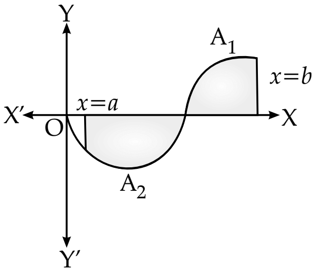
(ii) The area enclosed by the curve $x = f(y)$, the Y-axis and the lines at $y = c$ and $y = d$, is given by $\int_c^d x\,dy$.



(iii) If the curve $y = f(x)$ lies below the X-axis, then area bounded by the curve $y = f(x)$, X-axis and the lines at $x = a$ and $x = b$, is given by $\left|\int_a^b y dx\right|$.

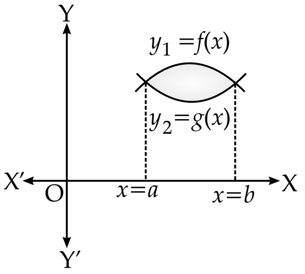


(iv) Generally, it may happen that some position of the curve is above X-axis and some is below the X-axis which is shown in the figure. The area A bounded by the curve $y = f(x)$, X-axis and the lines at $x = a$ and $x = b$, is given by $A = |A_2| + A_1$.

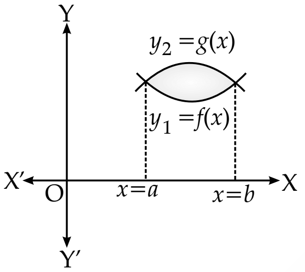


(v) Suppose, two curves are given by $y_1 = f(x)$ and $y_2 = g(x)$ and their points of intersections are given by $x = a$ and $x = b$.

(a) Suppose, $f(x) \geq g(x)$ in $[a, b]$, then $A = \int_a^b [f(x)-g(x)]dx$.

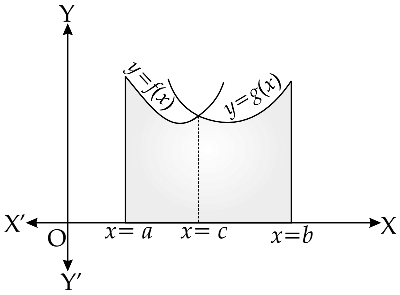


(b) Suppose, $f(x) \geq g(x)$ in $[a, c]$ and $f(x) \leq g(x)$ in $[c, b]$, then $A = \int_a^c [f(x) - g(x)]dx + \int_c^b [g(x) - f(x)]dx$.



(vi) Area bounded by the two curves, $y = f(x)$ and $y = g(x)$ between the lines at $x = a$ and $x = b$, is given by

$\int_a^c f(x)dx + \int_c^b g(x)dx$, where c is the intersection point between $x = a$ and $x = b$.



Chapter 13 – Application of Calculus

- **Cost function :**

  If x denotes the quantity produced of a certain commodity at total cost c, then the total cost function is given as $C = c(x)$, explicitly and $f(c, x) = 0$ is the implicit form. The total cost is divided into two parts, the fixed cost and the variable cost.

(1) Fixed Cost :

Fixed costs are those which incurred regardless of the level of production - like interest, rent, wages of permanent staff etc. Thus, Total Fixed Cost
$(TFC) = TC$, when $x = 0$
Fixed cost does not change whether there is any increase or decrease in level of production.

(2) Variable Cost :

Variable costs are those which vary with output.

e.g., Raw materials and wages of casual labour etc. Thus,

Total Cost = Total Fixed Cost + Total Variable Cost

or $TC = TFC + TVC$

(3) Average Cost :

The average cost represents the cost per unit, i.e.,

$$AC = \frac{C}{x} = \frac{\text{Total cost}}{\text{Number of commodities}}$$

(4) Demand Function :

The demand function is the functional relationship between demand and price of a commodity. If P denotes the price per unit and x is the number of units demanded by a consumer at that price, then demand function in explicit form will be written as $x = f(P)$ and in implicit form as $f(x, P) = 0$.

(5) Revenue Function :

Revenue is the amount received by a company on selling a certain number of units of a commodity. Let p be the price per unit and x be the number of units sold. Then, total revenue

$$\boxed{R \text{ or } R(x) = p \times x}$$

Total revenue = Selling price per unit of the commodity × Quantity sold

(6) Profit Function :

Profit function is the difference of revenue function and cost function, i.e.,

$$\boxed{P(x) = R(x) - C(x)}$$

(7) Break even Point :

The break even point is that point, where total revenue equals to the total cost incurred, i.e., $P(x) = 0$ or $R(x) = C(x)$. At break even point, a company begins to earn profit.

(8) Marginal Cost :

The marginal cost is the rate of change of the total cost with respect to x (Output).

$$\boxed{MC = \frac{d}{dx}(C) = \frac{dC}{dx},\ x > 0}$$

(9) Relation between Average Cost (AC) and Marginal Cost (MC) :

If C is the total cost of producing and marketing x units of commodity, then

$$\boxed{\frac{d}{dx}(AC) = \frac{1}{x}(MC - AC)}$$

Here, three cases arise:

- For $MC > AC \Rightarrow AC$ increases with x and AC curve is rising.
- For $MC = AC \Rightarrow AC$ is constant at all levels of output.
- For $MC < AC \Rightarrow AC$ decreases with x and AC curve is falling.

(10) Average Revenue and Marginal Revenue :

(i) Average Revenue is the revenue generated per unit of output sold. It is denoted by AR, i.e.,

$$AR = \frac{\text{Total revenue}}{\text{Number of units sold}} = \frac{R}{x} = \frac{p \cdot x}{x} = p$$

(ii) Marginal Revenue is the rate of change of total revenue with respect to quantity sold

i.e., $$\boxed{MR = \frac{dR}{dx} = \frac{d}{dx}(px)}$$

(11) Marginal Average Cost is called marginal average cost $\frac{d}{dx}(AC)$ may be denoted as (MAC). This is also called as slope of average cost curve.

➢ **Minimisation of AC and MC Functions :**

Using the concept of maxima and minima we can determine the level of output where per unit cost is minimum corresponding to a given total or average cost function. Following steps are used to find the optimal level of output:

Step 1:	From AC, determine $\frac{d}{dx}(AC)$.
Step 2:	Let $\frac{d}{dx}(AC) = 0$ and solve for x.
Step 3:	Find $\frac{d^2}{dx^2}(AC)$.
Step 4:	The value of x for which $\frac{d^2}{dx^2}(AC) > 0$ is the minimum per unit cost.
Step 5:	Minimum average cost can be determined by substituting this value of x in $AC(x)$. Similarly, we can find the minimum marginal cost or minimum cost.

➢ **Maximization of Total Revenue :**

It is possible to determine the level of output at which the total revenue is maximum, if demand function is given. Total revenue, $R(x)$, is the maximum when marginal revenue is zero.

To maximise total revenue, following steps are to be followed:

Step 1:	Determine $\frac{dR}{dx}$.
Step 2:	Let $\frac{dR}{dx} = 0$ and solve for x.
Step 3:	Determine $\frac{d^2R}{dx^2}$.
Step 4:	The value for which $\frac{d^2R}{dx^2} < 0$ gives the maximum total revenue.
Step 5:	The maximum revenue can be obtained by putting this value of x in $R(x)$. Similarly, maximum profit can be determined.

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Chapter 14 – Linear Regression

Linear Regression

The statistical method which help us in estimating or predicting the value of one variable for a given value of the related variable is called **regression**.

Let given bivariate data be $(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_n, y_n)$.

We can plot a scatter diagram and visualise a smooth curve approximating the data. This is called **curve filling**.

Line of best fit

The straight line having the property such that $d_1^2 + d_2^2 + \ldots + d_n^2$ is minimum is called the **line of best fit**. Using a bit of calculus, it can be shown that the least squares line approximating the set of points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ has equation

$$y = a + bx \qquad \ldots\text{(i)}$$

where the constants a and b are determined by solving simultaneously the equations

$$\Sigma y = an + b\Sigma x \qquad \ldots\text{(ii)}$$

$$\text{and } \Sigma xy = a\Sigma x + b\Sigma x^2 \qquad \ldots\text{(iii)}$$

which are called the normal equations for the least squares lines.

Solving eq. (ii) and (iii) simultaneously for a and b, we get

$$b = \frac{n\,\Sigma xy - (\Sigma x)\,(\Sigma y)}{n\,\Sigma x^2 - (\Sigma x)^2} \qquad \ldots\text{(iv)}$$

Now, eq. (ii) yields $\bar{y} = \frac{\Sigma y}{n} = a + b\frac{\Sigma x}{n} = a + b\bar{x}$,

So, $y - \bar{y} = (a + bx) - (a + b\bar{x}) = b(x - \bar{x})$

$$\therefore \quad y - \bar{y} = b(x - \bar{x}) \qquad \ldots\text{(v)}$$

Note: *b is the slope of line (v)* i.e., *of the regression line of y on x. It is also called the regression coefficient of y on x, denoted by b_{yx}. Thus, the line of best fit* i.e., *the regression line of y on x.*

$$y-\overline{y}=b_{yx}\ (x-\overline{x})$$

Similarly, the regression line of x on y is

$$x-\overline{x}=b_{xy}\ (y-\overline{y})$$

where, b_{xy} is the regression coefficient of x on y
The lines of best fit are called regression lines.

Regression Coefficients

$$b_{yx}=r\cdot\frac{\sigma_y}{\sigma_x}\text{ and }b_{xy}=r\cdot\frac{\sigma_x}{\sigma_y}$$

where, r is the coefficient of correlation and σ_x and σ_y are the standard deviations of y-variate and x-variate, respectively.

Some properties of lines of regression

(1) b_{yx}, b_{xy} and r or $f(x,y)$ are of the same sign

(2) $|r|$ is the geometric mean of b_{yx} and b_{xy}

(3) $0\le b_{yx}\cdot b_{xy}\le 1$

Note: $b_{yx}\cdot b_{xy}=r^2$ and $0\le r^2\le 1$

(4) If one of b_{yx} and b_{xy} is numerically greater than one, the other is numerically smaller than one.

(5) Two regression lines intersect at $(\overline{x},\ \overline{y})$

(6) The two regression lines coincide if and only if there is perfect linear relation between x and y i.e., if and only if $f(x,y)=\pm 1$

(7) If $r=0$, then the lines of regression are parallel to the coordinate axes.

Chapter 15 – Linear Programming

A linear programming problem is one that is concerned with finding the optimal value (maximum or minimum value) of a linear function (called **objective function**) of several variables (say x and y called **decision variable**), subject to the conditions that the variables are non-negative and satisfy a set of linear inequalities (called **linear constraints**).

Some Terms Related to LPP

(i) Constraints: The linear inequations or inequalities or restrictions on the variables of a linear programming problem are called **constraints**. The conditions $x\ge 0, y\ge 0$ are called non-negative restrictions.

(ii) Optimization Problem: A problem which seeks to maximise or minimise a linear function subject to certain constraints determined by a set of linear inequalities is called an **optimisation problem**. Linear programming problems are special type of **optimization problems**.

(iii) Objective Function: A linear function of two or more variables which has to be maximised or minimised under the given restrictions in the form of linear inequations (or **linear constraints**) is called the **objective function.** The variables used in the objective function are called **decision variables**.

(iv) Optimal Values: The maximum or minimum value of an objective function is known as its **optimal value**.

(v) Feasible Solution: Any solution to the given linear programming problem which also satisfies the non-negative restrictions of the problem is called a **feasible solution**. Any point outside the feasible region is called an **infeasible solution**.

(vi) Feasible Region: The common region determined by all the constraints including non-negative constraints x, y ≥ 0 of a linear programming problem is called the feasible region on solution region. Each point is this region represents a feasible choice. The region other than feasible region is called an infeasible region.

(vii) Bounded Region: A feasible region of a system of linear inequalities is said to be **bounded**, if it can be enclosed within a circle. Otherwise, it is said to be **unbounded region**.

(viii) Optimal Solution: A feasible solution at which the objective function has optimal value is called the optimal solution of the linear programming problem.

(ix) Optimization Technique: The process of obtaining the optimal solution is called **optimisation technique.**

Corner Point Method

It is a graphical method to solve the LPP.
The following steps are given below

Step I Find the feasible region of the linear programming problem and determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point.

Step II Evaluate the objective function $Z=ax+by$ at each corner point. Let M and m respectively denote the largest and smallest values of these points.

Step III When the feasible region is **bounded,** M and m are the maximum and minimum values of Z.

Step IV When, the feasible region is **unbounded**, then

(a) M is the maximum value of Z, if the open half plane determined by $ax+by>M$ has no point in common with the feasible region.

Otherwise, Z has no maximum value.

(b) Similarly, m is the minimum value of Z, if the open half plane determined by $ax + by < m$ has no point in common with the feasible region. Otherwise, Z has no minimum value.

■■■

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You never know what might be asked in the exam.
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Self Assessment Paper-1

MATHEMATICS

Class-12

UNSOLVED

Time Allowed : 3 hours **Maximum Marks : 80**

You are allowed an additional 15 minutes for only reading the paper.
You must NOT start writing during reading time.
The question paper has 12 printed pages.
The Question Paper is divided into three sections and has 22 questions in all.

Section A is compulsory and has fourteen questions.
You are required to attempt all questions either from Section B or Section C.
Section B and Section C have four questions each.
Internal choices have been provided in two questions of 2 marks, two questions of 4 marks and two questions of 6 marks in Section A.
Internal choices have been provided in one question of 2 marks and one question of 4 marks each in Section B and Section C.
While attempting Multiple Choice Questions in Section A, B and C, you are required to write only ONE option as the answer.
The intended marks for questions or parts of questions are given in the brackets [].
All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
Mathematical tables and graph papers are provided.
Instruction to Supervising Examiner:
Kindly read aloud the instructions given above to all the candidates present in the examination hall.

Section-A [65 marks]

1. In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed. **[15]**

(i) Which of the following relation describes the solution of system of equation in form of $AX = B$?

(a) $A^{-1}AX = A^{-1}B$ **(b)** $B^{-1}AX = A^{-1}B$ **(c)** $AB^{-1}AX = AB^{-1}X$ **(d)** $X^{-1}AB = A^{-1}BX$ **[Understanding]**

(ii) $\int \dfrac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta}$ is equal to

(a) $2(\sin x + x\cos\theta) + C$ **(b)** $2(\sin x - x\cos\theta) + C$
(c) $2(\sin x + 2x\cos\theta) + C$ **(d)** $2(\sin x - 2x\cos\theta) + C$ **[Recall]**

(iii) If $\sin^{-1}x + \sin^{-1}y = \dfrac{\pi}{2}$, then the value of $\cos^{-1}x + \cos^{-1}y$ is

(a) $\dfrac{\pi}{2}$ **(b)** π **(c)** 0 **(d)** $\dfrac{2\pi}{3}$ AI **[Application]**

(iv) Assertion (A): The degree of the differential equation $\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \dfrac{d^2y}{dx^2}$ is 2.

Reason (R): Degree of the differential equation is the power of the highest order derivative.
Which of the following is correct?

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true. **[Analysis]**

(v) If A and B are two events such that $P(A) \neq 0$ and $P(B \mid A) = 1$, then

(a) $A \subset B$ **(b)** $B \subset A$ **(c)** $B = \phi$ **(d)** $A = \phi$ **[Application]**

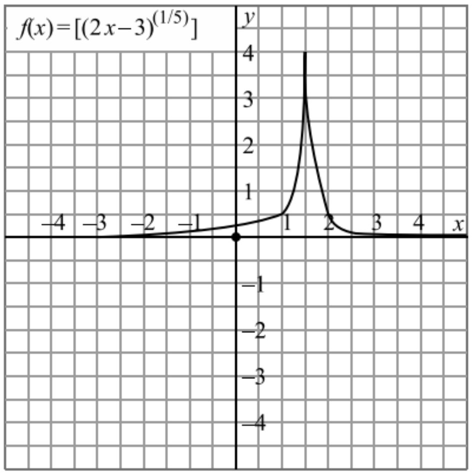
(vi) Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & x & y \\ 1 & x & -y \\ 1 & -x^2 & y^2 \end{bmatrix}$$

(a) $(x+1)$ **(b)** $-2xy(x+1)$ **(c)** $xy(x+1)$ **(d)** $2xy(x+1)$

[Understanding]

(vii) Shown below is the graph of a function $f(x)$.



Statement 1: $f(x)$ is continuous and differentiable at $x=\frac{3}{2}$.

Statement 2: $f(x)$ is continuous but not differentiable at $x=\frac{3}{2}$

Which of the following is correct?

(a) Statement 1 is true and Statement 2 is false. **(b)** Statement 2 is true and Statement 1 is false.
(c) Both the statements are true. **(d)** Both the statements are false. A I **[Application]**

(viii) If $y=\sqrt{x+\sqrt{x+\sqrt{x+\ldots\infty}}}$, then $\frac{dy}{dx}$ is equal to

(a) $2y-1$ **(b)** $2y+1$ **(c)** $\frac{1}{2y-1}$ **(d)** $\frac{1}{2y+1}$

(ix) Statement 1: An important property of an equivalence relation is that it divides the set into pair-wise disjoint subsets called equivalence classes whose collection is called a partition of the set.
Statement 2: Let R be a relation from A to B. Then B is called the co-domain of the relation R.
Which one of the following is correct?
(a) Statement 1 implies Statement 2. **(b)** Statement 2 implies Statement 1.
(c) Statement 1 is true only if Statement 2 is true. **(d)** Statement 1 and 2 are independent of each other.
[Understanding]

(x) Given A is an invertible square matrix and B is an invertible symmetric matrix, then
Assertion (A): A^T is invertible.
Reason (R): Inverse of B is a symmetric matrix.
Which of the following is correct?
(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true. **[Application]**

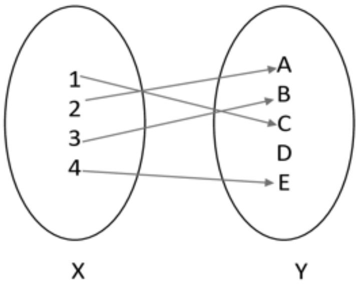
(xi) Two cards are drawn at random from a pack of 52 cards one-by-one without replacement. The probability of getting first card red and second card jack is

(a) $\frac{1}{26}$ **(b)** $\frac{1}{13}$ **(c)** $\frac{3}{13}$ **(d)** $\frac{3}{26}$ **[Understanding]**

(xii) If matrix $A=\left[a_{ij}\right]_{2\times 2}$, where $a_{ij}=1$ if $i\neq j$, and 0 if $i=j$ then find A^2. **[Analysis]**

(xiii) Consider the mapping $f:A\rightarrow B$ is defined by $f(x)=\frac{x-1}{x+1}$, such that $f(x)$ is one-one onto. Then find the range of $f(x)$. A I **[Understanding]**

(xiv) Is the given function is one-one? Give reason. **[Understanding]**



(xv) A bag contains 6 balls of which two are red and four are black. Two balls are drawn at random. What is the probability that they are of the different colours? **[Application]**

2. (i) If $y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$, then prove that $\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$.
[Understanding] [2]

OR

(ii) Show that the function *f* defined by $f(x)=(x-1)e^x+1$ is an increasing function for all $x>0$. **[Understanding] [2]**

3. Evaluate: $\int_{-1}^{2}\frac{|x|}{x}dx$ AI **[Evaluate] [2]**

4. Find the equation of normal to the curve $y=\tan x$ at (0, 0). **[Analysis] [2]**

5. (i) Find $\int\frac{\sin 2x}{\sqrt{9-\cos^4 x}}dx$. **[Evaluate] [2]**

OR

(ii) Find: $\int\frac{3+3\cos x}{x+\sin x}dx$ **[Evaluate]**

6. Express $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$; where $-\frac{\pi}{4}<x<\frac{\pi}{4}$ in the simplest form. **[Application] [2]**

7. Prove that $\cos\left[\tan^{-1}\left\{\cot\left(\sin^{-1}x\right)\right\}\right]=x$. **[Application] [4]**

8. Find: $\int\sqrt{\tan x}dx$ AI **[Application] [4]**

9. (i) If the rate of change of radius of a circle is 6 cm/s, then find the rate of change of area of the circle, when $r = 3$. **[Application] [4]**

OR

(ii) If $y = x\log\left(\frac{x}{a+bx}\right)$, then prove that: $\frac{x^3d^2y}{dx^2}=\left[x\frac{dy}{dx}-y\right]^2$ **[Analysis] [4]**

10. (i) A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%,60% and 35%, respectively.

(a) Calculate the probability that a randomly chosen seed will germinate.

(b) Calculate the probability that the seed is of type A_2, given that a randomly chosen seed germinates.

(c) A die is thrown and a card is selected at random from a deck of 52 playing cards. Then find the probability of getting an even number on the die and a spade card. **[Analysis] [4]**

OR

(ii) Two horses are considered for a race. The probability of selection of the first horse is $\frac{1}{4}$ and that of the second is $\frac{1}{3}$. What is the probability that :

(a) both of them will be selected

(b) only one of them will be selected

(c) none of them will be selected **[Analysis] [4]**

11. Word problems are mathematical problems that are delivered in ordinary words, instead of mathematical symbols. One of the challenges in dealing with word problems is that they first need to be translated into mathematical equations, which then need to be solved. Here is a word problem on the topic of matrices and determinants:

Divide ₹30,000 in two parts such that the total annual interest at 9% on the first part and 11% on the second part amounts ₹3060.

Answer the following question:

(a) Translate the problem into a system of equations.

(b) Solve the system of equation by using matrix method.

(c) Hence, find the amount of two parts. **[Application] [6]**

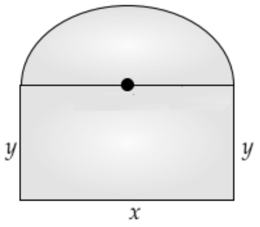
12. (i) Solve the differential equation:

$(y+\log x)dx-xdy=0$, given that $y=0$, when $x=1$. **[Application] [6]**

OR

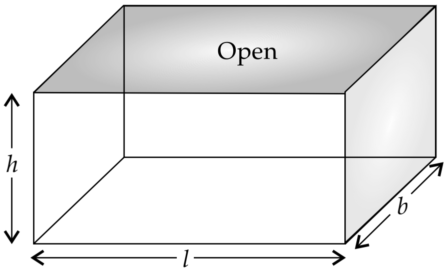
(ii) Evaluate: $\int(5x-1)\sqrt{6+5x-2x^2}\,dx$ **[Evaluate] [6]**

13. (i) A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the windows to admit maximum light through the whole opening. **[Application] [6]**

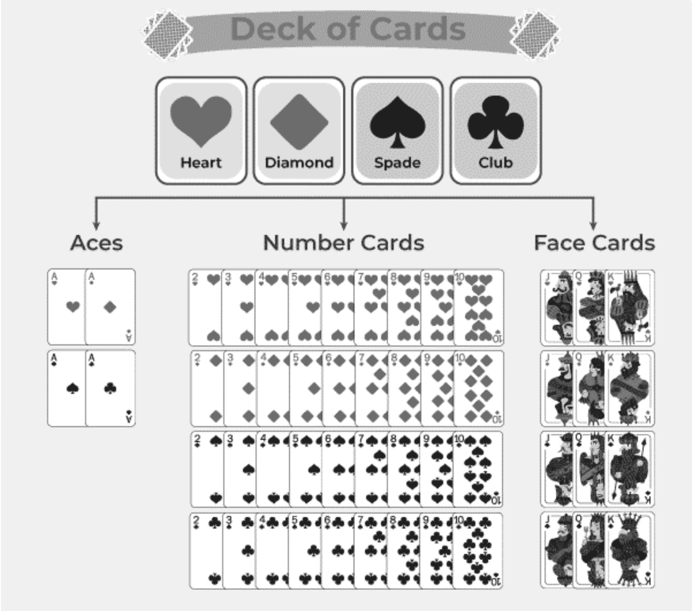


OR

(ii) A tank with rectangular base and rectangular sides open at the top is to be constructed so that its depth is 3 m and volume is 75 m^3. If building of the tank costs ₹100 per square metre for the base and ₹50 per square metre for the sides, find the cost of least expensive tank. AI **[Application] [6]**



14. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of diamond cards drawn. Also, find the mean of the distribution.



[Analysis] [6]

Section-B **[15 marks]**

15. In subparts (i) and (iii) choose the correct options and in subparts (ii) to (v), answer the questions as instructed. **[5]**

(i) Consider the following statements and choose the correct option:

Statement 1: The vector having initial and terminal points as (2, 5, 0) and (–3, 7, 4), respectively is $-\hat{i}+12\hat{j}+4\hat{k}$

Statement 2: The vector having initial and terminal points as (2, 5, 0) and (–3, 7, 4), respectively is $-5\hat{i}+2\hat{j}+4\hat{k}$

Which of the following is correct? **[Recall]**

(a) Only Statement 1 **(b)** Only Statement 2
(c) Both Statements 1 and 2 **(d)** Neither Statement 1 nor Statement 2

(ii) If the direction ratios of a line are 2, 3 and –6, then direction cosines of the line making obtuse angle with Y-axis are

(a) $\frac{2}{7},\frac{3}{7},\frac{6}{7}$ **(b)** $\frac{-2}{7},\frac{-3}{7},\frac{6}{7}$ **(c)** $\frac{-2}{7},\frac{3}{7},\frac{-6}{7}$ **(d)** $\frac{-2}{7},\frac{-3}{7},\frac{-6}{7}$

[Application]

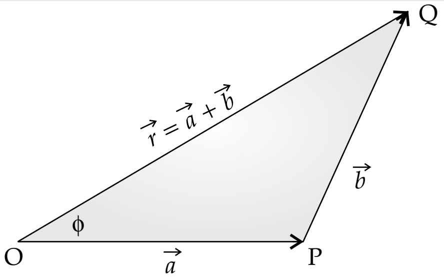
(iii) The length of the perpendicular from the point (3, –2, 1) to the plane $x-2y+2z=3$ is

(a) $\frac{2}{3}$ units **(b)** 3 units **(c)** 2 units **(d)** none of these

AI **[Understanding]**

(iv) If the scalar projection of $\vec{a}=\lambda\hat{i}+\hat{j}+4\hat{k}$ on $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units, then find value of λ. **[Understanding]**

(v) Write a unit vector in the direction of the sum of vectors $\vec{a}=2\vec{i}+2\vec{j}-5\vec{k}$ and $\vec{b}=2\vec{i}+\vec{j}-7\vec{k}$. **[Analysis]**



16. **(i)** Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$, then $c_1=1$ and $c_2=2$, find c_3 which makes $\vec{a},\vec{b}$ and $\vec{c}$ coplanar. **[Application] [2]**

OR

(ii) For any vector $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+p\hat{j}+q\hat{k})=\vec{0}$. Then find the values of p and q. **[Application] [2]**

17. **(i)** Find the equation of a line passing through the points P(–1, 3, 2) and Q(–4, 2, –2). Also, if the point $R(5,5,\lambda)$ is collinear with the points P and Q, then find the value of λ. **[Application] [4]**

OR

(ii) Find the equation of the plane through the point (–1, –1, 2) and perpendicular to the planes $3x+2y-3z=1$ and $5x-4y+z=5$. **[Application] [4]**

18. **(i)** Sketch the region bounded by $y=-1, y=2, x=y^3$ and $x=0$.

(ii) Evaluate: $\int_0^2 y^3\,dy$

(iii) Hence, find the area of the region bounded by $y=-1, y=2, x=y^3$ and $x=0$. **[4]**

Section-C **[15 marks]**

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed. **[5]**

(i) A company sells its product at ₹100 per unit. Fixed cost for the company is ₹35,000 and variable costs are estimated to run 30% of total revenue. The total cost function is

(a) $35000+100x$ **(b)** $35000+30x$ **(c)** $35000+3x$ **(d)** None of these **[Recall]**

(ii) Read the following statements and choose the correct option:

If the two lines of regression are $3x-2y+1=0$ and $2x-y-2=0$, then

(I) $\bar{x}$ is 5 and $\bar{y}$ is 8. **(II)** $\bar{x}$ is 8 and $\bar{y}$ is 5.

(III) $\bar{x}$ is 5 and $\bar{y}$ is 5. **(IV)** $\bar{x}$ is 0 and $\bar{y}$ is 8. **[Recall]**

Which of the following is correct?

(a) Only I is correct **(b)** Only I and III are correct
(c) Only I and IV are correct **(d)** Only III and IV are correct

(iii) Two lines of regression are $4x-2y=0$ and $3x-6y+5=0$. Find the correlation coefficient between x and y. **[Understanding]**

(iv) The demand function is $x=\frac{24-2p}{3}$, where x is the number of units demanded and p is the price per unit. Find the revenue function R in terms of p. **[Understanding]**

(v) A company practices a commodity with ₹ 24,000 as fixed cost. The variable cost estimated to be 25% of the total revenue received on selling the product is at the rate of ₹ 8 per unit. Find the break-even point. **[Understanding]**

20. **(i)** The average cost function associated with producing and marketing x units of an item is given by $AC=2x-11+\frac{50}{x}$. Find the range of values of the output x, for which AC is increasing. **[Application] [2]**

OR

(ii) The revenue function is given by $R(x)=100x-x^2-x^3$. Find

(a) The demand function. **(b)** Marginal revenue function. **[Application] [2]**

21. **(i)** The following results were obtained with respect to two variables x and y.

$$\Sigma x=15, \Sigma y=25, \Sigma xy=83, \Sigma x^2=55, \Sigma y^2=135 \text{ and } n=5$$

(a) Find the regression coefficient b_{xy} .

(b) Find the regression equation of x on y. **[Application] [4]**

OR

(ii) Find the equation of the regression line of y on x, if the observations (x, y) are as follows:
(1, 4), (2, 8), (3, 2), (4, 12), (5, 10), (6, 14), (7, 16), (8, 6), (9, 18)
Also, find the estimated value of y when $x = 14$. **[Application] [4]**

22. Aman has ₹1,500 to purchase rice and wheat for his grocery shop. Each sack of rice and wheat costs ₹180 and ₹120, respectively. He can store a maximum number of 10 bags in his shop. He will earn a profit of ₹11 per bag of rice and ₹9 per bag of wheat.

(a) Formulate a Linear Programming Problem to maximise Aman's profit.

(b) Calculate the maximum profit. AI **[Analysis] [4]**

■■

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SOLUTIONS

Self Assessment Paper - 1

Section-A

1. **(i) Option (d) is correct.**

Explanation: We have, $A^{-1}AX = A^{-1}B$ which is same as $X = A^{-1}B$, and this is the primary relation employed for finding inverse.

(ii) Option (a) is correct.

Explanation: Let

$$\begin{aligned} I &= \int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta}\,dx \\ &= \int \frac{(2\cos^2 x - 1 - 2\cos^2\theta + 1)}{\cos x - \cos\theta}\,dx \\ &= 2\int \frac{(\cos x + \cos\theta)(\cos x - \cos\theta)}{(\cos x - \cos\theta)}\,dx \\ &= 2\int (\cos x + \cos\theta)\,dx \\ &= 2(\sin x + x\cos\theta) + C \end{aligned}$$

(iii) Option (a) is correct.

Explanation: Given, $\sin^{-1} x + \sin^{-1} y = \dfrac{\pi}{2}$

$$\left[\left(\frac{\pi}{2}\right) - \cos^{-1} x\right] + \left[\left(\frac{\pi}{2}\right) - \cos^{-1} y\right] = \frac{\pi}{2}$$

$$\left(\frac{\pi}{2}\right) + \left(\frac{\pi}{2}\right) - \left(\frac{\pi}{2}\right) = \cos^{-1} x + \cos^{-1} y$$

Therefore, $\cos^{-1} x + \cos^{-1} y = \dfrac{\pi}{2}$.

(iv) Option (a) is correct.

Explanation:

Given, $$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$$

On squaring both sides, we get

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

So, the degree of differential equation is 2.

(v) Option (a) is correct.

Explanation: $P(A) \neq 0$

and $P(B \mid A) = 1$

$$\begin{aligned} P(B \mid A) &= \frac{P(B \cap A)}{P(A)} \\ 1 &= \frac{P(B \cap A)}{P(A)} \\ P(A) &= P(B \cap A) \\ \therefore \quad A &\subset B \end{aligned}$$

(vi) Option (b) is correct

Explanation: Given that,

$$A = \begin{bmatrix} 1 & x & y \\ 1 & x & -y \\ 1 & -x^2 & y^2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & x & y \\ 1 & x & -y \\ 1 & -x^2 & y^2 \end{vmatrix}$$

Taking x common from C_2 and y common from C_3, we get

$$|A| = xy\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -x & y \end{vmatrix}$$

Expanding along R_1, we get

$$\begin{aligned} |A| &= xy\{1(y - x) - 1(y + 1) + 1(-x - 1)\} \\ &= xy(y - x - y - 1 - x - 1) \\ &= xy(-2x - 2) \\ &= -2xy(x + 1). \end{aligned}$$

(vii) Option (b) is correct.

Explanation: For checking the continuity, we need to check the left hand and right hand limits and the value of the function at a point $x = a$.

$$\begin{aligned} \text{LHL} &= \lim_{x \to a^-} f(x) = \lim_{x \to \frac{3}{2}} (2x - 3)^{\frac{1}{5}} \\ &= \left(2 \times \frac{3}{2} - 3\right)^{\frac{1}{5}} \\ &= 0 \\ \text{RHL} &= \lim_{x \to a^+} f(x) = \lim_{x \to \frac{3}{2}} (2x - 3)^{\frac{1}{5}} \\ &= \left(2 \times \frac{3}{2} - 3\right)^{\frac{1}{5}} \\ &= 0 \\ \text{LHL} &= \text{RHL} = f(a) = 0 \end{aligned}$$

Thus, the function is continuous at about the point $x = \dfrac{3}{2}$.

Now, to check differentiability at the given point, we know

$$\begin{aligned} f'(a) &= \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \\ &= \lim_{h \to 0} \frac{f\left(\frac{3}{2} + h\right) - f\left(\frac{3}{2}\right)}{h} \\ &= \lim_{h \to 0} \frac{\left[2\left(\frac{3}{2} + h\right) - 3\right]^{\frac{1}{5}} - \left(2\left(\frac{3}{2}\right) - 3\right)^{\frac{1}{5}}}{h} \end{aligned}$$

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$$= \lim_{h\to 0}\frac{(3+2h-3)^{\frac{1}{5}}-(3-3)^{\frac{1}{5}}}{h}$$

$$= \lim_{h\to 0}\frac{(2h)^{\frac{1}{5}}-0}{h}$$

$$= \lim_{h\to 0}\frac{2^{\frac{1}{5}}}{h^{\frac{4}{5}}} = \infty$$

Thus, f is not differentiable at $x=\frac{3}{2}$.

(viii) Option (c) is correct.

Explanation:

$$y = \sqrt{x+\sqrt{x+\sqrt{x+\ldots\infty}}}$$

$$y = \sqrt{x+y}$$

Squaring both sides, we get

$$y^2 = x + y$$

$$y^2 - y = x.$$

$$\frac{d}{dx}\left(y^2-y\right)=\frac{d}{dx}(x)$$

$$2y\frac{dy}{dx}-\frac{dy}{dx}=1$$

$$\frac{dy}{dx}(2y-1)=1$$

$$\frac{dy}{dx}=\frac{1}{2y-1}$$

(ix) Option (d) is correct.

Explanation: Statement 1 and 2 are independent of each other.

(x) Option (b) is correct.

Explanation: Given: A is invertible.

So, $|A| \neq 0$

Also, $|A^T| \neq 0.$

Hence, A^T is also invertible.

So, Assertion is correct.

Now, taking B is an invertible symmetric matrix.

So, $|B| \neq 0$ and $B = B^T$

$$(B^{-1})^T = (B^T)^{-1} = B^{-1}$$

$\because$ $B^T = B$

$\Rightarrow$ B^{-1} is also symmetric matrix.

Hence, R is also true but does not give correct explanation of (a).

(xi) Option (a) is correct.

Explanation:

The required probability = P[(The first is a red jack card and the second is a jack card) + (The first is a red non-jack card and the second is a jack card)]

$$=\frac{2}{52}.\frac{3}{51}+\frac{24}{52}.\frac{4}{51}=\frac{1}{26}$$

(xii) We have,

$A = [a_{ij}]_{2\times 2}$, where $a_{ij} = 1$ if $i \neq j$ and 0 if $i = j$.

$$\therefore \quad A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

and
$$A^2=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}.\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$= I_2$$

(xiii) Let $y = f(x)$

$$y = \frac{x-1}{x+1}$$

$$xy + 2y = x - 1$$

$$xy - x = -2y - 1$$

$$x = \frac{-2y-1}{y-1}$$

$$y - 1 \neq 0$$

$$\Rightarrow \quad y \neq 1$$

Range $= R - \{1\}$

(xiv) Yes, A function $f : X \to Y$ is defined to be one-one, if the images of distinct elements of X under f are distinct. Here, the function has distinct elements having distinct images. Thus, it is a one-one.

(xv) Required probability $=\frac{{}^2C_1\times{}^4C_1}{{}^6C_2}=\frac{2\times 4}{\frac{6\times 5}{2}}=\frac{16}{30}=\frac{8}{15}$

2. (i) We have,

$$y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$$

Let $\sin^{-1} x = A$ and $\sin^{-1} y = B$. Then $x = \sin A$ and $y = \sin B$

$$\therefore \sin B\sqrt{1-\sin^2 A}+\sin A\sqrt{1-\sin^2 B}=1$$

$$\Rightarrow \quad \sin B \cos A + \sin A \cos B = 1$$

$$\Rightarrow \quad \sin (A + B) = 1$$

$$\Rightarrow \quad A + B = \sin^{-1} 1 = \frac{\pi}{2}$$

$$\Rightarrow \quad \sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$$

Differentiating w.r.t. x,

$$\frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$$

Hence Proved

OR

(ii) We have, $f(x) = (x-1)e^x + 1$

$$f'(x) = xe^x$$

Now $x > 0$ and $e^x > 0$ for all x

$\therefore$ $f'(x) > 0.$

$\Rightarrow f$ is an increasing function

3. Let, $$I = \int_{-1}^{2}\frac{|x|}{x}dx$$

Since,
$$\frac{|x|}{x} = \begin{cases}\frac{x}{x}, & x>0\\ \frac{-x}{x}, & x<0\end{cases}$$

$$= \begin{cases}1, & x>0\\ -1, & x<0\end{cases}$$

$$\therefore \quad I = \int_{-1}^{0}(-1)dx + \int_{0}^{2}(1)dx$$

$$= [-x]_{-1}^{0} + [x]_{0}^{2}$$

$$= -[0-(-1)] + (2-0)$$

$$= -1 + 2$$

$$= 1$$

4. Given, equation of the curve

$$y = \tan x$$

$$\therefore \quad \frac{dy}{dx} = \sec^2 x$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{at(0,0)} = \sec^2 0 = 1$$

$$\text{And} \quad -\frac{1}{\left(\frac{dy}{dx}\right)_{at(0,0)}} = -\frac{1}{1} = -1$$

$\therefore$ Equation of normal to the curve $y = \tan x$ at (0, 0) is

$$y - 0 = -1(x-0)$$

$$\Rightarrow \quad y + x = 0$$

5. (i) $I = \int \frac{\sin 2x}{\sqrt{9-\cos^4 x}}dx$

$$\text{Put} \quad \cos^2 x = t$$

$$\Rightarrow -2\cos x \sin x\ dx = dt$$

$$\sin 2x\ dx = -dt$$

The given integral

$$I = -\int \frac{dt}{\sqrt{3^2 - t^2}}$$

$$= -\sin^{-1}\frac{t}{3} + c$$

$$= -\sin^{-1}\frac{\cos^2 x}{3} + c$$

OR

(ii) Let, $x + \sin x = t$

So, $(1 + \cos x)dx = dt$

$$I = 3\int \frac{dt}{t} = 3\log|t| + c$$

$$= 3\log|x + \sin x| + c$$

or directly by writing formula

$$\int \frac{f'(x)}{f(x)}dx = \log|f(x)| + c$$

Examiner's Comment

- Some students split the numerator and apply by parts which is too lengthy.

Answering Tip

- Put the denominator as t; if numerator is the derivative of denominator.

6. $\sin^{-1}\left(\frac{\sin x}{\sqrt{2}} + \frac{\cos x}{\sqrt{2}}\right)$ if $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$$= \sin^{-1}\left(\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}\right)$$

$$= \sin^{-1}\left(\sin\left(x + \frac{\pi}{4}\right)\right) \text{ if } 0 < \left(x + \frac{\pi}{4}\right) < \frac{\pi}{2}$$

i.e., Principal values = $\left(x + \frac{\pi}{4}\right)$

7. To show, $\cos[\tan^{-1}\{\cot(\sin^{-1} x)\}] = x$

Solving LHS, $\cos[\tan^{-1}\{\cot(\sin^{-1} x)\}]$

$$\text{Let,} \quad \sin^{-1} x = \theta$$

$$\Rightarrow \quad \sin\theta = x$$

$$\Rightarrow \quad \cot\theta = \frac{\sqrt{1-x^2}}{x}$$

$$\Rightarrow \quad \theta = \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) \qquad \text{....(i)}$$

$$\text{Thus,} \quad \cos\left[\tan^{-1}\cot\left\{\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right\}\right]$$

[Putting value of θ in (i)]

$$\Rightarrow \cos\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

$$\text{Now, let} \quad \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = t$$

$$\Rightarrow \quad \tan t = \left(\frac{\sqrt{1-x^2}}{x}\right)$$

$$\Rightarrow \quad \cos t = x$$

$$\Rightarrow \quad t = \cos^{-1} x$$

$$\text{Thus,} \quad \cos t = \cos(\cos^{-1} x)$$

[Putting value in (ii)]

$$\Rightarrow \quad \cos t = x = \text{RHS}$$ **Hence proved**

8. $I = \int \sqrt{\tan x}\ dx,$

$$\text{Let} \quad \tan x = t^2$$

$$\sec^2 x\ dx = 2t\ dt$$

$$(1 + \tan^2 x)dx = 2t\ dt$$

$$(1 + t^4)dx = 2t\ dt$$

$$dx = \frac{2t}{1+t^4}dt$$

$$I = \int \sqrt{t^2}.\frac{2t}{1+t^4}dt$$

$$= \int \frac{2t^2}{1+t^4}dt$$

$$= \int \frac{2dt}{t^2 + \frac{1}{t^2}}dt \qquad [\text{divided by } t^2]$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right) + \left(1-\frac{1}{t^2}\right)}{t^2 + \frac{1}{t^2}}dt$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right)dt}{t^2 + \frac{1}{t^2} - 2 + 2} + \int \frac{\left(1-\frac{1}{t^2}\right)dt}{t^2 + \frac{1}{t^2} + 2 - 2}$$

$$I = \int \frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+\left(\sqrt{2}\right)^2} + \int \frac{\left(1-\frac{1}{t^2}\right)dt}{\left(t+\frac{1}{t}\right)^2-\left(\sqrt{2}\right)^2}$$

Let $t-\frac{1}{t}=u$ and $t+\frac{1}{t}=v$

$\left(1+\frac{1}{t^2}\right)dt = du$ and $\left(1-\frac{1}{t^2}\right)dt = dv$

$$= \int \frac{du}{u^2+\left(\sqrt{2}\right)^2} + \int \frac{dv}{v^2-\left(\sqrt{2}\right)^2}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\frac{u}{\sqrt{2}} + \frac{1}{2\sqrt{2}}\log\left|\frac{v-\sqrt{2}}{v+\sqrt{2}}\right| + c$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{t-\frac{1}{t}}{\sqrt{2}} + \frac{1}{2\sqrt{2}}\log\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}} + c$$

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\frac{\tan x-1}{\sqrt{2\tan x}} + \frac{1}{2\sqrt{2}}\log\frac{\tan x-\sqrt{2\tan x}+1}{\tan x+\sqrt{2\tan x}+1} + c$$

9. **(i)** The rate of change of radius of circle is $\frac{dr}{dt}=6$ cm/s

Area of the circle, $A = \pi r^2$

Differentiating w.r.t. 't', we get

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

$$\Rightarrow \frac{dA}{dt} = 2\pi r \times 6$$

$$\Rightarrow \frac{dA}{dt} = 12\pi r$$

$$\therefore \left(\frac{dA}{dt}\right)_{at\ r=3} = 12\pi \times 3 = 36\pi \text{ cm}^2/\text{s}$$

OR

(ii) Given, $y = x\log\left(\frac{x}{a+bx}\right)$

$$\Rightarrow \frac{y}{x} = \log\left(\frac{x}{a+bx}\right)$$

$$\Rightarrow \frac{y}{x} = \log x - \log(a+bx)$$

Differentiating w.r.t. x,

$$\frac{x\frac{dy}{dx}-y}{x^2} = \frac{1}{x} - \frac{b}{a+bx}$$

$$\Rightarrow x\frac{dy}{dx} - y = x - \frac{bx^2}{a+bx} = \left(\frac{ax}{a+bx}\right) \quad ...(i)$$

Differentiating again w.r.t. x

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = 1 - \frac{[(a+bx)2bx - bx^2(b)]}{(a+bx)^2}$$

$$\Rightarrow x\frac{d^2y}{dx^2} = \frac{(a+bx)^2 - 2bx(a+bx) + b^2x^2}{(a+bx)^2}$$

$$\Rightarrow x\frac{d^2y}{dx^2} = \frac{(a+bx)[a+bx-2bx] + b^2x^2}{(a+bx)^2}$$

$$\Rightarrow x\frac{d^2y}{dx^2} = \frac{(a+bx)(a-bx) + b^2x^2}{(a+bx)^2}$$

$$\Rightarrow x\frac{d^2y}{dx^2} = \frac{a^2 - b^2x^2 + b^2x^2}{(a+bx)^2}$$

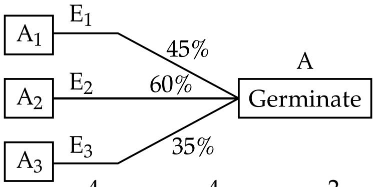
$$\Rightarrow x\frac{d^2y}{dx^2} = \frac{a^2}{(a+bx)^2}$$

$$\Rightarrow x\frac{d^2y}{dx^2} = \left(\frac{a}{a+bx}\right)^2$$

$$\Rightarrow x^3\frac{d^2y}{dx^2} = \left(\frac{ax}{a+bx}\right)^2 = \left(x.\frac{dy}{dx} - y\right)^2 \quad \text{[using equ. (i)]}$$

So, $x^3\frac{d^2y}{dx^2} = \left(x.\frac{dy}{dx} - y\right)^2$ **Hence Proved.**

10. (i) **(a)**



Here, $P(E_1) = \frac{4}{10}, P(E_2) = \frac{4}{10}, P(E_3) = \frac{2}{10}$

$$P\left(\frac{A}{E_1}\right) = \frac{45}{100}, P\left(\frac{A}{E_2}\right) = \frac{60}{100}, P\left(\frac{A}{E_3}\right) = \frac{35}{100}$$

$$\therefore P(A) = P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right) + P(E_3)\cdot P\left(\frac{A}{E_3}\right)$$

$$= \frac{4}{10}\times\frac{45}{100} + \frac{4}{10}\times\frac{60}{100} + \frac{2}{10}\times\frac{35}{100}$$

$$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{1000}$$

$$= \frac{490}{1000} = 0.49$$

(b) Required probability $= P\left(\frac{E_2}{A}\right)$

$$= \frac{P(E_2)\cdot P\left(\frac{A}{E_2}\right)}{P(A)}$$

$$= \frac{\frac{4}{10}\times\frac{60}{100}}{\frac{490}{1000}}$$

$$= \frac{240}{490} = \frac{24}{49}$$

(c) Let E_1 = Event for getting an even number on die and E_2 = Event that a spade card is selected

$\therefore$

$$P(E_1) = \frac{3}{6}$$

$$= \frac{1}{2}$$

and

$$P(E_2) = \frac{13}{52}$$

$$= \frac{1}{4}$$

Then, $P(E_1 \cap E_2) = P(E_1)\cdot P(E_2)$

$$= \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$$

OR

(ii) Given, $P(A) = P(\text{I horse selected}) = \dfrac{1}{4}$

$P(B) = P(\text{II horse selected}) = \dfrac{1}{3}$

$P(\bar{A}) = 1 - P(A)$

$= 1 - \dfrac{1}{4} = \dfrac{3}{4}$

and $P(\bar{B}) = 1 - P(B)$

$= 1 - \dfrac{1}{3} = \dfrac{2}{3}$

(i) $P(\text{Both of them selected}) = P(A \cap B) = P(A)P(B)$

($\because$ A and B are independent events)

$= \dfrac{1}{4} \times \dfrac{1}{3} = \dfrac{1}{12}$

(ii) $P(\text{only one selected}) = P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= P(A)P(\bar{B}) + P(\bar{A})P(B)$

$= \dfrac{1}{4} \times \dfrac{2}{3} + \dfrac{3}{4} \times \dfrac{1}{3} = \dfrac{5}{12}$

($\because$ A and B are independent events

$\Rightarrow$ A and $\bar{B}$, $\bar{A}$ and B are independent).

(iii) $P(\text{None of them selected}) = P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$

$= \dfrac{3}{4} \times \dfrac{2}{3} = \dfrac{1}{2}$

11. Let the two parts be ₹ x and ₹ $(30{,}000 - x)$, respectively.

Let A be the 1×2 matrix representing these two parts

$$\text{i.e.,}\quad A = \begin{array}{cc} \text{Part I} & \text{Part II} \\ [x & 30{,}000 - x] \end{array}$$

Let R denotes the 2×1 matrix representing the annual interest rates of interest on two parts, i.e.,

$$R = \begin{array}{cc} \text{Part I} & \multirow{2}{*}{} \\ \text{Part II} & \end{array}\begin{bmatrix} 0.09 \\ 0.11 \end{bmatrix}$$

The total annual interest on the two parts is given by the matrix multiplication AR

$\therefore$ $AR = 3{,}060$

$\Rightarrow$ $\begin{bmatrix} x & 30{,}000 - x \end{bmatrix}\begin{bmatrix} 0.09 \\ 0.11 \end{bmatrix} = 3{,}060$

$\Rightarrow$ $0.09x + 0.11(30{,}000 - x) = 3{,}060$

$\Rightarrow$ $\dfrac{9}{100}x + \dfrac{11}{100}(30{,}000 - x) = 3{,}060$

$\Rightarrow$ $9x + 3{,}30{,}000 - 11x = 3{,}06{,}000$

$\Rightarrow$ $x = 12{,}000$

Hence, two parts of ₹ 30,000 are ₹ 12,000 and ₹ 18,000, respectively.

12. (i) Given, $(y + \log x)dx - x\,dy = 0$

$\Rightarrow$ $\dfrac{dy}{dx} = \dfrac{y}{x} + \dfrac{\log x}{x}$

$\Rightarrow$ $\dfrac{dy}{dx} - \dfrac{y}{x} = \dfrac{\log x}{x}$

It is linear differential equation in the form of $\dfrac{dy}{dx} + Py = Q$.

Here, $P = -\dfrac{1}{x}$, $Q = \dfrac{\log x}{x}$

$\text{I.F.} = e^{\int P\,dx}$

$\Rightarrow$ $\text{I.F.} = e^{-\int \frac{1}{x}dx} = e^{-\log x} = \dfrac{1}{x}$

$\therefore$ Solution is given as,

$y \cdot \text{I.F.} = \int Q \times . \text{I.F.}\ dx$

$\Rightarrow$ $y \cdot \dfrac{1}{x} = \int \dfrac{\log x}{x} . \dfrac{1}{x} dx + C$

$\Rightarrow$ $\dfrac{y}{x} = \int \log x . \dfrac{1}{x^2} dx + C$

$\Rightarrow$ $\dfrac{y}{x} = \log x \int x^{-2} dx - \int \left(\dfrac{d}{dx} \log x \int x^{-2} dx \right) dx + C$

[using integration by parts]

$\Rightarrow$ $\dfrac{y}{x} = \log x \left(-\dfrac{1}{x} \right) - \int \dfrac{1}{x} \left(-\dfrac{1}{x} \right) dx + C$

$\Rightarrow$ $\dfrac{y}{x} = \log x \left(-\dfrac{1}{x} \right) + \left(-\dfrac{1}{x} \right) + C$

$\Rightarrow$ $y = -\log x - 1 + Cx$

$y = 0$, when $x = 1$ is given,

$0 = -\log_e 1 - 1 + C$

$\Rightarrow$ $C = 1$ ($\because$ $\log_e 1 = 0$)

$\therefore$ $y = \log x - 1 + x$

is the required solution.

OR

(ii) Let, $I = \int (5x - 1)\sqrt{6 + 5x - 2x^2}\, dx$

Take, $5x - 1 = A\dfrac{d}{dx}(6 + 5x - 2x^2) + B$

$= A(5 - 4x) + B$

$\Rightarrow$ $5 = -4A$ and $-1 = 5A + B$

$\Rightarrow$ $A = -\dfrac{5}{4}$, $B = \dfrac{21}{4}$

$$\therefore\quad I = -\frac{5}{4}\int (5 - 4x)\sqrt{6 + 5x - 2x^2}\, dx + \frac{21}{4}\int \sqrt{2}\sqrt{3 + \frac{5}{2}x - x^2}\, dx$$

$$= -\frac{5}{4} \times \frac{2}{3}(6 + 5x - 2x^2)^{3/2} + \frac{21\sqrt{2}}{4}\int \sqrt{\left(\frac{\sqrt{73}}{4}\right)^2 - \left(x - \frac{5}{4}\right)^2}\, dx$$

$$= -\frac{5}{6}(6 + 5x - 2x^2)^{3/2} + \frac{21\sqrt{2}}{4}\left[\frac{1}{2}\left(x - \frac{5}{4}\right)\sqrt{\left(\frac{\sqrt{73}}{4}\right)^2 - \left(x - \frac{5}{4}\right)^2} + \frac{1}{2}\left(\frac{\sqrt{73}}{4}\right)^2 \sin^{-1}\left(\frac{x - 5/4}{\sqrt{73}/4}\right)\right] + C$$

$$= -\frac{5}{6}(6 + 5x - 2x^2)^{3/2} + \frac{21\sqrt{2}}{4}\left[\left(\frac{4x - 5}{8}\right)\sqrt{\left(\frac{73}{4}\right)^2 - \left(x - \frac{5}{4}\right)^2} + \frac{73}{32}\sin^{-1}\left(\frac{4x - 5}{\sqrt{73}}\right)\right] + C$$

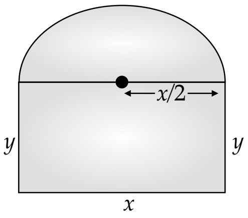
13. **(i)** Let length and breadth of rectangle be x and y.

$\therefore$ $P = 2y + x + \pi\dfrac{x}{2}$

and $$A = xy + \frac{1}{2}\pi\frac{x^2}{4}$$

$$= \frac{x}{2}\left[P - x - \frac{\pi x}{2}\right] + \frac{\pi x^2}{8}$$

$$= P\frac{x}{2} - \frac{x^2}{2} - \pi\frac{x^2}{4} + \pi\frac{x^2}{8}$$



$$\frac{dA}{dx} = \frac{P}{2} - x - \frac{\pi x}{2} + \frac{\pi x}{4}$$

$$= \frac{P}{2} - x - \frac{\pi x}{4}$$

$$\frac{dA}{dx} = 0$$

or $$x = \frac{2P}{4+\pi}$$

and $$y = \frac{P}{4+\pi}$$

$$\frac{d^2A}{dx^2} = -1 - \frac{\pi}{4} < 0$$

or Area is maximum, when length

$$= \frac{2P}{4+\pi},$$

and breadth $$= \frac{P}{4+\pi}$$

OR

(ii) Given, l, b and h are the length, breadth and depth of the tank, respectively.

$\therefore$ $l \times b \times 3 = 75$

$\Rightarrow$ $l \times b = 25$

Let C be the cost, then

$$C = 100(l \times b) + 100\,[h(b + l)]$$

$$= 100\left(l \times \frac{25}{l}\right) + 300\left(\frac{25}{l} + l\right)$$

$$= 2500 + 300\left(\frac{25}{l} + l\right)$$

Differentiating w.r.t. l,

$$\frac{dC}{dl} = 0 + 300\left(\frac{-25}{l^2} + 1\right)$$

Putting, $$\frac{dC}{dl} = 0$$

$\Rightarrow$ $$300\left(-\frac{25}{l^2} + 1\right) = 0$$

$\Rightarrow$ $l^2 = 25$ or $l = 5$

Getting $$\left(\frac{d^2C}{dl^2}\right) = 300\left(\frac{50}{l^3}\right)$$

$\Rightarrow$ $$\left(\frac{d^2C}{dl^2}\right)_{\text{at } l=5} = \frac{15000}{125} > 0$$

i.e., C is minimum, when $l = 5$

$\Rightarrow$ $b = 5$

$\therefore$ $C = 100(25) + 300(10)$

$= 2{,}500 + 3{,}000$

$= 5{,}500$

Hence, the minimum cost is ₹ 5,500.

14. Let X = denote the random variable. $X = 0, 1, 2$, $n = 2$, $p = \frac{1}{4}, q = \frac{3}{4}$.

x_i	**0**	**1**	**2**	**Total**
p_i	${}^2C_0\left(\frac{3}{4}\right)^2 = \frac{9}{16}$	${}^2C_1\frac{1}{4}\left(\frac{3}{4}\right) = \frac{6}{16}$	${}^2C_2\left(\frac{1}{4}\right)^2 = \frac{1}{16}$	
x_ip_i	0	$\frac{6}{16}$	$\frac{2}{16}$	$\frac{1}{2}$

$$\text{Mean} = \Sigma x_ip_i = \frac{1}{2}$$

Section-B

15. (i) Option (b) is correct.

Explanation:

Required vector $= (-3-2)\hat{i} + (7-5)\hat{j} + (4-0)\hat{k}$

$= -5\hat{i} + 2\hat{j} + 4\hat{k}$

(ii) Option (b) is correct.

Explanation: Direction cosines of the line, whose direction ratios are 2, 3, –6 are:

$$\frac{2}{\sqrt{2^2+3^2+(-6)^2}}, \frac{3}{\sqrt{2^2+3^2+(-6)^2}}, \frac{-6}{\sqrt{2^2+3^2+(-6)^2}}$$

or $\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}$

Since, line makes obtuse angle with Y-axis, then

$$\cos\beta < 0$$

Therefore, direction cosines are $\frac{-2}{7}, \frac{-3}{7}, \frac{6}{7}$.

(iii) Option (c) is correct.

Explanation: Length of perpendicular is

$$\left|\frac{3 - 2(-2) + 2(1) - 3}{\sqrt{1^2 + (-2)^2 + (2)^2}}\right| = \frac{6}{3} = 2 \text{ units}$$

(iv) A.T.Q., $$\frac{2\lambda + 6 + 12}{\sqrt{2^2 + 6^2 + 3^2}} = 4$$

$\Rightarrow$ $2\lambda + 18 = 28$

$\therefore$ $\lambda = 5$

Examiner's Comment

- Most of the candidates calculated dot product instead of applying projection formula.

Answering Tip

- Clarify the concept of scalar projection of vectors thoroughly.

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(v) Let $\vec{r} = \vec{a} + \vec{b} = 4\vec{i} + 3\vec{j} - 12\vec{k}$

$$|\vec{r}| = \sqrt{16+9+144} = \sqrt{169} = 13$$

So, unit vector

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{4\hat{i}+3\hat{j}-12\hat{k}}{13}$$

$$= \frac{4}{13}\hat{i} + \frac{3}{13}\hat{j} - \frac{12}{13}\hat{k}$$

16. (i)

$$\begin{bmatrix}\vec{a} & \vec{b} & \vec{c}\end{bmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ c_1 & c_2 & c_3 \end{vmatrix} = c_2 - c_3$$

$$c_1 = 1, c_2 = 2$$

$$\begin{bmatrix}\vec{a} & \vec{b} & \vec{c}\end{bmatrix} = 2 - c_3$$

$\because$ $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar then $\begin{bmatrix}\vec{a} & \vec{b} & \vec{c}\end{bmatrix} = 0$ or $c_3 = 2$

OR

(ii) Given $(2\hat{i}+6\hat{j}+27\hat{k}) \times (\hat{i}+p\hat{j}+q\hat{k}) = \vec{0}$

i.e.,
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & p & q \end{vmatrix} = \vec{0}$$

$\Rightarrow (6q - 27p)\hat{i} + (27 - 2q)\hat{j} + (2p - 6)\hat{k} = \vec{0}$

$\Rightarrow 27 - 2q = 0$ and $2p - 6 = 0$

$$\Rightarrow \quad q = \frac{27}{2}, p = 3$$

17. (i) The equation of a line passing through two points P(–1, 3, 2) and Q(–4, 2, –2) is given by:

$$\frac{x+1}{-4+1} = \frac{y-3}{2-3} = \frac{z-2}{-2-2}$$

Or,
$$\frac{x+1}{-3} = \frac{y-3}{-1} = \frac{z-2}{-4}$$

Or,
$$\frac{x+1}{3} = \frac{y-3}{1} = \frac{z-2}{4} \quad \text{...(i)}$$

Now, if the three points P(–1, 3, 2), Q(–4, 2, –2) and R(5, 5, λ) are collinear, then the coordinates of point R must satisfy the equation (i).

$$\frac{5+1}{3} = \frac{5-3}{1} = \frac{\lambda-2}{4}$$

$$\Rightarrow \quad \frac{6}{3} = \frac{2}{1} = \frac{\lambda-2}{4}$$

i.e., $\lambda = 10$

Examiner's Comment

- After expressing the equation of the line in Cartesian symmetric form, a few candidates failed to find λ by substituting coordinates of (5, 5, λ) in the equation of the line. Candidates who used the vector form of the line also made a similar mistake.

Answering Tip

- Understanding the equation of a line in all its forms is essential. Clarity is needed regarding the direction ratios of a line, as well as the concepts of parallel and perpendicular lines.

OR

(ii) Given that, required plane passes through the point (–1, –1, 2).

$\therefore a(x+1) + b(y+1) + c(z-2) = 0$ ...(i)

is the required equation of the plane.

As (i) is perpendicular to the planes

$3x + 2y - 3z = 1$ and $5x - 4y + z = 5$,

We have $3a + 2b - 3c = 0$ ...(ii)

and $5a - 4b + c = 0$ ...(iii)

($\because$ Condition of perpendicularity, $a_1a_2 + b_1b_2 + c_1c_2 = 0$)

Solving eq. (ii) and (iii), we get

$$\frac{a}{2-12} = \frac{b}{-3-15} = \frac{c}{-12-10}$$

$$\Rightarrow \quad \frac{a}{-10} = \frac{b}{-18} = \frac{c}{-22}$$

$$\Rightarrow \quad a = \frac{5c}{11}, b = \frac{9c}{11}$$

Substituting these values in eq. (i), we get

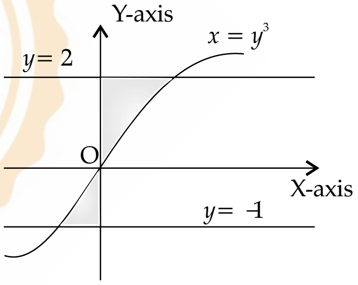
$$\frac{5c}{11}(x+1) + \frac{9c}{11}(y+1) + c(z-2) = 0$$

$$\Rightarrow \quad \frac{c}{11}[5x+5+9y+9+11z-22] = 0$$

$$\Rightarrow \quad 5x + 9y + 11z - 8 = 0$$

is the required equation of the plane.

18. (i) Graph of the region bounded by $y = -1$, $y = 2$, $x = y^3$ and $x = 0$ is shown below.



(ii) $$\int_0^2 y^3 dy = \left[\frac{y^4}{4}\right]_0^2 = \frac{1}{4}(2^4 - 0) = \frac{16}{4} = 4 \quad \text{....(i)}$$

(iii) $$\text{Required area} = \left|\int_{-1}^{0} x\, dy\right| + \int_0^2 x\, dy$$

$$= \left|\int_{-1}^{0} y^3\, dy\right| + \int_0^2 y^3\, dy$$

$$= \left|\left[\frac{y^4}{4}\right]_{-1}^{0}\right| + 4 \quad \text{[From (i)]}$$

$$= \left|\frac{1}{4}(0) - (-1)^4\right| + 4$$

$$= \frac{1}{4} + 4 = \frac{17}{4} \text{ sq.units}$$

Section-C

19. (i) Option (b) is correct.

Explanation: Revenue function $R(x) = 100x$

So total cost function $C(x) = 35{,}000 + 30\%$ of $100x$

$$\Rightarrow \quad C(x) = 35{,}000 + \frac{30}{100} \times 100x$$

$$\Rightarrow \quad C(x) = 35{,}000 + 30x$$

(ii) Option (a) is correct.

Explanation: Given, lines of Regression:

$$3x-2y+1=0 \quad \ldots(i)$$
$$2x-y-2=0 \quad \ldots(ii)$$
$$\therefore \quad 3x-2y=-1$$
$$(2x-y=2)\times 2$$

(Multiplying eq. (ii) by 2 and subtracting it from eq. (i))

$$\Rightarrow \quad 3x-2y=-1$$
$$4x-2y=4$$
$$-x=-5$$
$$\therefore \quad x=5$$
$$3(5)-2y=-1$$

(Substituting value of x in eq. (i))

$$\therefore \quad y=8$$

Thus, $\bar{x}=5,\ \bar{y}=8$

(iii) Let $4x - 2y = 0$ be the regression equation of x on y. And $3x - 6y + 5 = 0$ the regression equation of y on x

$$4x = 2y$$
$$x = \frac{y}{2}$$

Therefore, $b_{xy} = \frac{1}{2}$

$$6y = 3x+5$$
$$y = \frac{x}{2}+\frac{5}{6}$$
$$b_{yx} = \frac{1}{2}$$

Now, $r^2 = b_{yx} \times b_{xy} = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{4}$

$$|r| = \frac{1}{2}$$

(iv) Revenue function $R = p \times x$

$$\Rightarrow \quad R = p\left(\frac{24-2p}{3}\right)$$
$$\Rightarrow \quad R = p\left(8-\frac{2}{3}p\right)$$
$$\Rightarrow \quad R = 8p-\frac{2}{3}p^2$$

(v) Suppose that x number of units are produced and sold. As each unit's variable cost is 25% of revenue.

The variable cost of x units = 25% of ₹ $8x$ = ₹ $2x$.

Total cost of producing x units:

$C(x) = TFC + TVC$ = ₹ $(24000 + 2x)$

⇒ Price of one unit = ₹ 8

Total revenue on selling x units = $R(x)$ = ₹ $8x$

At break-even values

$$C(x) = R(x)$$
$$\Rightarrow \quad 24000 + 2x = 8x$$
$$\Rightarrow \quad 24000 = 6x$$
$$\Rightarrow \quad x = 4000$$

20. (i) Given, average cost function,

$$AC = 2x-11+\frac{50}{x}$$

Now, $\frac{d}{dx}(AC) = \frac{d}{dx}\left(2x-11+\frac{50}{x}\right)$

$$= 2-\frac{50}{x^2}$$

AC increases, $\frac{d}{dx}(AC) > 0$

$$\Rightarrow \quad 2-\frac{50}{x^2} > 0$$
$$\Rightarrow \quad x^2-25 > 0$$
$$\Rightarrow \quad (x-5)(x+5) > 0$$
$$\Rightarrow \quad x < -5 \text{ or } x > 5$$

∵ x is positive, AC increases, when $x > 5$.

Examiner's Comment

- Most of the candidates attempted the first part correctly but while calculating the value of x when AC was increasing, some candidates included the negative value of x.

Answering Tip

- Analyse in detail about conditions of increasing and decreasing functions and simplification of problems based on those concepts.

OR

(ii) Given, $R(x) = 100x - x^2 - x^3$

(a) Demand's function $= \frac{R(x)}{x}$

$$= \frac{100x-x^2-x^3}{x} = 100 - x - x^2$$

(b) M.R. $= \frac{d}{dx}(R(x)) = 100-2x-3x^2$

21. **(i)** Given, $\sum x = 15, \sum y = 25, \sum xy = 83,$

$\sum x^2 = 55, \sum y^2 = 135$ and $n = 5$

(a) Regression coefficient

$$b_{xy} = \frac{\sum xy - \frac{\sum x \cdot \sum y}{n}}{\sum y^2 - \frac{\left(\sum y\right)^2}{n}}$$

$$= \frac{83-\frac{15\times 25}{5}}{135-\frac{(25)^2}{5}}$$

$$\frac{83-75}{135-125} = \frac{8}{10} = 0.8$$

(b) Regression equation x on y given by

$$x-\bar{x} = b_{xy}\left(y-\bar{y}\right)$$

Hence, $\bar{x} = \frac{\sum x}{n} = \frac{15}{5} = 3,\ \bar{y} = \frac{\sum y}{n} = \frac{25}{5} = 5$

$\therefore x - 3 = 0.8\,(y - 5)$

$\Rightarrow x-3 = \frac{4}{5}(y-5)$

$\Rightarrow 5x - 15 = 4y - 20$

$\Rightarrow 5x - 4y + 5 = 0$

OR

(ii)

x_i	y_i	x_iy_i	x_i^2
1	4	4	1
2	8	16	4

3	2	6	9
4	12	48	16
5	10	50	25
6	14	84	36
7	16	112	49
8	6	48	64
9	18	162	81
$\Sigma x_i = 45$	$\Sigma y_i = 90$	$\Sigma x_i y_i = 530$	$\Sigma x_i^2 = 285$

$$\therefore \quad b_{yx} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2}$$

$$= \frac{9\times 530 - 45\times 90}{9\times 285 - (45)^2} = \frac{4}{3}$$

$\therefore$ Regression equation of y on x

$$y - 10 = \frac{4}{3}(x-5)$$

$$y = \frac{4}{3}x + \frac{10}{3}$$

Now, when $x = 14$

$$y = \frac{4}{3}\times 14 + \frac{10}{3} = 22$$

Examiner's Comment

- Many of the candidates wrote incorrect formula for regression coefficients which caused error in finding the regression equation y on x. Several candidates were unable to interpret the result of y for $x = 14$.

Answering Tip

- Practice a variety of problems based on regression.

22.

$$Z = 11x + 9y$$

$$x + y \le 10$$

$$180x + 120y \le 1500$$

$$18x + 12y \le 150$$

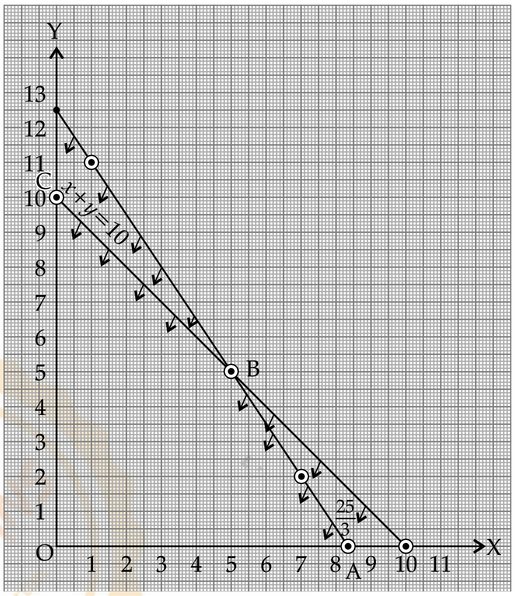
$$3x + 2y \le 25$$

$x + y = 10$

x	0	10	5
y	10	0	5

$3x + 2y = 25$

x	1	5	7
y	11	5	2



Feasible region is OABCO

$Z = 11x + 9y$

At (0, 0) $\quad Z = 11\times 0 + 9\times 0 = 0$

At A(25/3, 0) $\quad Z = 11\times \frac{25}{3} + 9\times 0 = \frac{275}{3} = 91.67$

At B(5, 5) $\quad Z = 11\times 5 + 9\times 5 = 55 + 45 = 100$

At C(0, 10) $\quad Z = 11\times 0 + 9\times 10 = 0 + 90 = 90$

Maximum profit at (5, 5) is ₹ 100.

❑❑

Self Assessment Paper-2

MATHEMATICS

Class-12

UNSOLVED

Time Allowed : 3 hours **Maximum Marks : 80**

You are allowed an additional 15 minutes for only reading the paper.
You must NOT start writing during reading time.
The question paper has 12 printed pages.
The Question Paper is divided into three sections and has 22 questions in all.

Section A is compulsory and has fourteen questions.
You are required to attempt all questions either from Section B or Section C.
Section B and Section C have four questions each.
Internal choices have been provided in two questions of 2 marks, two questions of 4 marks and two questions of 6 marks in Section A.
Internal choices have been provided in one question of 2 marks and one question of 4 marks each in Section B and Section C.
While attempting Multiple Choice Questions in Section A, B and C, you are required to write only ONE option as the answer.
The intended marks for questions or parts of questions are given in the brackets [].
All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
Mathematical tables and graph papers are provided.
Instruction to Supervising Examiner:
Kindly read aloud the instructions given above to all the candidates present in the examination hall.

Section-A [65 marks]

1. In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed **[15]**

(i) Let L be a set of all straight lines in a plane. The relation R on L defined as 'perpendicular to' is :

(a) Symmetric and Transitive **(b)** Transitive

(c) Symmetric **(d)** Equivalence **[Understanding]**

(ii) $\int \frac{dx}{e^x + e^{-x}}$ is equal to

(a) $\tan^{-1}(e^x) + C$ **(b)** $\tan^{-1}(e^{-x}) + C$

(c) $\log(e^x - e^{-x}) + C$ **(d)** $\log(e^x + e^{-x}) + C$ AI **[Application]**

(iii) $\forall\, x \in R$, $\cot^{-1}(-x) =$

(a) $\pi - \cot^{-1} x$ **(b)** $-\tan^{-1} x$ **(c)** $-\cot^{-1} x$ **(d)** $\pi + \cot^{-1} x$ **[Recall]**

(iv) **Assertion (A):** For the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} + x^{1/5} = 0$, the order and degree are 2 and 4, respectively.

Reason (R): The order of a differential equation is defined to be that of the highest order derivative it contains. The degree of a differential equation is defined as the power to which the highest order derivative is raised.

Which of the following is correct?

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true. **[Analysis]**

(v) If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

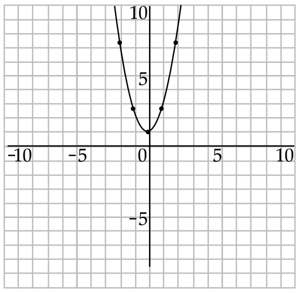
(a) $P(A/B) = P(B)/P(A)$ **(b)** $P(A/B) < P(A)$

(c) $P(A/B) \geq P(A)$ **(d)** $P(A/B) \leq P(A)$ **[Application]**

(vi) For what value of k inverse does not exist for the matrix $\begin{bmatrix} 1 & 2 \\ k & 6 \end{bmatrix}$:

(a) 0 **(b)** 3 **(c)** 6 **(d)** 2 **[Understanding]**

(vii) Consider the graph of the function $f(x) = e^{|x|}$.



Statement 1: $f(x)$ is continuous everywhere but not differentiable at $x = 0$

Statement 2: $f(x)$ is continuous and differentiable everywhere.

Which of the following is correct?

(a) Statement 1 is true and Statement 2 is false. **(b)** Statement 2 is true and Statement 1 is false.

(c) Both the statements are true. **(d)** Both the statements are false. AI **[Application]**

(viii) What is the value of $\frac{\pi}{2} + \tan^{-1}\frac{1}{\sqrt{2}}$?

(a) $\frac{1}{\sqrt{2}}$ **(b)** $\frac{\pi}{2}$ **(c)** -1 **(d)** $-\frac{1}{\sqrt{2}}$

(ix) **Statement 1:** A relation R from A to B is said to be a universal relation, if $R = A \times B$.

Statement 2: A relation R from A to B is called an empty relation or a void relation from A to B if $R = \phi$.

Which one of the following is correct?

(a) Statement 1 implies Statement 2. **(b)** Statement 2 implies Statement 1.

(c) Statement 1 is true only if Statement 2 is true. **(d)** Statement 1 and 2 are independent of each other.

[Understanding]

(x) Consider matrices, $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}$, then

Assertion: $(A + B)^2 = A^2 + 2AB + B^2$

Reason: $AB = BA$

Which of the following is correct?

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true. AI **[Application]**

(xi) If Z is a random variable defined as the number of heads minus number of tails in a simultaneous toss of 6 coins, all possible values of Z are

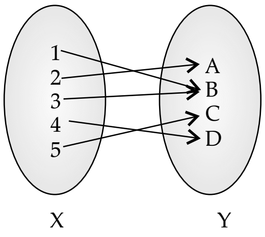
(a) $\{-6, -4, -2, 0, 2, 4, 6\}$ **(b)** $\{0, 2, 4, 6\}$

(c) $\{-6, -4, -2, 0, 2\}$ **(d)** $\{-4, -2, 0, 2, 4\}$ **[Understanding]**

(xii) Find the value of $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$. **[Analysis]**

(xiii) An equivalence relation R in A divides it into equivalence classes A_1, A_2, A_3. What is the value of $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$? AI **[Understanding]**

(xiv) The following figure represents which type of function? **[Understanding]**



(xv) A problem of statistics is given to two students whose chances of solving it are $\frac{1}{3}$ and $\frac{1}{5}$. What is the probability that the problem is solved? **[Application]**

2. (i) If $y = e^{\sin x^2}$, then find $\frac{dy}{dx}$. **[Understanding] [2]**

OR

(ii) Show that the function $f(x) = \frac{x}{3} + \frac{3}{x}$ decreases in the intervals $(-3, 0) \cup (0, 3)$. AI **[Understanding] [2]**

3. Find the value of $\int_{-\pi}^{\pi} (1 - x^2) \sin x . \cos^2 x . dx$. **[Application] [2]**

4. Find the equation of the normal lines to the curve $3x^2 - y^2 = 8$ which are parallel to the line $x + 3y = 8$. **[Analysis] [2]**

5. (i) Evaluate : $\int \frac{x\sin^{-1}(x^2)}{\sqrt{1-x^4}}$ **AI**

OR

(ii) Evaluate : $\int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2}\right] e^{2x} dx$ **[Evaluate] [2]**

6. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), \frac{-3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form. **[Application] [2]**

7. If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then find x. **[Application] [4]**

8. Evaluate: $\int \frac{x+1}{(x+2)(x+3)} dx$ **[Application] [4]**

9. (i) Separate the intervals $\left[0, \frac{\pi}{2}\right]$ into sub-intervals in which $f(x) = \sin 4x + \cos 4x$ is increasing or decreasing. **[Application] [4]**

OR

(ii) If $x = a(\cos 2\theta + 2\theta \sin 2\theta)$ and $y = a(\sin 2\theta - 2\theta \cos 2\theta)$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{8}$. **[Analysis] [4]**

10. (i) There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an aeroplane are 0.3 and 0.2, respectively. Both of them fired one shell at an aeroplane at the same time.

(a) What is the probability that the shell fired from exactly one of them hit the plane?

(b) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B ?

(c) How is Bayes' theorem different from conditional probability?

(d) Write the rule of Total Probability. **[Analysis] [4]**

OR

(ii) A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots.

(a) Let the target is hit by A, B and C. Then find the probability that A, B and C all will hit.

(b) What is the probability that B, C will hit and A will lose?

(c) What is the probability that any two of A, B and C will hit?

(d) What is the probability that none of them will hit the target? **AI** **[Analysis] [4]**

11. An orphanage is a residential institution, total institution or group home, devoted to the care of orphans and children who, for various reasons, cannot be cared for by their biological families. Seema lives near to Mother Care orphanage and spend her time with orphanage children. On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However if there were 16 children more, everyone would have got ₹10 less.

(a) Translate the problem into a system of equations.

(b) Solve: the system of equation by using matrix method.

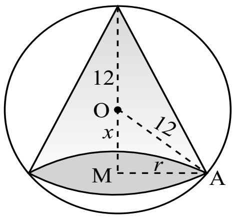
(c) Hence, find the number of children and the amount distributed by Seema. **[Application] [6]**

12. (i) Solve: $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$. When, $x = 1, y = \frac{\pi}{4}$ **AI** **[Application] [6]**

OR

(ii) Evaluate: $\int_0^{\pi} e^{2x}\sin\left(\frac{\pi}{4} + x\right) dx$ **[Evaluate] [6]**

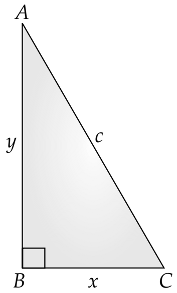
13. (i) A cone is inscribed in a sphere of radius 12 cm. If the volume of the cone is maximum, find its height. **AI** **[Application] [6]**



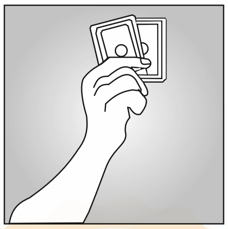
OR

(ii) Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

[Application] [6]



14. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards.



(a) If X denotes the random variable "number of kings", then what are the possible values X can take?

(b) Find the probability distribution of this random variable X.

(c) Find the mean of the number of kings. **[Analysis] [6]**

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Section-B [15 marks]

15. In subparts (i) and (iii) choose the correct options and in subparts (ii) to (v), answer the questions as instructed.

[5]

(i) Consider the following statements and choose the correct option:

Statement 1: Let P be a point on the line joining the points $A(0, 5, -4)$ and $B(4, -2, 1)$. If X-coordinate of P is 3, then its Y-coordinate is $-\frac{1}{4}$.

Statement 2: The equation of line passing through two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by

$$\frac{x - x_1}{x_2 + x_1} = \frac{y - y_1}{y_2 + y_1} = \frac{z - z_1}{z_2 + z_1}$$

Which of the following is correct?

(a) Only Statement 1
(b) Only Statement 2
(c) Both Statements 1 and 2
(d) Neither Statement 1 nor Statement 2 **[Recall]**

(ii) The two planes $x - 2y + 4z = 10$ and $18x + 17y + kz = 50$ are perpendicular, if k is

(a) – 4 **(b)** 4 **(c)** 2 **(d)** –2 **[Application]**

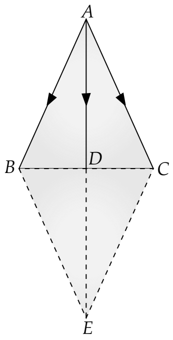
(iii) The sum of intercepts cut off by the plane $2x + y - z = 5$ on the coordinate axes is:

(a) $\frac{5}{2}$ **(b)** $\frac{3}{2}$ **(c)** $\frac{7}{2}$ **(d)** None of these

[Understanding]

(iv) Find the vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9. **[Understanding]**

(v) The two vectors $\left(\hat{j} + \hat{k}\right)$ and $\left(3\hat{i} - \hat{j} + 4\hat{k}\right)$ represent the two sides AB and AC, respectively of ΔABC. Find the length of the median through A. **[Analysis]**



16. **(i)** If $\overrightarrow{a}=\hat{i}-\hat{j}+7\hat{k}$ and $\overrightarrow{b}=5\hat{i}-\hat{j}+\lambda\hat{k}$, then find the value of λ so that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular vectors. **[Application] [2]**

OR

(ii) If $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}\right|$, then prove that $2\vec{a}+\vec{b}$ is perpendicular to $\vec{b}$. **[Application] [2]**

17. **(i)** Find the image of the point (3, –2, 1) in the plane $3x - y + 4z = 2$. **A I** **[Application] [4]**

OR

(ii) Find the value of p, so that the lines $l_1 = \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $l_2 = \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. Also, find the equation of a line passing through a point (3, 2, –4) and parallel to line l_1. **[Application] [4]**

18. **(i)** Sketch the region enclosed by the parabola $y^2=4ax$ and the chord $y = mx$.

(ii) Evaluate: $\int_0^{4a/m^2} 2\sqrt{ax}\,dx$

(iii) Hence, find the region bounded by the parabola $y^2=4ax$ and the chord $y = mx$. **[4]**

Section-C [15 marks]

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed. **[5]**

(i) The cost-function of a firm is given by $C(x)=3x^2<-2x+6$. The average cost of the firm at $x = 3$ is

(a) 11 **(b)** 17 **(c)** 9 **(d)** 27 **[Recall]**

(ii) Read the following statements and choose the correct option:

(I) Regression allows researchers to predict or explain the variation in one variable based on another variable.

(II) The statistical method which helps us to estimate or predict the unknown value of one variable from the known value of related variable is called correlation.

(III) In regression, the order of the variables is very important.

(IV) In regression, the explanatory variable (or the independent variable) always belongs on the x-axis. **[Recall]**

Which of the following is correct?

(a) Only I and IV **(b)** Only I, III and IV

(c) Only I, II and IV **(d)** All are correct.

(iii) If the regression line of x on y is $9x + 3y - 46 = 0$ and y on x is, $3x + 12y - 7 = 0$, then find the correlation coefficient 'r'. **A I** **[Understanding]**

(iv) For a product, a manufacturer spends ₹10,000 on the infra structure and variable cost of ₹15 per unit of the product. The sale price per unit was fixed at ₹20. Find the cost and revenue function. **[Understanding]**

(v) The total cost function for a production is given by $C(x) = \frac{3}{4}x^2 - 7x + 27$. Find the number of units produced for which $M.C. = A.C.$ ($M.C.$= Marginal Cost and $A.C.$ = Average Cost.) **[Understanding]**

20. **(i)** Find the cost of increasing from 100 to 200 units if the marginal cost in rupees per unit is given by the function, $MC=0.003x^2-0.01x+2.5$. **A I** **[Application] [2]**

OR

(ii) The total cost function is given by $C(x)=x+2x^3-3.5x^2$, find the marginal average cost function (MAC). Also, find the points where the MAC curve cuts the X-axis and Y-axis. **[Application] [2]**

21. **(i)** The following table shows the mean and standard deviation of the marks of Mathematics and Physics scored by the students in a school:

	Mathematics	Physics
Mean	**84**	**81**
Standard Deviation	**7**	**4**

The correlation coefficient between the given marks is 0.86. Estimate the likely marks in Physics if the marks in Mathematics are 92. **[Application] [4]**

OR

(ii) The correlation coefficient between x and y is 0.6. If the variance of x is 225, the variance of y is 400, mean of x is 10 and mean of y is 20, find:

(a) the equations of two regression lines.

(b) the expected value of y when $x = 2$. **[Application] [4]**

22. A company produces soft drinks that have a contract which requires that a minimum of 80 units of chemical A and 60 units of the chemical B go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs ₹ 10. The supplier T has a packet of mix of 1 unit of A and 1 unit of B that costs ₹ 4.

(a) Formulate LPP and solve it graphically.

(b) How many packets of mixes from S and T should the company purchase to honour the contract requirement and yet maintain the minimum cost ? **[Analysis] [4]**

■■

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Self Assessment Paper - 2

Section-A

1. **(i) Option (c) is correct.**

Explanation: $L_1 \perp L_2$, $L_2 \perp L_3$ then $L_1 \,||\, L_3$
Therefore, relation is not transitive
$L_1 \perp L_2$ then $L_2 \perp L_1$
So, relation is symmetric.

(ii) Option (a) is correct.

Explanation:

Let,
$$I = \int \frac{dx}{e^x + e^{-x}}$$
$$= \int \frac{e^x}{e^{2x}+1} dx$$

Also, let $e^x = t$

$\Rightarrow e^x dx = dt$
$$I = \int \frac{dt}{1+t^2}$$
$$= \tan^{-1} t + C$$
$$= \tan^{-1}\left(e^x\right) + C$$

(iii) Option (a) is correct.

Explanation: $\cot^{-1}(-x) = \pi - \cot^{-1} x$

(iv) Option (a) is correct.

Explanation:

Given that,
$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{1/5} = 0$$
$$\Rightarrow \quad \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} = -x^{1/5}$$
$$\Rightarrow \quad \left(\frac{dy}{dx}\right)^{\frac{1}{4}} = -\left(x^{1/5} + \frac{d^2y}{dx^2}\right)$$

On squaring both sides, we get
$$\left(\frac{dy}{dx}\right)^{\frac{1}{2}} = \left(x^{1/5} + \frac{d^2y}{dx^2}\right)^2$$

Again, on squaring both sides, we get
$$\frac{dy}{dx} = \left(x^{\frac{1}{5}} + \frac{d^2y}{dx^2}\right)^4$$

Order = 2, degree = 4

(v) Option (c) is correct.

Explanation:

If $A \subset B$, then $A \cap B = A$

$\Rightarrow \quad P(A \cap B) = P(A)$

Also, $\quad P(A) < P(B) \quad$ ...(i)

Consider, $\quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \quad$ ...(ii)

It is known that, $\quad P(B) \le 1$

$$\Rightarrow \quad \frac{1}{P(B)} \ge 1$$
$$\Rightarrow \quad \frac{P(A)}{P(B)} \ge P(A)$$

From equation (ii), we obtain
$$\Rightarrow \quad P\left(\frac{A}{B}\right) \ge P(A)$$

(vi) Option (b) is correct.

Explanation: Given inverse of $\begin{bmatrix} 1 & 2 \\ k & 6 \end{bmatrix}$ does not exist.

$\Rightarrow$ Matrix is singular.
$$\begin{vmatrix} 1 & 2 \\ k & 6 \end{vmatrix} = 0$$
$$\Rightarrow \quad 6 - 2k = 0$$
$$\Rightarrow \quad k = 3$$

(vii) Option (a) is correct.

Explanation: Given that,
$$f(x) = e^{|x|}$$

The functions e^x and $|x|$ are continuous functions for all real value of x.

Since, e^x is differentiable everywhere but $|x|$ is non-differentiable at $x = 0$.

Thus, the given functions $f(x) = e^{|x|}$ is continuous everywhere but not differentiable at $x = 0$.

(viii) Option (d) is correct.

Explanation:
$$\cot\left(\frac{\pi}{2} + \tan-1\frac{1}{\sqrt{2}}\right) = \tan\left(\tan-1\frac{1}{\sqrt{2}}\right)$$
$$\cot\left(\frac{\pi}{2} + \tan-1\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$$

(ix) Option (d) is correct.

Explanation: Statement 1 and 2 are independent of each other because a universal relation and an empty relation are mutually exclusive and do not imply each other.

(x) Option (a) is correct.

Explanation: Given $\quad A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}$
$$AB = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 0 \end{bmatrix}$$
$$BA = \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 0 \end{bmatrix}$$
$$\Rightarrow \quad AB = BA$$

Now,
$$(A+B)^2 = (A+B)(A+B)$$
$$= A^2 + AB + BA + B^2$$
$$= A^2 + AB + AB + B^2 \quad \{AB = BA\}$$
$$= A^2 + 2AB + B^2$$

Given Assertion (A) is true.

Also, $AB = BA$

Reason (R) is true and correct explanation of Assertion (A).

(xi) Option (a) is correct.

Explanation: Here, Z = no. of heads – no. of tails

No. of heads	No. of tails	Values of Z
0	6	$0 - 6 = -6$
1	5	$1 - 5 = -4$
2	4	$2 - 4 = -2$
3	3	$3 - 3 = 0$
4	2	$4 - 2 = 2$
5	1	$5 - 1 = 4$
6	0	$6 - 0 = 6$

(xii) Let,

$$\Delta = \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 7 & 0 \\ 3 & 8 & 0 \\ 5 & 9 & 0 \end{vmatrix} [C_3 \to C_3 - 9C_2 - C_1]$$

$$= 0$$

Since, all the values of C_3 are zero so determinant value is zero.

(xiii) $A_1 \cup A_2 \cup A_3 = A$ and $A_1 \cap A_2 \cap A_3 = \phi$

(xiv) The function in the graph is onto or surjective. A function $f : X \to Y$ is said to be surjective or onto if, every element of Y is the image of some elements in X.

The condition for a surjective function is for every $y \in Y$, there is an element in X such that $f(x) = y$.

(xv) Let A be the event that problem solved by 1st student.

Let B be the event that problem solved by 2nd student.

$$P(A) = \frac{1}{3} \text{ and } P(A) = \frac{1}{5}$$

Both the events are independent and problem is solved if atleast one student solves the problem.

$$\text{Required probability} = P(\overline{A \cup B})$$

$$= 1 - P(\overline{A})P(\overline{B})$$

$$= 1 - \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right) = \frac{7}{15}$$

2. (i) Given, $y = e^{\sin x^2}$

Let $y = e^z$, where $z = \sin t$ and $t = x^2$

On differentiating both sides of t w.r.t. x, we get

$$\frac{dt}{dx} = 2x \quad \ldots(i)$$

On differentiating both sides of z w.r.t. t, we get

$$\frac{dz}{dt} = \cos t \quad \ldots(ii)$$

On differentiating both sides of y w.r.t. z, we get

$$\frac{dy}{dz} = e^z \quad \ldots(iii)$$

Using chain rule of derivative,

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dt} \times \frac{dt}{dx} = e^z \times \cos t \times 2x$$

[from eqs. (i), (ii) and (iii)]

$$= 2xe^{\sin t} \cos t = 2xe^{\sin x^2} \cos x^2$$

OR

(ii) Given,

$$f(x) = \frac{x}{3} + \frac{3}{x}$$

$$f'(x) = \frac{1}{3} - \frac{3}{x^2}$$

For decreasing, $f'(x) < 0 \Rightarrow \frac{1}{3} - \frac{3}{x^2} < 0$

$\Rightarrow x^2 < 9 \Rightarrow -3 < x < 3$

Since, $f(x)$ is not defined at $x = 0$.

So, $f(x)$ decreasing in $(-3, 0) \cup (0, 3)$.

3. Let $f(x) = (1 - x^2) \cdot \sin x . \cos^2 x$

as $f(-x) = -f(x)$

$\Rightarrow f$ is an odd function.

$$\therefore I = \int_{-\pi}^{\pi} (1 - x^2) \sin x \cdot \cos^2 x \cdot dx = 0$$

4. We have, the equation of the curve is

$$3x^2 - y^2 = 8 \quad \ldots(i)$$

Also, the given equation of the line is $x + 3y = 8$.

$$\Rightarrow \quad 3y = 8 - x$$

$$\Rightarrow \quad y = -\frac{x}{3} + \frac{8}{3}$$

Thus, slope of the line is $-\frac{1}{3}$ which should be equal to the slope of the equation of normal to the curve. On differentiating equation (i) with respect to x, we get

$$6x - 2y\frac{dy}{dx} = 0.$$

$\Rightarrow \frac{dy}{dx} = \frac{6x}{2y} = \frac{3x}{y}$ = Slope of the tangent to the curve

Now, slope of normal to the curve

$$= -\frac{1}{\left(\frac{dy}{dx}\right)} = -\frac{1}{\left(\frac{3x}{y}\right)} = -\frac{y}{3x}$$

$$\therefore \quad -\left(\frac{y}{3x}\right) = -\frac{1}{3}$$

$$\Rightarrow \quad -3y = -3x$$

$$\Rightarrow \quad y = x$$

On substituting the value of the given equation of the curve, we get

$$3x^2 - x^2 = 8$$

$$\Rightarrow \quad 2x^2 = 8$$

$$\Rightarrow \quad x^2 = 4$$

$$\Rightarrow \quad x = \pm 2$$

For, $x = 2$

$$3(2)^2 - y^2 = 8$$

$$\Rightarrow \quad y^2 = 4$$

$$\Rightarrow \quad y = \pm 2$$

So, the points at which normal is parallel to the given line are (±2, ±2).

Hence, the equation of normal at (±2, ±2) is

$$\Rightarrow \quad y-(\pm 2) = -\frac{1}{3}\left[x-(\pm 2)\right]$$

$$\Rightarrow \quad 3[y-(\pm 2)] = -[x-(\pm 2)]$$

$$\therefore \quad x+3y \pm 8 = 0$$

5. **(i)** Given that $\int \frac{x\sin^{-1}\left(x^2\right)}{\sqrt{1-x^4}}dx$

Put $\sin^{-1}(x^2) = y$

$$\Rightarrow \quad \frac{x}{\sqrt{1-x^4}}dx = \frac{dy}{2}$$

$$\Rightarrow \quad \frac{1}{2}\int y\,dy = \frac{1}{4}y^2 + C$$

$$\therefore \quad \int \frac{x\sin^{-1}\left(x^2\right)}{\sqrt{1-x^4}}dx = \frac{1}{4}\left(\sin^{-1}\left(x^2\right)\right)^2 + C$$

OR

(ii) Put $2x = t$,

$$\therefore \quad dx = \frac{1}{2}dt$$

$$\therefore \quad I = \int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2}\right]e^{2x}dx$$

$$= \int_2^4 \left[\frac{1}{t} - \frac{1}{t^2}\right]e^t dt$$

$$= \left[\frac{1}{t}e^t\right]_2^4 = \frac{e^4}{4} - \frac{e^2}{2}$$

Since, $\int \left(f(x)+f'(x)\right).e^x dx = e^x f(x) + C$

6.
$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right]$$

$$= \tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$$

$$= \tan^{-1}\left[\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]$$

$$= \tan^{-1}\left[\tan\left\{\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right] = \frac{\pi}{4}+\frac{x}{2}$$

7.
$$x^2\left(\tan^{-1}x\right)^2 + \left(\cot^{-1}x\right)^2 = \frac{5\pi^2}{8}$$

$$\Rightarrow \quad \left(\tan^{-1}x\right)^2 + \left(\frac{\pi}{2} - \tan^{-1}x\right)^2 = \frac{5\pi^2}{8}$$

$$\Rightarrow \quad 2\left(\tan^{-1}x\right)^2 - \pi\tan^{-1}x + \frac{\pi^2}{4} - \frac{5\pi^2}{8} = 0$$

$$\Rightarrow \quad 2\left(\tan^{-1}x\right)^2 - \pi\tan^{-1}x - \frac{3\pi^2}{8} = 0$$

$$\Rightarrow \quad \tan^{-1}x = \frac{\pi \pm \sqrt{\pi^2 + 3\pi^2}}{4}$$

$$\Rightarrow \quad \tan^{-1}x = \frac{3\pi}{4}, \frac{-\pi}{4}$$

$$\therefore \quad x = -1$$

8.
$$\int \frac{x+1}{(x+2)(x+3)}dx = \int \frac{x+2-1}{(x+2)(x+3)}dx$$

$$= \int \frac{1}{(x+3)}dx - \int\left\{\frac{1}{(x+2)} - \frac{1}{(x+3)}\right\}dx$$

$$= \log|x+3| - \log|x+2| + \log|x+3| + C$$

$$\Rightarrow \int \frac{x+1}{(x+2)(x+3)}dx = 2\log|x+3| - \log|x+2| + C$$

9. **(i)** Given $f(x) = \sin^4 x + \cos^4 x$

$$\therefore \quad f'(x) = 4\sin^3 x\cos x - 4\cos^3 x\sin x$$

$$= -4\sin x\cos x[-\sin^2 x + \cos^2 x]$$

$$= -2\sin 2x\cos 2x = -\sin 4x$$

On equating

$$f'(x) = 0 \Rightarrow -\sin 4x = 0$$

$$\Rightarrow \quad 4x = 0, \pi, 2\pi, \ldots$$

$$\Rightarrow \quad x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \ldots$$

Sub-intervals are $\left[0,\frac{\pi}{4}\right], \left[\frac{\pi}{4},\frac{\pi}{2}\right]$

Now, $f'(x) < 0$ in $\left[0,\frac{\pi}{4}\right]$

$\Rightarrow$ $f(x)$ is decreasing in $\left[0,\frac{\pi}{4}\right]$

and, $f'(x) > 0$ in $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$

$\therefore$ $f'(x)$ is increasing in $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$.

OR

(ii)
$$x = a(\cos 2\theta + 2\theta\sin 2\theta)$$

$$\Rightarrow \quad \frac{dx}{d\theta} = a(-2\sin 2\theta + 2\sin 2\theta + 4\theta\cos 2\theta)$$

$$\Rightarrow \quad \frac{dx}{d\theta} = a(4\theta\cos 2\theta)$$

and
$$y = a(\sin 2\theta - 2\theta\cos 2\theta)$$

$$\Rightarrow \quad \frac{dy}{d\theta} = a(2\cos 2\theta + 4\theta\sin 2\theta - 2\cos 2\theta)$$

$$\Rightarrow \quad \frac{dy}{d\theta} = a(4\theta\sin 2\theta)$$

Using (i) and (ii),

$$\frac{dy}{dx} = \frac{a\left(4\theta\sin 2\theta\right)}{a\left(4\theta\cos 2\theta\right)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$$

Differentiating again with respect to x, we get

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2\sec^2 2\theta \cdot \frac{d\theta}{dx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2\sec^2 2\theta \cdot \frac{1}{a\left(4\theta\cos 2\theta\right)}$$

$$\Rightarrow \left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{8}} = 2\sec^2\frac{\pi}{4}\cdot\frac{1}{a\left(4\frac{\pi}{8}\cos\frac{\pi}{4}\right)}$$

$$= \frac{8\sqrt{2}}{\pi a}$$

10. **(i)** **(a)** Let P be the event that the shell fired from hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial with the guns operating independently:

$$E_1 = PQ, E_2 = \overline{P}\,\overline{Q}, E_3 = \overline{P}Q, E_4 = P\overline{Q}$$

Let E = The shell fired from exactly one of them hits the plane.

$$P(E_1) = 0.3 \times 0.2 = 0.06,$$
$$P(E_2) = 0.7 \times 0.8 = 0.56$$
$$P(E_3) = 0.7 \times 0.2 = 0.14$$
$$P(E_4) = 0.3 \times 0.8 = 0.24$$

$$P\left(\frac{E}{E_1}\right) = 0, P\left(\frac{E}{E_2}\right) = 0, P\left(\frac{E}{E_3}\right) = 1, P\left(\frac{E}{E_4}\right) = 1$$

$$P(E) = P(E_1).P\left(\frac{E}{E_1}\right) + P(E_2).P\left(\frac{E}{E_2}\right) + P(E_3).P\left(\frac{E}{E_3}\right) + P(E_4).P\left(\frac{E}{E_4}\right)$$

$$= 0.14 + 0.24 = 0.38$$

(b) By Bayes' Theorem,

$$P\left(\frac{E_3}{E}\right) = \frac{P(E_3)\cdot P\left(\frac{E}{E_3}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right) + P(E_3)\cdot P\left(\frac{E}{E_3}\right) + P(E_4)\cdot P\left(\frac{E}{E_4}\right)}$$

$$= \frac{0.14}{0.38} = \frac{7}{19}$$

(c) Bayes' theorem defines the probability of an event based on the prior knowledge of the conditions related to the event whereas in case of the conditional probability, we find the reverse probabilities using Bayes' theorem.

(d) Consider an event E which occurs via two different events A and B. The probability of E is given by the value of total probability as:

$$P(E) = P(A \cap E) + P(B \cap E)$$

$$P(E) = P(A)P\left(\frac{E}{A}\right) + P(B)\left(\frac{E}{B}\right)$$

OR

(ii) **(a)** $P(A) = \frac{4}{5}, P(B) = \frac{3}{4}, P(C) = \frac{2}{3}$

Probability that A, B and C all will hit the target

$$= P(A \cap B \cap C)$$
$$= P(A)P(B)P(C)$$
$$= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$$
$$= \frac{2}{5}$$

(b) $$P(\overline{A}) = 1 - \frac{4}{5} = \frac{1}{5}$$

Probability that B, C will hit and A will lose

$$= P(\overline{A} \cap B \cap C)$$
$$= P(\overline{A})\cdot P(B)\cdot P(C)$$
$$= \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3}$$
$$= \frac{1}{10}$$

(c) $$P(\overline{B}) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$P(\overline{C}) = 1 - \frac{2}{3} = \frac{1}{3}$$

Probability that any two of A, B and C will hit

$$= P(\overline{A})P(B)P(C) + P(A)P(\overline{B})P(C) + P(A)P(B)P(\overline{C})$$

$$= \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3}$$

$$= \frac{1}{10} + \frac{2}{15} + \frac{1}{5}$$

$$= \frac{3+4+6}{30} = \frac{13}{30}$$

(d) Probability that none of them will hit the target $= P(\overline{A})P(\overline{B})P(\overline{C}) = \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{60}$

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11. **(a)** Let the number of children be x and the amount distributed by Seema for one student be ₹y.

So, $(x - 8)(y + 10) = xy$

$\Rightarrow$ $5x - 4y = 40$

and $(x + 16)(y - 10) = xy$

$\Rightarrow$ $5x - 8y = -80$

Here, $A = \begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

(b) $$AX = B \Rightarrow X = A^{-1}B$$

$$A^{-1} = -\frac{1}{20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= -\frac{1}{20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}\begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$$

$\Rightarrow$ $x = 32, y = 30$

(c) No. of students = 32

Amount given to each student = ₹ 30.

12. **(i)** The given D.E. is:

$$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$$

When, $x = 1, y = \frac{\pi}{4}$

$$\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = \frac{y}{x}\cos\left(\frac{y}{x}\right) + 1 \qquad \text{....(i)}$$

Put, $\frac{y}{x} = v$

or, $$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

From (i), $\cos v\left(v+x\dfrac{dv}{dx}\right)=v\cos v+1$

Or $\quad v\cos v+x\cos v\dfrac{dv}{dx}=v\cos v+1$

Or, $\quad x\cos v\dfrac{dv}{dx}=1$

Or, $\quad \int\cos v\,dv=\int\dfrac{dx}{x}$

Or, $\quad \sin v=\log x+C$

Or, $\quad \sin\left(\dfrac{y}{x}\right)=\log x+C$

Put $y=\dfrac{\pi}{4},x=1$

Or, $\quad \sin\dfrac{\pi}{4}=\log 1+C$

Or, $\quad \dfrac{1}{\sqrt{2}}=C$

The particular solution is

$$\sin\left(\frac{y}{x}\right)=\log x+\frac{1}{\sqrt{2}}$$

OR

(ii) $\displaystyle\int_0^{\pi}e^{2x}\sin\left(\frac{\pi}{4}+x\right)dx$

Let $I=\displaystyle\int_0^{\pi}e^{2x}\cdot\sin\left(\frac{\pi}{4}+x\right)dx$

Integrating by parts, without limits

$$I=\sin\left(\frac{\pi}{4}+x\right).\frac{e^{2x}}{2}-\int\cos\left(\frac{\pi}{4}+x\right).\frac{e^{2x}}{2}dx$$

$$=\sin\left(\frac{\pi}{4}+x\right).\frac{e^{2x}}{2}-\frac{1}{2}\int e^{2x}\cos\left(\frac{\pi}{4}+x\right)dx$$

$$=\sin\left(\frac{\pi}{4}+x\right).\frac{e^{2x}}{2}-\frac{1}{2}\left[\cos\left(\frac{\pi}{4}+x\right).\frac{e^{2x}}{2}-\int-\sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}dx\right]$$

$$I\left(1+\frac{1}{4}\right)=\frac{1}{4}e^{2x}\left\{2\sin\left(\frac{\pi}{4}+x\right)-\cos\left(\frac{\pi}{4}+x\right)\right\}$$

$$\Rightarrow\quad I=\frac{1}{5}e^{2x}\left\{2\sin\left(\frac{\pi}{4}+x\right)-\cos\left(\frac{\pi}{4}+x\right)\right\}$$

Now, $\displaystyle\int_0^{\pi}e^{2x}\sin\left(\frac{\pi}{4}+x\right)dx$

$$=\frac{1}{5}\left[e^{2x}\left\{2\sin\left(\frac{\pi}{4}+x\right)-\cos\left(\frac{\pi}{4}+x\right)\right\}\right]_0^{\pi}$$

$$=\frac{1}{5}\left[e^{2\pi}\left\{2\sin\left(\pi+\frac{\pi}{4}\right)-\cos\left(\pi+\frac{\pi}{4}\right)\right\}-2\sin\frac{\pi}{4}+\cos\frac{\pi}{4}\right]$$

$$=\frac{1}{5}\left(e^{2\pi}\left(-2\sin\frac{\pi}{4}+\cos\frac{\pi}{4}\right)-\frac{2}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)$$

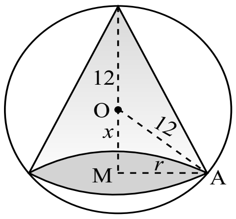
$$=\frac{1}{5}\left[-\frac{e^{2\pi}}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right]$$

$$\therefore\int_0^{\pi}e^{2x}\sin\left(\frac{\pi}{4}+x\right)dx=-\frac{1}{5\sqrt{2}}[e^{2\pi}+1]$$

13. (i) Given, r is the radius of cone, h is the height of cone, and x is the distance between centre of sphere and centre of base of cone,

Given, radius of sphere R = 12 cm

$\therefore\quad h=12+x \quad$ ...(i)



Volume of cone (V) $=\dfrac{1}{3}\pi r^2h$

$=\dfrac{1}{3}\pi(12^2-x^2)(12+x)$

$[\because$ In ΔOMA, $12^2=x^2+r^2]$

$=\dfrac{1}{3}\pi(144-x^2)(12+x)$

$\therefore\quad\dfrac{dV}{dx}=\dfrac{1}{3}\pi\,[(144-x^2).1+(12+x)(-2x)]$

$=\dfrac{1}{3}\pi\,[144-x^2-24x-2x^2]$

$=\dfrac{1}{3}\pi\,[144-24x-3x^2]$

For maximum or minimum volume, put $\dfrac{dV}{dx}=0$

i.e., $\dfrac{1}{3}\pi\left[144-24x-3x^2\right]=0$

$\Rightarrow\quad 144-24x-3x^2=0$

$\Rightarrow\quad x^2+8x-48=0$

$\Rightarrow\quad (x+12)(x-4)=0$

$\Rightarrow\quad x=-12$ and $x=4$

Now, $\quad\dfrac{d^2V}{dx^2}=\dfrac{1}{3}\pi[-24-6x]$

$\therefore\quad\left(\dfrac{d^2V}{dx^2}\right)_{x=4}=\dfrac{1}{3}\pi[-24-24]<0$

Hence, at x = 4 volume of cone is maximum and height of cone $(h)=12+x=12+4=16$ cm.

OR

(ii) Given ΔABC is right angle triangle and hypotenuse is c unit.

To Prove: Area is maximum when triangle is an isosceles triangle.

Proof: In ΔABC, $\quad\angle B=90°$

$\therefore\quad c^2=x^2+y^2$

Area of $\Delta ABC=\dfrac{1}{2}$ base × height

$A=\dfrac{1}{2}xy$

$A^2=\dfrac{1}{4}x^2y^2 \qquad$ (Let $A^2=Z$)

$$Z = \frac{1}{4}x^2(c^2 - x^2)$$

$$Z = \frac{c^2x^2}{4} - \frac{x^4}{4}$$

$$\frac{dZ}{dx} = \frac{d}{dx}\frac{c^2x^2}{4} - \frac{d}{dx}\frac{x^4}{4}$$

$$\frac{dZ}{dx} = \frac{2c^2x}{4} - \frac{4x^3}{4}$$

$$\frac{dZ}{dx} = \frac{c^2x}{2} - x^3$$

For maximum/minimum,

$$\frac{dZ}{dx} = 0$$

$$\frac{c^2x}{2} - x^3 = 0$$

$$x\left(\frac{c^2}{2} - x^2\right) = 0$$

Either $x = 0$ (Not possible because $x > 0$)

So, $$\frac{c^2}{2} - x^2 = 0$$

$$\frac{c^2}{2} = x^2$$

$$c^2 = 2x^2$$

Now, from equation (i) $x^2 + y^2 = 2x^2$

$$y^2 = x^2 \Rightarrow y = x$$

Again, $$\frac{dZ}{dx} = \frac{c^2x}{2} - x^3$$

Again, $$\frac{d^2Z}{dx^2} = \frac{c^2}{2} - 3x^2$$

$$\frac{d^2Z}{dx^2} = \frac{2x^2}{2} - 3x^2 = -2x^2$$

$$\frac{d^2Z}{dx^2} < 0$$

∴ Area is maximum when $x = y$.

∴ ΔABC is an isosceles right angle triangle.

Hence Proved.

14. **(a)** X = no. of kings = 0, 1, 2

(b) $P(X = 0) = P(\text{no king}) = \frac{48}{52} \times \frac{47}{51} = \frac{188}{221}$

$P(X = 1) = P(\text{one king and one non-king})$

$$= \frac{4}{52} \times \frac{48}{51} \times 2 = \frac{32}{221}$$

$P(X = 2) = P\text{ (two kings)}$

$$= \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$$

Probability distribution is given by

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

(c) Mean $= \sum X \cdot P(X) = \frac{32}{221} + \frac{2}{221} = \frac{34}{221}$ or $\frac{2}{13}$

Section-B

15. **(i) Option (a) is correct.**

Explanation:

Equation of line passing through the points A(0, 5, –4) and B(4, –2, 1) is

$$\frac{x-0}{4-0} = \frac{y-5}{-2-5} = \frac{z+4}{1+4} = r$$

Or, $$\frac{x}{4} = \frac{y-5}{-7} = \frac{z+4}{5} = r$$

Given, X-coordinate of P is 3.

∴ $$4r = 3 \Rightarrow r = \frac{3}{4}$$

Therefore, Y-coordinate $= -7r + 5$

$$= -7 \times \frac{3}{4} + 5$$

$$= \frac{-21+20}{4} = \frac{-1}{4}$$

(ii) Option (b) is correct.

Explanation:

$P_1 : x - 2y + 4z - 10 = 0$

$P_2 : 18x + 17y + kz - 50 = 0$

P_1 and P_2 are perpendicular if $18 - 34 + 4k = 0$

$\Rightarrow \quad 4k = 16$

$\Rightarrow \quad k = 4$

(iii) Option (a) is correct.

Explanation:

Equation of the plane is $2x + y - z = 5$

Dividing by 5

$$\Rightarrow \quad \frac{2x}{5} + \frac{y}{5} - \frac{z}{5} = 1$$

$$\Rightarrow \quad \frac{x}{\frac{5}{2}} + \frac{y}{5} + \frac{z}{-5} = 1$$

Sum of intercepts

$$= a + b + c$$

$$= \frac{5}{2}, \left[\text{Since, } a = \frac{5}{2},\ b = 5, c = -5\right]$$

(iv) Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

Any vector in the direction of a vector $\vec{a}$ is given by

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{1^2 + 2^2 + 2^2}} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$

∴ Vector in the direction of $\vec{a}$ with magnitude 9

$$= 9\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} = 3\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

$$= 3\hat{i} - 6\hat{j} + 6\hat{k}$$

(v) Given, $\overline{AB} = \left(\hat{j} + \hat{k}\right), \overline{AC} = \left(3\hat{i} - \hat{j} + 4\hat{k}\right)$

Now, ABEC represent a parallelogram with AE as the diagonal.

$$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{AC}$$

$$= \left(\hat{j} + \hat{k}\right) + \left(3\hat{i} - \hat{j} + 4\hat{k}\right) = 3\hat{i} + 5\hat{k}$$

Now, $$\left|\overrightarrow{AE}\right| = \left|\sqrt{(3)^2 + (5)^2}\right| = \left|\sqrt{9+25}\right| = \sqrt{34}$$

∴ $$\left|\overrightarrow{AD}\right| = \frac{1}{2}\sqrt{34} \text{ units}$$

16. **(i)** Given,

$$\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$$

and $$\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$$

So, $$\vec{a} + \vec{b} = \left(\hat{i} - \hat{j} + 7\hat{k}\right) + \left(5\hat{i} - \hat{j} + \lambda\hat{k}\right)$$

$$= 6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}$$

$$\vec{a} - \vec{b} = \left(\hat{i} - \hat{j} + 7\hat{k}\right) - \left(5\hat{i} - \hat{j} + \lambda\hat{k}\right)$$

$$= -4\hat{i} + (7-\lambda)\hat{k}$$

Since, $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$ are perpendicular vectors,

$\therefore$ $$\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right) = 0$$

$$\left\{6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}\right\}\cdot\left\{-4\hat{i} + (7-\lambda)\hat{k}\right\} = 0$$

or $$-24 + (7+\lambda)(7-\lambda) = 0$$

or $$49 - \lambda^2 - 24 = 0$$

or $$\lambda^2 = 49 - 24 = 25$$

or $$\lambda = \pm 5 \text{ units}$$

OR

(ii) Given, $$\left|\vec{a}+\vec{b}\right| = \left|\vec{a}\right|$$

$$\left|\vec{a}+\vec{b}\right|^2 = \left|\vec{a}\right|^2$$

Or $$\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\vec{a}\cdot\vec{b} = \left|\vec{a}\right|^2$$

$$\left|\vec{b}\right|^2 + 2\vec{a}\cdot\vec{b} = 0$$

Now, $$\left(2\vec{a}+\vec{b}\right)\cdot\vec{b} = 2\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b}$$

$$\vec{b}\cdot\left(2\vec{a}+\vec{b}\right) = 0 \quad \text{....(i)}$$

From eqn. (i), $2\vec{a}+\vec{b}$ is $\perp$ to $\vec{b}$.

17. **(i)** Equation of the plane

$$3x - y + 4z = 2$$

$\therefore$ Direction ratio of perpendicular line to the plane is (3, – 1, 4).

Equation of the line is passing through (3, – 2, 1) and direction ratios are (3, – 1, 4) is

$$\frac{x-3}{3} = \frac{y+2}{-1} = \frac{z-1}{4} = \lambda$$

General point on the line is $(3\lambda + 3, -\lambda - 2, 4\lambda + 1)$

Foot of perpendicular lies in the plane

$\therefore$ $3(3\lambda + 3) - (-\lambda - 2) + 4(4\lambda + 1) = 2$

$$9\lambda + 9 + \lambda + 2 + 16\lambda + 4 = 2$$

$$26\lambda = 2 - 15$$

$$\lambda = \frac{-13}{26} = \frac{-1}{2}$$

Hence, point of intersection of the line and plane is

$$\left(\frac{-3}{2} + 3, \frac{1}{2} - 2, -2 + 1\right) = \left(\frac{3}{2}, \frac{-3}{2}, -1\right)$$

Let the image of the point (3, – 2, 1) be (x, y, z).

$\therefore$ $$\frac{x+3}{2} = \frac{3}{2} \Rightarrow x = 0$$

$$\frac{y-2}{2} = \frac{-3}{2} \Rightarrow y = -1$$

$$\frac{z+1}{2} = -1 \Rightarrow z = -3$$

Hence, coordinate of image is (0, – 1, – 3).

Examiner's Comment

- Most of the candidates applied the condition for perpendicularity incorrectly and used incorrect direction ratios. It led to calculate incorrect foot of the perpendicular and incorrect image of the given point.

Answering Tip

- Do enough practice of problems based on applications of distance of a point with reference to a line or a plane.

OR

(ii) Given lines can be written as

$$l_1 = \frac{x-1}{-3} = \frac{y-2}{\frac{p}{7}} = \frac{z-3}{2}$$

and $$l_2 = \frac{x-1}{\left(\frac{-3p}{7}\right)} = \frac{y-5}{1} = \frac{z-6}{-5}$$

Since, the lines are perpendicular.

$$\Rightarrow \quad (-3)\left(-\frac{3p}{7}\right) + \left(\frac{p}{7}\right)(1) + (2)(-5) = 0$$

$$\frac{9p}{7} + \frac{p}{7} - 10 = 0$$

$\therefore$ $$p = 7$$

Equation of line passing through (3, 2, –4) and parallel to l_1 is

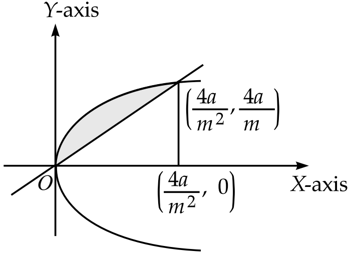
$$\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}$$

18. **(i)** Equation of the parabola $y^2 = 4ax$ represents a parabola with vertex at the origin.

The chord $y = mx$ passes through the origin.

The point of intersection of the curve and the line is $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$

Shaded region represents the required area


Y-axis
$\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$
O
$\left(\frac{4a}{m^2}, 0\right)$
X-axis

(ii) Let, $I_1 = \int_0^{4a/m^2} 2\sqrt{ax}\,dx$

$$= 2\sqrt{a}\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^{4a/m^2}$$

$$= 2\sqrt{a}\times\frac{2}{3}\left(\frac{4a}{m^2}\right)^{3/2}$$

$$= \frac{4\sqrt{a}}{3}\,\frac{8a\sqrt{a}}{m^3}$$

$$= \frac{32}{3}\,\frac{a^2}{m^3}$$

(iii) Required area $= \int_0^{\frac{4a}{m^2}} 2\sqrt{ax}\,dx - \int_0^{\frac{4a}{m^2}} mx\,dx$

$$= I_1 - m\left[\frac{x^2}{2}\right]_0^{\frac{4a}{m^2}}$$

$$= I_1 - \frac{m}{2}\left(\frac{4a}{m^2}\right)^2$$

$$= \frac{32}{3}\,\frac{a^2}{m^3} - \frac{m}{2}\,\frac{16a^2}{m^4} \quad \text{[from (ii)]}$$

$$= \frac{32}{3}\,\frac{a^2}{m^3} - \frac{8a^2}{m^3}$$

$$= \frac{a^2}{m^3}\left(\frac{32}{3} - 8\right)$$

$$= \frac{a^2}{m^3}\left(\frac{32-24}{3}\right)$$

$$= \frac{8a^2}{3m^3}\text{ sq. units}$$

Section-C

19. (i) Option (c) is correct.

Explanation:

Cost function,

$$C(x) = 3x^2 - 2x + 6$$

$$\text{Average cost} = \frac{3x^2 - 2x + 6}{x}$$

$$= 3x - 2 + \frac{6}{x}$$

At $x = 3$, Average cost $= 9 - 2 + 2 = 9$

(ii) Option (b) is correct

Explanation: The correct statements are:

I: Regression predicts/explains the variation in one variable based on another.

III: The order of variables is crucial in regression.

IV: The independent variable is on the x-axis in regression.

(iii) Option (b) is correct.

Given, $9x + 3y - 46 = 0$

Regression line of x on y:

$$9x = -3y + 46$$

$$x = -\frac{1}{3}y + \frac{46}{9}$$

i.e., $b_{xy} = -\frac{1}{3}$

and $3x + 12y - 7 = 0$

Regression line of y on x :

$$12y = -3x + 7$$

i.e., $y = -\frac{1}{4}x + \frac{7}{12}$

i.e., $b_{yx} = -\frac{1}{4}$

$\therefore$ correlation coefficient,

$$r = -\sqrt{b_{yx}b_{xy}}$$

$$= -\sqrt{\left(-\frac{1}{3}\right)\times\left(-\frac{1}{4}\right)}$$

$$= -\sqrt{\frac{1}{12}}$$

$$= -\frac{1}{2\sqrt{3}}$$

(iv) Let x be the number of units produced and sold.

$$C(x) = \text{Fixed cost} + \text{Variable cost}$$

$$C(x) = 10{,}000 + 15x$$

$R = px$ is the total revenue

$\therefore$ $R(x) = 20x$

(v)

$$C(x) = \frac{3}{4}x^2 - 7x + 27$$

$$A.C. = \frac{C(x)}{x} = \frac{3}{4}x - 7 + \frac{27}{x}$$

$$M.C. = \frac{d}{dx}(C(x)) = \frac{3}{4}.2x - 7$$

$$= \frac{3}{2}x - 7$$

$\because$ $AC = MC$

$$\frac{3}{4}x - 7 + \frac{27}{x} = \frac{3}{2}x - 7$$

$$3x^2 + 108 = 6x^2$$

$$3x^2 = 108$$

$$x^2 = 36$$

$$x = 6$$

20. **(i)** Given, $MC = 0.003x^2 - 0.01x + 2.5$

$\therefore$ Increase in cost (from 100 to 200 units)

$$C = \int_{100}^{200}(0.003x^2 - 0.01x + 2.5)dx$$

$$= \left[\frac{0.003x^3}{3} - \frac{0.01x^2}{2} + 2.5x\right]_{100}^{200}$$

$$= [0.001x^3 - 0.005x^2 + 2.5x]_{100}^{200}$$

$$= 0.001[(200)^3 - (100)^3] - 0.005[(200)^2 - (100)^2] + 2.5(200 - 100)$$

$$= 7000 - 150 + 250$$

$$= 7100$$

OR

(ii) Given, total cost function,

$$C(x) = x + 2x^3 - 3.5x^2$$

Here, $AC = \frac{C}{x}$

$$= \frac{1}{x}(x + 2x^3 - 3.5x^2)$$

$$= 1 + 2x^2 - 3.5x$$

$$= 2x^2 - 3.5x + 1$$

We know that, marginal average cost (MAC) is

$$MAC = \frac{d}{dx}(AC)$$

$$= \frac{d}{dx}(2x^2 - 3.5x + 1)$$

$$\Rightarrow \quad MAC = 4x - 3.5$$

Since, MAC is linear, it is a straight line.

$\Rightarrow$ It will cut X-axis at (0.875, 0) and Y-axis at (0, – 3.5)

21. **(i)** Let x represents the marks in Mathematics and y represents the marks in Physics.

Given, $\quad \bar{x} = 84, \ \bar{y} = 81$

$$\sigma_x = 7, \sigma_y = 4$$

And $\quad r = 0.86$

$$\therefore \quad b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x}$$

$$= 0.86 \times \frac{4}{7} = 0.49$$

$\therefore$ Regression equation of y on x

$$\Rightarrow \quad y - \bar{y} = b_{yx}(x - \bar{x})$$

$$\Rightarrow \quad y - 81 = 0.49(x - 84)$$

$$\Rightarrow \quad y - 81 = 0.49x - 41.16$$

$$\Rightarrow \quad y = 0.49x + 39.84$$

Putting $x = 92$,

$$y = 45.08 + 39.84 = 84.92$$

Hence, the likely marks in Physics are 84.92.

OR

(ii) Given,

Correlation coefficient (r) = 0.6

$$\text{Var}(x) = 225$$

$$\therefore \quad \sigma_x = 15$$

$$\text{and} \quad \text{Var}(y) = 400$$

$$\therefore \quad \sigma_y = 20$$

and mean of $x(\bar{x}) = 10$

mean of $y(\bar{y}) = 20$

(i) Regression line y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$\text{or} \quad y - \bar{y} = r.\frac{\sigma_y}{\sigma_x}(x - \bar{x}).$$

$$\Rightarrow \quad y - 20 = 0.6 \times \frac{20}{15}(x - 10)$$

$$\Rightarrow \quad y - 20 = 0.8(x - 10)$$

$$\Rightarrow \quad y - 20 = 0.8x - 8$$

$$\Rightarrow \quad y = 0.8x + 12$$

Regression line x on y

$$\text{or} \quad x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - \bar{x} = r.\frac{\sigma_x}{\sigma_y}(y - \bar{y})$$

$$x - 10 = 0.6 \times \frac{15}{20}(y - 20)$$

$$x - 10 = 0.45(y - 20)$$

$$\Rightarrow \quad x = 0.45\,y - 9 + 10$$

$$\Rightarrow \quad x = 0.45\,y + 1$$

(ii) Expected value of y, when $x = 2$

$$y = 0.8x + 12$$

$$\Rightarrow \quad y = 0.8 \times 2 + 12$$

$$\Rightarrow \quad y = 13.6$$

22. **(a)** Let x and y units of packet of mixes are purchased from S and T respectively. If Z is the total cost, then

$$Z = 10x + 4y \quad \text{....(i)}$$

is objective functions which we have to minimise.

Here, Constraints are:

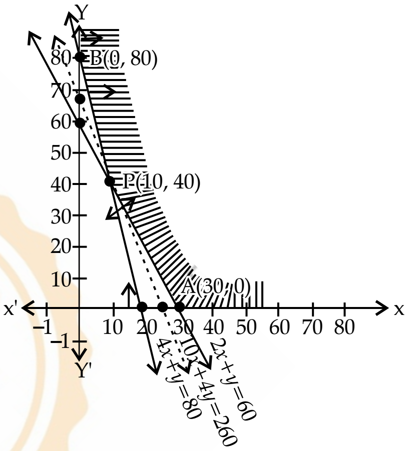
$$4x + y \geq 80 \quad \text{....(ii)}$$

$$2x + y \geq 60 \quad \text{....(iii)}$$

Also, $\quad x \geq 0 \quad$ ....(iv)

$$y \geq 0 \quad \text{.....(v)}$$

On plotting the graph of above constraints or inequalities (ii), (iii), (iv) and (v) we get shaded region having corner points A, P, B on possible region. For coordinates of P



Point of Intersection of:

$$2x + y = 60 \quad \text{….(vi)}$$

And $\quad 4x + y = 80 \quad$ ….(vii)

From eqn. (vi) – eqn. (vii)

$$2x + y - 4x - y = 60 - 80$$

$$\Rightarrow \quad -2x = -20$$

$$\Rightarrow \quad x = 10$$

$$\Rightarrow \quad y = 40$$

$\therefore$ Co-ordinates of P = (10, 40)

(b) Now the value of 2 is evaluated at corner point in the following table:

Corner Point	$Z = 10x + 4y$
A(30, 0)	300
P(10, 40)	260
B(0, 80)	320

⟵ Minimum

Since, possible region is unbounded. Therefore, we have to draw the graph of the inequality.

$$10x + 4y < 260 \quad \text{….(viii)}$$

Since, the graph of inequality (viii) does not have any point common.

So, the minimum value of 2 is 260 at (10, 40)

i.e., minimum cost of each bottle is ₹260 if the company purchases 10 packets of mixes from S and 40 packets of mixes from supplier T.

❑❑

Sample Question Paper-3

MATHEMATICS

Class-12

SOLVED

Time Allowed : 3 hours **Maximum Marks : 80**

You are allowed an additional 15 minutes for only reading the paper.
You must NOT start writing during reading time.
The question paper has 12 printed pages.
The Question Paper is divided into three sections and has 22 questions in all.

Section A is compulsory and has fourteen questions.
You are required to attempt all questions either from Section B or Section C.
Section B and Section C have four questions each.
Internal choices have been provided in two questions of 2 marks, two questions of 4 marks and two questions of 6 marks in Section A.
Internal choices have been provided in one question of 2 marks and one question of 4 marks each in Section B and Section C.
While attempting Multiple Choice Questions in Section A, B and C, you are required to write only ONE option as the answer.
The intended marks for questions or parts of questions are given in the brackets [].
All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
Mathematical tables and graph papers are provided.
Instruction to Supervising Examiner:
Kindly read aloud the instructions given above to all the candidates present in the examination hall.

Section-A [65 marks]

1. In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed. **[15]**

(i) Matrices A and B will be inverse of each other only if

(a) $AB = BA$ **(b)** $AB = BA = 0$ **(c)** $AB = 0, BA = I$ **(d)** $AB = BA = I$

[Understanding]

(ii) The value of $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ is

(a) $\frac{\pi}{2}$ **(b)** $\frac{2\pi}{3}$ **(c)** $\frac{\pi}{6}$ **(d)** $\frac{\pi}{12}$ **[Recall]**

(iii) The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is

(a) $\frac{\pi}{2}$ **(b)** $\frac{3\pi}{2}$ **(c)** $\frac{5\pi}{2}$ **(d)** $\frac{7\pi}{2}$ **[Application]**

(iv) Assertion (A): The order and the degree of differential equation $2+\left(\frac{dy}{dx}\right)^3 = \frac{d^3y}{dx^3}$ are 3 and 1 respectively.

Reason (R): The order of a differential equation is the order of the highest derivative appearing in the differential equation. The degree of the differential equation is the power of the highest order derivative.
Which of the following is correct?
(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true. **[Analysis]**

(v) If $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then the value of $P(B/A)$ is

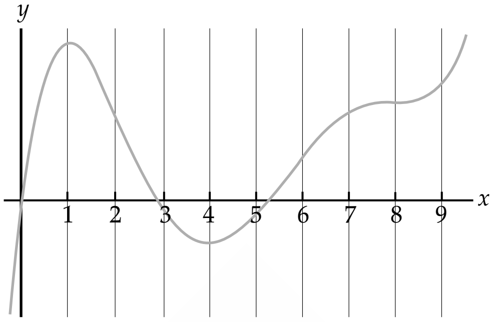
(a) $\frac{1}{8}$ **(b)** $\frac{4}{7}$ **(c)** $\frac{5}{7}$ **(d)** $\frac{7}{8}$ AI **[Application]**

(vi) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = x$, then what is the value of $\begin{vmatrix} a & 2b & c \\ 4d & 8e & 4f \\ g & 2h & i \end{vmatrix}$?

(a) $8x$ **(b)** $12x$ **(c)** $4x$ **(d)** $2x$

[Understanding]

(vii) Shown below is the graph of a function, $f(x)$.



Statement 1: $(-\infty, 1)$, $(4, 8)$ & $(8, \infty)$ are the intervals on which the $f(x)$ increases.

Statement 2: $(-\infty, 1)$, $(4, 8)$ & $(-8, -\infty)$ are the intervals on which the $f(x)$ increases.

Which of the following is correct?

(a) Statement 1 is true and Statement 2 is false.
(b) Statement 2 is true and Statement 1 is false.
(c) Both the statements are true.
(d) Both the statements are false. **[Application]**

(viii) If $x = a\cos\theta$, $y = a\sin\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$ will be

(a) -1 **(b)** 1 **(c)** 0 **(d)** 2

(ix) **Statement 1:** Let A be the set of all 50 students of Class – IX in a school. Let $f : A \to N$ be a function defined by $f(x)$ = roll number of student 'x'. Then the function 'f' is one-one.

Statement 2: Let A be the set of all 50 students of Class – IX in a school. Let $f : A \to N$ be a function defined by $f(x)$ = roll number of student 'x'. Then the function 'f' is onto.

Which one of the following is correct?

(a) Statement 1 implies Statement 2.
(b) Statement 2 implies Statement 1.
(c) Statement 1 is true only if Statement 2 is true.
(d) Statements 1 and 2 are independent of each other. **[Understanding]**

(x) **Assertion (A):** Let the matrices $A = \begin{bmatrix} -3 & 2 \\ -5 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 \\ 5 & -3 \end{bmatrix}$ be such that $A^{100}B = BA^{100}$

Reason (R): $AB = BA$ implies $A^nB = BA^n$ for all positive integers n.

Which of the following is correct?

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true. **[Application]**

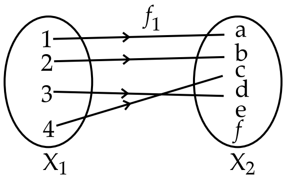
(xi) If two balls are drawn from a bag containing 3 white, 4 black and 5 red balls. Then, the probability that the drawn balls are of different colours is

(a) $\frac{1}{66}$ **(b)** $\frac{3}{66}$ **(c)** $\frac{19}{66}$ **(d)** $\frac{47}{66}$ **[Understanding]**

(xii) If $A = \begin{vmatrix} -8 & -2 & 1 \\ -3 & -5 & 0 \\ 6 & -9 & 4 \end{vmatrix}$, write the minor of the element a_{31}. **[Analysis]**

(xiii) Let L be a set of all straight lines in a plane. What is the relation R on L defined as 'perpendicular to' ?

[Understanding]



(xiv) The function $f : X \to Y$ is one-one. Give reason. **[Understanding]**

(xv) 50 tickets in a box are numbered 00, 01, 02, ..., 49. One ticket is drawn randomly from the box. Find the probability of the ticket having the product of its digits 7, given that the sum of the digits is 8. **[Application]**

2. **(i)** Find $\frac{dy}{dx}$, if $x^3 + y^3 = 3axy$. AI **[Understanding] [2]**

OR

(ii) The edge of a variable cube is increasing at the rate of 10 cm/s. How fast is the volume of the cube increasing when the edge is 5 cm long ? **[Understanding]**

3. Evaluate: $\int xe^x \, dx$ **[Evaluate] [2]**

4. Prove that the curves $y^2 = 4x$ and $x^2 + y^2 - 6x + 1 = 0$ touch each other at the point (1, 2). **[Application] [2]**

5. **(i)** Evaluate: $\int_{-1}^{2} \frac{|x|}{x} dx$. AI **[Evaluate] [2]**

OR

(ii) Evaluate: $\int_4^5 |x - 5| \, dx$. **[Evaluate]**

6. Solve for : $5 \tan^{-1} x + 3 \cot^{-1} x = 2\pi$ **[Analysis] [2]**

7. Prove that : $\tan^{-1}\frac{1}{2} = \frac{\pi}{4} - \tan^{-1}\frac{1}{3}$. AI **[Application] [4]**

8. Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$ AI **[Application] [4]**

9. **(i)** Water is dripping out from a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at the uniform rate of 2 cm^2/s in the surface, through a tiny hole at the vertex of bottom. When the slant height of water level is 4 cm, find the rate of decrease of the slant height of the water. **[Application] [4]**

OR

(ii) Solve the following differential equation:

$$\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$$ **[Analysis]**

10. **(i)** A group of Class XI students designed a game for Class XII students to be played at the farewell programme. They kept two sets of cards on the stage. Each set contained four cards numbered 1, 2, 3 and 4. Each student of Class XII had to draw one card from each set simultaneously.

The score a student received was defined as the difference between the numbers shown on the cards. The possible outcome of the score is given in the table below:

Numbers on Card 1

Numbers on Card 2

	1	2	3	4
1	0	p	2	3
2	1	0	p	2
3	q	1	0	p
4	3	q	1	0

(a) Calculate the values of p and q in the above table.

(b) Reshma plays the game once. Let the random variable 'S' represents Reshma's score. Construct the probability distribution for 'S'

s			
$P(S = s)$			

(c) Find the probability that Reshma's score is at most 2. AI **[Analysis] [4]**

OR

(ii) For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

- 960 of the total applications were the folk genre.
- 192 of the folk applications were for the below 18 category.
- 104 of the classical applications were for the 18 and above category.

(a) What is the probability that an application selected at random is for the 18 and above category provided it is under the classical genre? Show your work.

(b) An application selected at random is found to be under the below 18 category. Find the probability that it is under the folk genre. Show your work.

(c) Write the rule of Total Probability.

11. Garima owns a cafe that serves burgers, samosas and juice bottles. On a particular day, 100 items in total were sold. The number of burgers sold was thrice the number of samosas sold. Also, the number of samosas sold was 10 more than the number of juice bottles sold.

(i) Form a set of simultaneous equations for the above information.

(ii) Solve the set of equations formed in (i) by matrix method.

(iii) Hence, find the number of items sold in each category. **[Application] [6]**

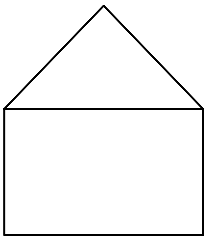
12. **(i)** Solve the following differential equation:

$$2ye^{x/y}\,dx + (y - 2xe^{x/y})dy = 0, \text{ given } x = 0 \text{ and } y = 1$$

AI **[Application]**

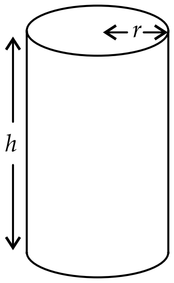
OR

(ii) Evaluate: $\int_0^{\pi} e^{2x} \sin\left(\frac{\pi}{4} + x\right) dx$ **AI** **[Evaluate] [6]**



13. **(i)** Mrs. Roy designs a window in her son's study room so that the room gets maximum sunlight. She designs the window in the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the window that will admit maximum sunlight into the room. **AI** **[Application] [6]**

OR



(ii) Sumit has bought a closed cylindrical dustbin. The radius of the dustbin is 'r' cm and height is 'h' cm. It has a volume of $20\pi \text{cm}^3$.

(a) Express 'h' in terms of 'r', using the given volume.

(b) Prove that the total surface area of the dustbin is $2\pi r^2 + \frac{40\pi}{r}$

(c) Sumit wants to paint the dustbin. The cost of painting the base and top of the dustbin is 2 per cm^2 and the cost of painting the curved side is ₹ 25 per cm^2. Find the total cost in terms of 'r', for painting the outer surface of the dustbin including the base and top.

(d) Calculate the minimum cost for painting the dustbin. **[Application]**

14. A biased four-sided die with faces labelled 1, 2, 3 and 4 is rolled and recorded. Let X be the result obtained when the die is rolled. The probability distribution for X is given in the following table where p and q are constants.

x	1	2	3	4
$P(X = x)$	p	0.3	q	0.1

For the probability distribution, it is known that $E(X) = 2$.

(i) Find p and q.

(ii) Also, find $P(X > 2)$.

(iii) Ajay play a game with this four-sided die. In this game he is allowed a maximum of five rolls. His score is calculated by adding the results of each roll. He wins the game if his score is at least 10. After 3 rolls, Ajay has score of four points. Assuming the rolls of the die are independent, find the probability that Ajay wins the game. **[Analysis] [6]**

Section-B [15 marks]

15. In subparts (i) to (iii) choose the correct options and in subparts (iv) and (v), answer the questions as instructed.**[5]**

(i) Consider the following statements and choose the correct option :

Statement 1: The position vector of the point which divides the join of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$ in the ratio 3 : 1. is $\frac{5\vec{a}}{4}$.

Statement 2: The position vector of a point R dividing the line segments joining the points P and Q, whose position vectors are $\vec{p}$ and $\vec{q}$ in the ratio $m : n$ internally, is given by $\frac{m\vec{p} - n\vec{q}}{m - n}$.

Which of the following is correct?

(a) Only Statement 1

(b) Only Statement 2

(c) Both Statements 1 and 2

(d) Neither Statement 1 nor Statement 2 **AI** **[Recall]**

(ii) If a line makes an angle α, β and γ with positive direction of the coordinate axes, then the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$ will be:

(a) 1 **(b)** 3 **(c)** –2 **(d)** 2 **[Application]**

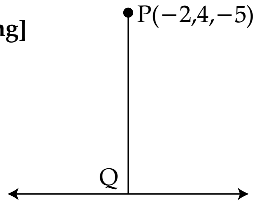
(iii) The direction cosines of x-axis are-

(a) <1, 1, 1> **(b)** <0, 0, 0> **(c)** <1, 0, 0> **(d)** <0, 1, 1>

A I **[Understanding]**

(iv) If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 5\hat{i} - 3\hat{j} + \hat{k}$ Find the projection of $\vec{b}$ on $\vec{a}$ A I **[Understanding]**

(v) Find the distance of the point (–2, 4, –5) from the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$. **[Analysis]**



16. (i) Write a vector of magnitude of 52 units in the direction of the vector $4\hat{i} - 3\hat{j} + 12\hat{k}$.

A I **[Application] [2]**

OR

(ii) The position vectors of the points A and B are (2, −3 , 2)and (2, 3, 1) respectively, then find the area of triangle OAB.

A I **[Application]**

17. (i) Find the equation of the lines passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$. A I **[Application] [4]**

OR

(ii) Find the image of the point (2, –1, 5) in the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$. Also, find the length of the perpendicular from the point (2, –1, 5) to the line. A I **[Application]**

18. (i) Sketch the region enclosed bounded by the curve, $y^2 = 4x$ and $x^2 = 4y$

(ii) Evaluate: $\int (y_2 - y_1)dx$

(iii) Hence find the area bounded by the curve, $y^2 = 4x$ and $x^2 = 4y$. A I **[4]**

Section-C [15 marks]

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.**[5]**

(i) Which condition is true if AC is increases with x and AC curve is rising.

(a) $MC < AC$ **(b)** $MC = AC$ **(c)** $MC > AC$ **(d)** None of these **[Recall]**

(ii) Read the following statements and choose the correct option:

(I) The slope of the regression line of Y on X is also called the correlation coefficient of X on Y.

(II) The slope of the regression line of Y on X is also called the correlation coefficient of Y on X.

(III) The slope of the regression line of Y on X is also called the regression coefficient of X on Y.

(IV) The slope of the regression line of Y on X is also called the regression coefficient of Y on X.

Which of the following is correct?

(a) Only IV is correct **(b)** Only III and II are correct

(c) Only II and IV are correct **(d)** Only III and IV are correct **[Recall]**

(iii) By using the data $\bar{x} = 25$, $\bar{y} = 30$; $b_{yx} = 1.6$ and $b_{xy} = 0.4$, What is the regression equation of y on x? What is the most likely value of y when $x = 60$? **[Understanding]**

(iv) The marginal revenue of a manufacturing facility is given by the function $MR(x) = 408 - 17x$. The manufacturer should limit the production to 24 units. Why? A I **[Understanding]**

(v) Find the marginal cost function (MC), if the cost function is: $C(x) = \frac{x^3}{3} + 5x^2 - 16x + 2$. **[Understanding]**

20. (i) A company produces a commodity with ₹ 24,000 as fixed cost. The variable cost estimated to be 25% of the total revenue received on selling the product, is at the rate of ₹ 8 per unit. Find the break-even point.

A I **[Application] [2]**

OR

(ii) The average cost function associated with producing and marketing x units of an item is given by $AC = 2x - 11 + \frac{50}{x}$. Find the range of values of the output x, for which AC is increasing. A I **[Application]**

21. (i) Find the regression coefficients b_{yx}, b_{xy} and correlation coefficient 'r' for the following data: (2, 8), (6, 8), (4, 5), (7, 6), (5, 2). **[Application] [4]**

OR

(ii) The following table shows the mean, the standard deviation and the coefficient of correlation of two variables x and y.

Series	x	y
Mean	8	6
Standard deviation	12	4
Coefficient of correlation	0.6	

Calculate:

(a) the regression coefficient b_{xy} and b_{yx}.

(b) the probable value of y when $x = 20$. **[Application]**

22. The standard weight of a special purpose brick is 5 kg and it must contain two basic ingredients B_1 and B_2 cost ₹ 5 per kg and B_2 cost ₹ 8 per kg. Strength considerations dictate that the brick should not contain more than 4 of B_1 and a minimum of 2 kg of B_2. Since the demand for the product is likely to be related to the price of the bricks, find the minimum cost of the brick satisfying the above conditions.

(a) Formulate this situation as an L.P.P.

(b) Solve it graphically. **[Analysis] [4]**

❑❑

SOLUTIONS

Self Assessment Paper - 3

Section-A

1. **(i) Option (c) is correct.**

Explanation:

$$\begin{vmatrix}1+\cos\theta & \sin\theta\\ \sin\theta & 1-\cos\theta\end{vmatrix}$$

$$=(1+\cos\theta)(1-\cos\theta)-(\sin\theta)(\sin\theta)$$

$$=1-\cos^2\theta-\sin^2\theta = 0$$

(ii) Option (d) is correct.

Explanation: Let $I=\int_a^b xf(x)dx$ ...(i)

$$I=\int_a^b(a+b-x)f(a+b-x)dx$$

$$\left[\because \int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$$

$$\Rightarrow \quad I=\int_a^b(a+b-x)f(x)dx$$

$$\Rightarrow \quad I=(a+b)\int_a^b f(x)dx-I \quad \text{[Using Eq.(i)]}$$

$$\Rightarrow \quad I+I=(a+b)\int_a^b f(x)dx$$

$$\Rightarrow \quad 2I=(a+b)\int_a^b f(x)dx$$

$$\Rightarrow \quad I=\left(\frac{a+b}{2}\right)\int_a^b f(x)dx$$

(iii) Option (a) is correct.

Explanation: $\sin\left[\frac{\pi}{2}-\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

$$=\sin\left[\frac{\pi}{2}+\sin^{-1}\left(\sin\frac{\pi}{3}\right)\right]=\sin\left[\frac{\pi}{2}+\frac{\pi}{3}\right]$$

$$=\cos\frac{\pi}{3}=\frac{1}{2}$$

(iv) Option (c) is correct.

Explanation: Every identity relation on a non-empty set A is a reflexive relation but not conversely.

(v) Option (b) is correct.

Explanation: Given, $P(A) = 0.7$, $P(B) = 0.4$ and $P(B|A) = 0.3$

By conditional probability, we have;

$P(B|A) = P(A \cap B)/P(A)$

$\Rightarrow P(A \cap B) = P(B|A).P(A) = 0.3 \times 0.7 = 0.21$

(vi) Option (b) is correct.

Explanation: It is given that

$$\begin{bmatrix}3x+7 & 5\\ y+1 & 2-3x\end{bmatrix}=\begin{bmatrix}0 & y-2\\ 8 & 4\end{bmatrix}$$

Equating the corresponding elements, we get:

$$3x+7=0$$

$$\Rightarrow \quad x=-\frac{7}{3}$$

$$5=y-2$$

$$\Rightarrow \quad y=7$$

$$y+1=8$$

$$\Rightarrow \quad y=7$$

$$2-3x=4$$

$$\Rightarrow \quad x=-\frac{2}{3}$$

We find that on comparing the corresponding elements of the two matrices, we get two different values of x, which is not possible.

Hence, it is not possible to find the values of x and y for which the given matrices are equal.

(vii) Option (d) is correct.

Explanation: Figure (i) shows infinite discontinuity $[\lim_{x\to a+} f(x) = \infty$ & $\lim_{x\to a-} f(x) = -\infty]$.

Figure (ii) shows non removable discontinuity $[\lim_{x\to a} f(x)$ doesn't exist$]$

(viii) Option (b) is correct.

Explanation:

$$y=\cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)+\text{cosec}^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$$

$$=\cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)+\sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$$

$$(\text{cosec}^{-1}\theta=\sin^{-1}\theta)$$

$$y=\frac{\pi}{2} \qquad \left(\cos^{-1}\theta+\sin^{-1}\theta=\frac{\pi}{2}\right)$$

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{\pi}{2}\right)=0$$

(ix) Option (d) is correct.

(x) Option (a) is correct.

Explanation :

Given that, $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

Also $A + A' = I$

$$\Rightarrow \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Equating corresponding entries, we have

$$2\cos\alpha=1$$

$$\cos\alpha=\frac{1}{2}$$

$$\cos\alpha=\cos\frac{\pi}{3}$$

$$\therefore \quad \alpha=\frac{\pi}{3}$$

(xi) Option (a) is correct.

Explanation: Given, $E = \{1, 3, 5\}$, $F = \{2, 3\}$, $G = \{2, 3, 4, 5\}$

$$P(E) = \frac{3}{6} = \frac{1}{2},\ P(F) = \frac{2}{6} = \frac{1}{3},\ P(G) = \frac{4}{6} = \frac{2}{3}$$

Now, $(E \cup F) \cap G = \{2, 3, 5\}$

$$\therefore \quad P[(E \cup F) \cap G] = \frac{3}{6} = \frac{1}{2}$$

Therefore,
$$P\left[\frac{(E \cup F)}{G}\right] = P\frac{[(E \cup F) \cap G]}{P(G)} = \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{3}{4}$$

(xii) $|adj\ A| = |A|^{n-1}$, where n is the order of the matrix

Since, given $|adj\ A| = 64$

$$\therefore \quad |A|^{3-1} = |64|$$
$$|A|^2 = 64 \Rightarrow |A| = 8$$

(xiii) We know that, for a relation to be transitive,

$(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$.

Here, $(1, 2) \in R$ and $(2, 1) \in R$ but $(1, 1) \notin R$.

∴ R is not transitive.

(xiv) The given function is bijective i.e., both one-one and onto function.

One – One Function: Every element in the domain X has a distinct image in the co-domain Y. Thus, the given function is one- one.

Onto Function: Every element in the co- domain Y has a pre- image in the domain X. Thus, the given function is onto.

(xv) We know that, $\sum_{i=1}^{n} P_i = 1$, where $P_i \geq 0$

$$P_1 + P_2 + P_3 + P_4 = 1$$
$$k + k^2 + 2k^2 + k = 1$$
$$3k^2 + 2k - 1 = 0$$
$$(3k - 1)(k + 1) = 0$$
$$k = \frac{1}{3}$$
$$k + 1 = 0$$
$$k = -1$$

$k < 0$ is not possible

2. (i) Let $u = x^{\log x}$ and $v = \log x$

Now,
$$\frac{1}{u}\frac{du}{dx} = 2\log x \times \frac{1}{x}$$
$$\Rightarrow \quad \frac{du}{dx} = \frac{2\log x}{x} \cdot x^{\log x}$$

Again, $v = \log x$
$$\Rightarrow \quad \frac{dv}{dx} = \frac{1}{x}$$
$$\therefore \quad \frac{du}{dv} = 2x^{\log x}\log x$$

OR

(ii) Let
$$y = x - \frac{4}{x}$$
$$\therefore \frac{dy}{dx} = 1 - 4(-1)x^{-2} = 1 + \frac{4}{x^2}$$

which is always positive as x^2 is always positive for all real values of x.

$$\therefore \quad \frac{dy}{dx} > 0 \text{ for all real values of } x.$$

$\therefore \quad y = x - \frac{4}{x}$ always increases as x increases.

Hence Proved.

3. Let
$$I = \int_0^1 \frac{e^x}{1 + e^{2x}}dx$$

Let, $e^x = y$

or $e^x dx = dy$

When, $x = 0$; $y = 1$ and $x = 1$; $y = e$

$$\therefore \quad I = \int_1^e \frac{dy}{1 + y^2}$$
$$= \left[\tan^{-1} y\right]_1^e$$
$$= \tan^{-1} e - \tan^{-1} 1$$
$$= \tan^{-1} e - \frac{\pi}{4}$$

4. The equation of the curve is given by $y = e^{2x}$

Since, it passes through the point (0,1).

$$\therefore \quad \frac{dy}{dx} = e^{2x}.2 = 2.e^{2x}$$
$$\left(\frac{dy}{dx}\right)_{(0,1)} = 2.e^{2\times 0} = 2$$

= Slope of tangent to the curve.

∴ Equation of tangent is
$$y - 1 = 2(x - 0)$$
$$y = 2x + 1$$

Since, tangent to the curve $y = e^{2x}$ at the point (0,1) meets x-axis, i.e., $y = 0$.

$$\therefore \quad 0 = 2x + 1$$
$$\Rightarrow \quad x = -\frac{1}{2}$$

So, the required point is $(-\frac{1}{2}, 0)$.

5. (i) Given, $\int \frac{x^2}{x^2 - 4}dx$

$$\therefore \quad \int \frac{x^2 - 4 + 4}{x^2 - 4}dx = \int \left(\frac{x^2 - 4}{x^2 - 4} + \frac{4}{x^2 - 4}\right)dx$$
$$\Rightarrow \int \left(1 + \frac{4}{x^2 - 4}\right)dx = x + 4 \times \frac{1}{2 \times 2}\log\left|\frac{x - 2}{x + 2}\right| + C$$
$$= x + \log\left|\frac{x - 2}{x + 2}\right| + C.$$

OR

(ii)
$$\int_0^2 \sqrt{4 - x^2}\,dx = \int_0^2 \sqrt{2^2 - x^2}\,dx$$
$$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$
$$= [0 + 2\sin^{-1}(1)] - [0 + 2\sin^{-1} 0]$$
$$= 2 \times \frac{\pi}{2} - (0 + 2 \times 0) = \pi$$

6. Let, $2x = \tan\theta$

$$\text{LHS} = \tan^{-1}\left(\frac{3\tan\theta - \tan 3\theta}{1 - 3\tan^2\theta}\right) - \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 3\theta) - \tan^{-1}(\tan 2\theta)$$

$$= 3\theta - 2\theta$$

$$= \theta \text{ or } \tan^{-1} 2x$$

$\therefore$ LHS = RHS **Hence Proved**

Examiner's Comment

- Students apply the formula for $\tan^{-1} x + \tan^{-1} y$ and make it complicated, even though the answer is obtained.

Answering Tip

- The right substitution gives the answer in a few steps.

7. $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$

$$= (\sin^{-1}x + \cos^{-1}x)^2 - 2\sin^{-1}x\cos^{-1}x$$

$$= \left(\frac{\pi}{2}\right)^2 - 2\sin^{-1}x\left(\frac{\pi}{2} - \sin^{-1}x\right)$$

$$= \frac{\pi^2}{4} - \pi\sin^{-1}x + 2(\sin^{-1}x)^2$$

$$= 2\left[(\sin^{-1}x)^2 - \frac{\pi}{2}\sin^{-1}x + \frac{\pi^2}{8}\right]$$

$$= 2\left[\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$$

$\therefore$ Least value

$$= 2\left(\frac{\pi^2}{16}\right) = \frac{\pi^2}{8}$$

and Greatest value

$$= 2\left[\left(-\frac{\pi}{2} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right] = \frac{5\pi^2}{4}$$

8. Let $I = \int_0^1 x\tan^{-1}x\,dx$

$$I = \left[\tan^{-1}x \cdot \frac{x^2}{2}\right]_0^1 - \int_0^1 \left(\frac{1}{1+x^2}\cdot\frac{x^2}{2}\right)dx$$

[Using integration by parts]

$$= \left[\frac{x^2}{2}\tan^{-1}x\right]_0^1 - \frac{1}{2}\int_0^1 \frac{x^2+1-1}{1+x^2}dx$$

$$= \left[\frac{1}{2}\tan^{-1}1 - 0\right] - \frac{1}{2}\left[\int_0^1 dx - \int_0^1 \frac{1}{1+x^2}dx\right]$$

$$= \frac{\pi}{8} - \frac{1}{2}\int_0^1 1dx + \frac{1}{2}\int_0^1 \frac{1}{1+x^2}dx$$

$$= \frac{\pi}{8} - \left[\frac{x}{2}\right]_0^1 + \left[\frac{1}{2}\tan^{-1}x\right]_0^1$$

$$= \frac{\pi}{8} - \frac{1}{2} - 0 + \frac{1}{2}\left[\tan^{-1}1 - 0\right]$$

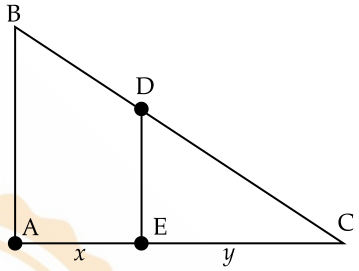
$$= \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}$$

$$= \frac{\pi}{4} - \frac{1}{2}.$$

9. (i) Let AB represent the height of the street light from the ground. At any time t seconds, let the man represented as ED of height 1.6 m be at a distance of x m from AB and the length of his shadow EC by y m.

Using similarity of triangles, we have $\frac{4}{1.6} = \frac{x+y}{y}$

$$3y = 2x$$



Differentiating both sides w.r.t. t, we get

$$3\frac{dy}{dt} = 2\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{2}{3}\times 0.3 \Rightarrow \frac{dy}{dt} = 0.2$$

At any time t seconds, the tip of his shadow is at a distance of $(x+y)$m from AB.

The rate at which the tip of his shadow moving

$$= \left(\frac{dx}{dt} + \frac{dy}{dt}\right)\text{m/s} = 0.5 \text{ m/s}$$

The rate at which his shadow is lengthening

$$= \frac{dy}{dt}\text{m/s} = 0.2 \text{ m/s}$$

OR

(ii) Considers $e^x = t$ and finds $dx = \frac{dt}{t}$.

Writes the function to be integrated as $\int \frac{2}{(t-1)(2t+3)}dt.$

Uses partial fractions to rewrite the function as follows:

$$\frac{2}{5}\int\left(\frac{1}{t-1} - \frac{2}{2t+3}\right)dt$$

Integrates the function to obtain the following:

$$\frac{2}{5}\left[\ln|t-1| - \ln|2t+3|\right] + C$$

where C is the constant of integration.

Replaces t with e^x to obtain $\frac{2}{5}\ln\left|\frac{e^x-1}{2e^x+3}\right| + C.$

10. (i) (a) P(Diya drawing dark chocolate from Shanti's bag) $= \frac{7}{20}$

(b) E_1 = drawing dark chocolate from Shanti's bag
E_2 = drawing milk chocolate from Shanti's bag
E_3 = drawing white chocolate from Shanti's bag
A = after transfer chocolate from Shanti's bag to Diya's bag then draw 1 dark chocolate from Diya's bag

$$P(E_1) = \frac{7}{20}$$

$$P(E_2) = \frac{8}{20}$$

$$P(E_3) = \frac{5}{20}$$

$$P(A/E_1) = \frac{8}{21}$$

$$P(A/E_2) = \frac{7}{21}$$

$$P(A/E_3) = \frac{7}{21}$$

$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)$$

$$P(A) = \frac{7}{20}\times\frac{8}{21}+\frac{8}{20}\times\frac{7}{21}+\frac{5}{20}\times\frac{7}{21}$$

$$= \frac{56+56+35}{20\times 21} = \frac{147}{420} = \frac{7}{20}$$

(c) P(drawing a dark chocolate from Diya's bag after transfer) $= \frac{7}{20}$.

P(drawing a dark chocolate from Shanti's bag before transfer) $= \frac{7}{20}$

Both are same.

OR

(ii)(a) E = Person selected has COVID

F = Does not have COVID

G = Test is judged COVID positive

Probability of the person to be tested as COVID positive given that he is actually having COVID

$$= P(G/E) = 90\% = \frac{90}{100} = 0.9$$

(b) Probability of person to be tested as COVID positive given that he is actually not having COVID

$$= P(G/F) = 1\% = \frac{1}{100} = 0.01$$

(c) $P(F) = 1 - P(E) = 1 - 0.001$

$$\left[\because P(E) = 0.1\% = \frac{0.1}{100} = 0.001\right]$$

$$= 0.999$$

(d)
$$P(E/G) = \frac{0.001\times 0.9}{0.001\times 0.9 + 0.999\times 0.01}$$

$$= \frac{9\times 10^{-4}}{9\times 10^{-4} + 99.9\times 10^{-4}}$$

$$= \frac{9\times 10^{-4}}{10^{-4}(9+99.9)}$$

$$= \frac{9}{108.9} = \frac{10}{121}$$

$= 0.083$ (approx)

11. Let no. of burgers $= x$
Let no. of samosas $= y$
Let no. of juice bottles $= z$
A.T.Q.
Total items sold were 100.

i.e., $x + y + z = 100$ ...(i)

The number of burgers sold was thrice the no. of samosas sold.

i.e., $x = 3y$

$\Rightarrow$ $x - 3y = 0$ ...(ii)

The no. of samosas sold was 10 more than the number of juice bottles.

i.e., $y = z + 10$

$\Rightarrow$ $y - z = 10$ ...(ii)

(a)
$$x + y + z = 100$$
$$x - 3y = 0$$
$$y - z = 10$$

(b)
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -3 & 0 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 10 \end{bmatrix}$$

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -3 & 0 \\ 0 & 1 & -1 \end{bmatrix} \quad ....(i)$$

$$|A| = (1)(3–0)–1(–1–0)+1(1+0)$$
$$A = 3 + 1 + 1 = 5 \neq 0$$

$\Rightarrow$ A^{-1} exists

$$\text{Adj. } A = \begin{bmatrix} 3 & 2 & 3 \\ 1 & -1 & 1 \\ 1 & -1 & -4 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj. } A}{|A|} = \frac{1}{5}\begin{bmatrix} 3 & 2 & 3 \\ 1 & -1 & 1 \\ 1 & -1 & -4 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{5}\begin{bmatrix} 3 & 2 & 3 \\ 1 & -1 & 1 \\ 1 & -1 & -4 \end{bmatrix}\begin{bmatrix} 100 \\ 0 \\ 10 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 300+30 \\ 100+10 \\ 100-40 \end{bmatrix} = \begin{bmatrix} 66 \\ 22 \\ 12 \end{bmatrix}$$

(c) $x = 66$, $y = 22$ and $z = 12$.

12. **(i)** The differential equation a linear differential equation.

On comparing the given differential equation with

$$\frac{dy}{dx} + Py = Q$$

We get, $P = \cot x$, $Q = \frac{2}{1+\sin x}$

$$\text{I.F.} = e^{\int P dx} = e^{\int \cot x\, dx} = e^{\log \sin x} = \sin x$$

The general solution is given by $y \times \text{I.F.}$

$$= \int Q \times \text{I.F.}\, dx + c$$

$$\Rightarrow \quad y \sin x = \int 2\frac{\sin x}{1+\sin x}dx + c$$

$$\Rightarrow \quad y \sin x = 2\int \frac{\sin x + 1 - 1}{1+\sin x}dx + c$$

$$\Rightarrow \quad y \sin x = 2\int \left[1 - \frac{1}{1+\sin x}\right]dx + c$$

$$\Rightarrow \quad y \sin x = 2\int \left[1 - \frac{1}{1+\cos\left(\frac{\pi}{2}-x\right)}\right]dx + c$$

$$\Rightarrow \quad y \sin x = 2\int \left[1 - \frac{1}{2\cos^2\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]dx + c$$

$$\Rightarrow \quad y \sin x = 2\int \left[1 - \frac{1}{2}\sec^2\left(\frac{\pi}{2}-\frac{x}{2}\right)\right]dx + c$$

$$\Rightarrow \quad y \sin x = 2\left[x + \tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right] + c$$

Given that, $y = 0$, when $x = \frac{\pi}{4}$

Hence, $0 = 2\left[\frac{\pi}{4} + \tan\frac{\pi}{8}\right] + c$

$$c = -2\left[\frac{\pi}{4} + \tan\frac{\pi}{8}\right]$$

Hence, the particular solution is

$$y = \text{cosec}x\left[2\left\{x + \tan\left(\frac{\pi}{4}-\frac{x}{2}\right) - \left(\frac{\pi}{4} + \tan\frac{\pi}{8}\right)\right\}\right]$$

OR

(ii) Let $I = \int_0^{\frac{3}{2}} |x \cos \pi x| dx$

In $0 \le x \le \frac{1}{2}, x\cos\pi x \le 0,\ x \ge 0$

In $\frac{1}{2} \le x \le \frac{3}{2}, x\cos\pi x \le 0,\ x \ge 0$

$$\therefore \ I = \int_0^{\frac{1}{2}} x\cos\pi x\, dx - \int_{\frac{1}{2}}^{\frac{3}{2}} x \cos \pi x\, dx$$

$$= \left[x\frac{(\sin \pi x)}{\pi} - \int(1)\cdot\frac{(\sin \pi x)}{\pi}dx\right]_0^{\frac{1}{2}}$$

$$- \left[x\frac{(\sin \pi x)}{\pi} - \int(1)\cdot\frac{(\sin \pi x)}{\pi}dx\right]_{\frac{1}{2}}^{\frac{3}{2}}$$

$$\therefore \ I = \left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{(\pi)^2}\right]_0^{\frac{1}{2}} - \left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{(\pi)^2}\right]_{\frac{1}{2}}^{\frac{3}{2}}$$

$$= \left[\frac{\frac{1}{2}\sin\frac{\pi}{2}}{\pi} + \frac{\cos\frac{\pi}{2}}{\pi^2} - 0 - \frac{\cos 0}{\pi^2}\right]$$

$$- \left[\frac{\frac{3}{2}\sin\frac{3\pi}{2}}{\pi} + \frac{\cos\frac{3\pi}{2}}{\pi^2} - \frac{\frac{1}{2}\sin\frac{\pi}{2}}{\pi} - \frac{\cos\frac{\pi}{2}}{\pi^2}\right]$$

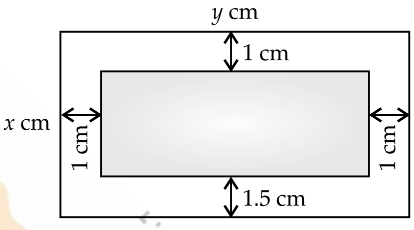
$$\therefore \ I = \left[\frac{1}{2\pi} - \frac{1}{\pi^2}\right] - \left[\frac{3}{2\pi}(-1) + 0 - \frac{1}{2\pi} - 0\right]$$

$$= \left[\frac{1}{2\pi} - \frac{1}{\pi^2} + \frac{3}{2\pi} + \frac{1}{2\pi}\right]$$

$$\therefore \ I = \frac{1}{\pi}\left[\frac{5}{2} - \frac{1}{\pi}\right].$$

13. **(i)** Let x cm and y cm be the dimensions of the printed page.

$\therefore \quad xy = 80 \quad \ldots\text{(i)}$



Let A (in sq. cm) be the printed area then,

$$A = (x-2)\left(y - \frac{5}{2}\right)$$

$$\Rightarrow \quad A = xy - \frac{5x}{2} - 2y + 5$$

$$\Rightarrow \quad A = 80 - \frac{5x}{2} - \frac{2 \times 80}{x} + 5$$

[using (i)]

$$\Rightarrow \quad A = 85 - \frac{5x}{2} - \frac{160}{x}$$

Differentiating w.r.t. x, we get

$$\frac{dA}{dx} = -\frac{5}{2} + \frac{160}{x^2}$$

Differentiating once again w.r.t. x

$$\frac{d^2A}{dx^2} = -\frac{320}{x^3}$$

For maximum and minimum area,

$$\therefore \quad \frac{dA}{dx} = 0$$

$$-\frac{5}{2} + \frac{160}{x^2} = 0$$

$$\Rightarrow \quad \frac{160}{x^2} = \frac{5}{2}$$

$$\Rightarrow \quad x^2 = 64$$

$$\Rightarrow \quad x = \pm 8$$

Also, $\left(\frac{d^2A}{dx^2}\right)_{x=8} = -\frac{320}{512} < 0$

$x = 8$

⇒ A is maximum at $x = 8$

From (i), $xy = 80$

$\Rightarrow \quad 8 \times y = 80$

$\Rightarrow \quad y = 10$

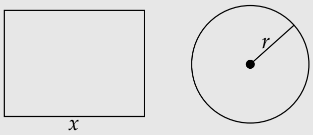
Therefore, the dimensions of the printed page are 10 cm and 8 cm.

OR

(ii) Given, the side of the square is x and radius of the circle is r.

$$\Rightarrow \quad 4x + 2\pi r = 50$$

$$2\pi r = 50 - 4x$$



$$\therefore \quad r = \frac{50-4x}{2\pi}$$

$$\Rightarrow \quad A = x^2 + \pi r^2$$

$$= x^2 + \pi\left(\frac{50-4x}{2\pi}\right)^2$$

$$= x^2 + \frac{1}{4\pi}(50-4x)^2$$

For maximum and minimum value of A,

$$\frac{dA}{dx} = 0$$

$$\frac{dA}{dx} = 2x + \frac{1}{4\pi}2(50-4x)(-4) = 0$$

$$\Rightarrow \quad 2\pi x - 100 + 8x = 0$$

$$\Rightarrow \quad x = \frac{50}{\pi+4}$$

$\frac{d^2A}{dx^2} =$ +ve, A is minimum at $x = \frac{50}{\pi+4}$

$\therefore$ Length of square $= \frac{200}{\pi+4}$ m

Length of circle $= 50 - \frac{200}{\pi+4} = \frac{50\pi}{\pi+4}$ m

Examiner's Comment

- The required sum of areas of the two parts had to be expressed correctly in terms of a single variable. Some candidates failed to do this. Differentiation and simplification errors were also made by several candidates.

Answering Tip

- Differentiation rules for different functions and forms need continuous revision and practice. Obtaining the result in the required form needs simplification, cross multiplication, substitution, etc. Hence, enough revision and practices is required.

14. The first six positive integers as 1, 2, 3, 4, 5 and 6.
Where X can take values of 3,4, 5, 6.
[$\because$ 1 and 2 cannot be selected since among 3 numbers the greatest is required]

$P(X = 3) = P$(3 and other numbers which are less than 3) i.e., (1, 2, 3)

$$P(X = 3) = \frac{1}{^6C_3} = \frac{1}{20}$$

$P(X = 4) = P$(4 and other numbers which are less than 4) i.e., (1, 2, 4), (2, 3, 4), (1, 3, 4)

$$P(X = 4) = \frac{3}{^6C_3} = \frac{3}{20}$$

$P(X = 5) = P$(5 and other numbers which are less than 5)
i.e., (1, 2, 5), (1, 3, 5), (1, 4, 5), (2, 3, 5), (2, 4, 5), (3, 4, 5)

$$P(X = 5) = \frac{6}{^6C_3} = \frac{6}{20}$$

$P(X = 6) = P$(6 and other numbers which are less than 6)

$$P(X = 6) = \frac{10}{^6C_3} = \frac{10}{20}$$

X	3	4	5	6
P(X)	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{10}{20}$
XP(X)	$\frac{3}{20}$	$\frac{12}{20}$	$\frac{30}{20}$	$\frac{60}{20}$

$$\text{Mean} = \Sigma XP(X)$$

$$= 3\times\frac{1}{20} + 4\times\frac{3}{20} + 5\times\frac{6}{20} + 6\times\frac{10}{20}$$

$$= \frac{3}{20} + \frac{12}{20} + \frac{30}{20} + \frac{60}{20} = \frac{105}{20} = 5.25$$

Section-B

15. (i) Option (c) is correct.

Explanation:

$$\left(\hat{i}\times\hat{j}\right)\cdot\hat{k} + \hat{i}\cdot\hat{j} = \hat{k}\cdot\hat{k} + \hat{i}\cdot\hat{j}$$

$$= 1 + 0 = 1$$

$$\left(\hat{k}\times\hat{j}\right)\cdot\hat{i} + \hat{j}\cdot\hat{k} = -\hat{i}\cdot\hat{i} + 0$$

$$= -1 + 0 = -1$$

(ii) Option (a) is correct.

Explanation: Z-axis makes an angle 90° with X-axis, 90° with Y-axis and 0° with Z-axis

$\therefore$ Direction cosines of Z-axis: cos 90°, cos 90°, cos 0° i.e., 0, 0, 1

Therefore, sum of the direction cosines = 0 + 0 + 1 = 1

(iii) Option (d) is correct.

Explanation:

The foot of perpendicular from point $P(\alpha, \beta, \gamma)$ on Y-axis is $Q(0, \beta, 0)$.

$\therefore$Required distance,

$$PQ = \left|\sqrt{(\alpha-0)^2 + (\beta-\beta)^2 + (\gamma-0)^2}\right|$$

$$= \left|\sqrt{\alpha^2+\gamma^2}\right| \text{ units}$$

(iv) $\vec{u} = \hat{i}$

$\vec{v} = \hat{j}$

$\vec{w} = \hat{k}$

$$|\vec{u}| = |\vec{v}| = |\vec{w}| = 1$$

$$\therefore \quad (\vec{w} \times \vec{v}) = \hat{k} \times \hat{j} = -\hat{i}$$

$$\Rightarrow \quad \vec{u} = \hat{i}$$

$\Rightarrow$ Angle between $-\hat{i}$ and $\hat{i}$ = 180°

(v) Equation of diagonal of cube from O(0,0,0) to F(2,2,2)

$$\vec{r} = \left(0\hat{i}+0\hat{j}+0\hat{k}\right) + \lambda\left(2\hat{i}+2\hat{j}+2\hat{k}-0\hat{i}-0\hat{j}-0\hat{k}\right)$$

$$\vec{r} = \lambda\left(2\hat{i}+2\hat{j}+2\hat{k}\right)$$

$$= 2\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$\vec{r} = \mu\left(\hat{i}+\hat{j}+\hat{k}\right), \text{ Where } \mu = 2\lambda$$

16. (i) $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$

Hence

$$\vec{a}+\vec{b} = 6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k} \text{ and } \vec{a}-\vec{b} = -4\hat{i} + (7-\lambda)\hat{k}$$

$\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ will orthogonal if,

$(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 0$

$$-24 + (49 - \lambda^2) = 0$$

$$\Rightarrow \quad \lambda^2 = 25$$

$$\lambda = \pm 5$$

OR

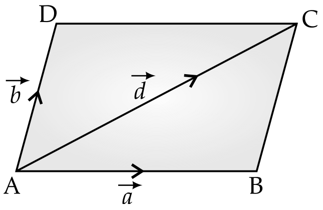
(ii) Let $\vec{a} = \hat{i} - \hat{j} + \hat{k}$

$$\vec{d} = 4\hat{i} + 5\hat{k}$$

$$\because \quad \vec{a}+\vec{b} = \vec{d}$$

$$\therefore \quad \vec{b} = \vec{d} - \vec{a}$$

$$= 3\hat{i} + \hat{j} + 4\hat{k}$$



$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 3 & 1 & 4 \end{vmatrix}$$

$$= -5\hat{i} - 1\hat{j} + 4\hat{k}$$

Area of parallelogram $= |\vec{a} \times \vec{b}|$

$$= \left|\sqrt{25+1+16}\right|$$

$$= \sqrt{42} \text{ sq. units}$$

Examiner's Comment

- Mostly students get confused in deciding the formula to be used since a side and a diagonal are given.

Answering Tip

- Practice more problems related to the area of triangle and parallelogram.

17. (i) Eliminating t between the equations, we obtain the equation of the path $\frac{x}{2} = \frac{y}{-4} = \frac{z}{4}$, which are the equations of the line passing through the origin having direction ratios (2, –4, 4). This line is the path of the rocket.

When t = 10 seconds, the rocket will be at the point (20, -40, 40). Hence, the required distance from the origin at 10 seconds = $|\sqrt{20^2 + 40^2 + 40^2}|$ km

= 20 × 3 km = 60 km

The distance of the point (20, – 40, 40) from the given line

$$= \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|} = \frac{|-30\hat{j} \times (10\hat{i} - 20\hat{j} + 10\hat{k})|}{|10\hat{i} - 20\hat{j} + 10\hat{k}|} \text{ km}$$

$$= \frac{|-300\hat{i} + 300\hat{k}|}{|10\hat{i} - 20\hat{j} + 10\hat{k}|} \text{km}$$

$$= \frac{300\sqrt{2}}{10\sqrt{6}} \text{ km}$$

$$= 10\sqrt{3} \text{ km}$$

OR

(ii) Let equation of required line is

$$\frac{x-1}{a} = \frac{y-1}{b} = \frac{z-1}{c} \qquad \text{...(i)}$$

Since, the line is perpendicular to

$$\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$$

and
$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

$$a + 2b + 4c = 0 \qquad \text{...(ii)}$$

$$2a + 3b + 4c = 0 \qquad \text{...(iii)}$$

Solving (ii) and (iii),

$$\frac{a}{4} = \frac{b}{-4} = \frac{c}{-1}$$

$\therefore$ DR's of line in cartesian form is: 4, –4, –1

Equation of line in Cartesian form is:

$$\frac{x-1}{4} = \frac{y-1}{-4} = \frac{z-1}{-1}$$

Vector form of line is

$$\vec{r} = (\hat{i} + j + k) + \lambda(4\hat{i} - 4j - k)$$

Let θ be the angle between given lines.

$$\cos\theta = \frac{1(2)+2(3)+4(4)}{\sqrt{1+4+16}\sqrt{4+9+16}}$$

$$= \frac{24}{\sqrt{21}\sqrt{29}}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{24}{\sqrt{609}}\right)$$

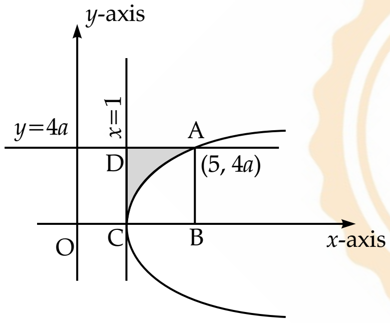
Examiner's Comment

- Most often students get wrong with the conditions for perpendicularity.

Answering Tip

- The angle between lines and their various conditions should be learned thoroughly.

18. **(i)** Given curve $y^2 = 4a^2(x-1)$ represents a parabola with vertex at (1, 0), which opens on the right side.
Given parabola $y^2 = 4a^2(x-1)$ and the line $y = 4a$ intersect at $(5, 4a)$.



(ii) $$\int 2a\sqrt{x-1}\,dx = 2a\left[\frac{(x-1)^{3/2}}{\frac{3}{2}}\right] = 4a(x-1)^{3/2}$$

(iii) Shaded region represents the required area.

$\therefore$ Required area = Area of $(CBAD)$

$$= 4\times 4a - \int_1^5 2a\sqrt{x-1}\,dx$$

$$= 16a - 2a\left[\frac{(x-1)^{3/2}}{\frac{3}{2}}\right]_1^5$$

$$= 16a - 2a\times\left[\frac{2}{3}(4^{3/2}-0)\right]$$

$$= \frac{16a}{3} \text{ sq. units.}$$

Section-C

19. **(i) Option (d) is correct.**

Explanation: $MC(201) = C(201-200)$

(ii) Option (a) is correct.

Explanation: Because it is a least scattered diagram.

(iii) $2x - 4 = 0$

$$y = \frac{1}{2}x$$

$$\therefore b_{yx} = \frac{1}{2}$$

$$6x - 3y + 5 = 0$$
$$6x = 3y - 5$$
$$x = \frac{1}{2}y - \frac{5}{6}$$

$$\therefore b_{xy} = \frac{1}{2}$$

$$r = \pm\sqrt{b_{yx}\times b_{xy}} = \pm\sqrt{\frac{1}{2}\times\frac{1}{2}}$$

$$\therefore r = \frac{1}{2}$$

(iv) $$AR = \frac{R}{x}$$

$$= \frac{24\,x + 4x^2 + 12}{x}$$

$$= 24 + 4x + \frac{12}{x}$$

(v) Cost function $C(x) = 5x + 700$
Profit function, $P(x) = 10x - (5x + 700)$
$= 5x - 700$
Revenue function $= 10x$

20. **(i)** Given fixed cost = ₹ 24,000
Let number of units be: x
Selling price per unit $p(x)$= ₹ 8
$\therefore$Revenue function $R(x) = 8x$
Cost function $C(x) = 24{,}000 + 25\%$ of $8x$
$= 24{,}000 + 2x$
For break even points: $R(x) = C(x)$
$8x = 24{,}000 + 2x$
$6x = 24{,}000$
$x = 4{,}000$ units

[ISC Marking Scheme, 2019]

OR

(ii) $$\text{Total variable cost} = 3x + \frac{x^5}{25}$$

$$\text{Average variable cost} = \frac{\text{Total variable cost}}{\text{Output}}$$

$$\Rightarrow \text{Average variable cost} = \frac{3x + \frac{x^5}{25}}{x}$$

$$= 3 + \frac{x^4}{25}$$

$\Rightarrow$ Average variable cost > 0 (as $x^4 > 0$)
Hence, average variable cost increases with output x.

21. **(i)** $\Sigma x = 510$, $\Sigma y = 7140$, $\Sigma x^2 = 4150$, $\Sigma y^2 = 7{,}40{,}200$

$$\text{Mean } \bar{x} = \frac{\Sigma x}{n} = \frac{510}{102} = 5$$

$$\text{Mean } \bar{y} = \frac{\Sigma y}{n} = \frac{7{,}140}{102} = 70$$

Regression Coefficient of y on x

$$b_{yx} = \frac{n \cdot \Sigma xy - \Sigma x \cdot \Sigma y}{n \cdot \Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{102 \times 54,900 - 510 \times 7,140}{102 \times 4,150 - (510)^2}$$

$$= \frac{55,99,800 - 36,41,400}{4,23,300 - 2,60,100}$$

$$= \frac{1958400}{163200} = 12$$

Regression equation of y on x

$y - \bar{y} = b_{yx}\,(x - \bar{x})$

$\Rightarrow y - 70 = 12(x - 5)$

$\Rightarrow y = 12x - 60 + 70$

$\Rightarrow y = 12x + 10$

OR

(ii) $\bar{x} = 8, \sigma_x = 12$

$\bar{y} = 6, \sigma_y = 4$

$r = 0.6$

$$\Rightarrow b_{yx} = r\frac{\sigma_y}{\sigma_x} = 0.6\left(\frac{4}{12}\right) = 0.2$$

$$\Rightarrow b_{xy} = r\frac{\sigma_x}{\sigma_y} = 0.6\left(\frac{12}{4}\right) = 1.8$$

Regression equation of y on x

$y - \bar{y} = b_{yx}\,(x - \bar{x})$

$\Rightarrow y - 6 = 0.2\,(x - 8)$

$\Rightarrow y = 0.2x + 4.4$

Putting $x = 20$ gives

$y = 0.2(20) + 4.4 = 8.4$

So, probable value of y on x is 20 is 8.4 .

22. **(i)**

Corner Points	$Z = 3x - 4y$
O(0,0)	0
A(0,8)	-32
B(4,10)	-28
C(6,8)	-14
D(6,5)	-2
E(4,0)	12

Max $Z = 12$ at E(4, 0)

Min $Z = -32$ at A(0, 8)

(ii) Since, maximum value of Z occurs at B(4,10) and C(6,8).

$\therefore \quad 4p + 10q = 6p + 8q$

$\Rightarrow \quad 2q = 2p$

$\Rightarrow \quad p = q$

Number of optimal solutions are infinite.

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Self Assessment Paper-4

MATHEMATICS
Class-12

UNSOLVED

Time Allowed : 3 hours **Maximum Marks : 80**

You are allowed an additional 15 minutes for only reading the paper.
You must NOT start writing during reading time.
The question paper has 12 printed pages.
The Question Paper is divided into three sections and has 22 questions in all.

Section A is compulsory and has fourteen questions.
You are required to attempt all questions either from Section B or Section C.
Section B and Section C have four questions each.
Internal choices have been provided in two questions of 2 marks, two questions of 4 marks and two questions of 6 marks in Section A.
Internal choices have been provided in one question of 2 marks and one question of 4 marks each in Section B and Section C.
While attempting Multiple Choice Questions in Section A, B and C, you are required to write only ONE option as the answer.
The intended marks for questions or parts of questions are given in the brackets [].
All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
Mathematical tables and graph papers are provided.
Instruction to Supervising Examiner:
Kindly read aloud the instructions given above to all the candidates present in the examination hall.

Section-A [65 marks]

1. In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed.

(i) A = $\begin{bmatrix} 3 & 4 \\ -5 & 2 \end{bmatrix}$ and B is the inverse of A. Which of the following is AB ? **[15]**

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **(c)** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **[Understanding]**

(ii) $\int_0^{\pi/2} \sqrt{1-\sin 2x}\, dx$ is equal to

(a) $2\sqrt{2}$ **(b)** $2\left(\sqrt{2}-1\right)$ **(c)** 2 **(d)** $2\left(\sqrt{2}-1\right)$ **[Recall]**

(iii) If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then the value of x is

(a) 1 **(b)** $\frac{1}{5}$ **(c)** –1 **(d)** 0 **[Application]**

(iv) **Assertion (A):** The order of differential equation $\left[1+\left(\frac{dy}{dx}\right)^2+\cos\left(\frac{dy}{dx}\right)\right]^{3/4}=\frac{d^3y}{dx^3}$ is 3.

Reason (R): The degree of differential equation $\left[1+\left(\frac{dy}{dx}\right)^2+\cos\left(\frac{dy}{dx}\right)\right]^{3/4}=\frac{d^3y}{dx^3}$ is not defined.

Which of the following is correct?

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true. **[Analysis]**

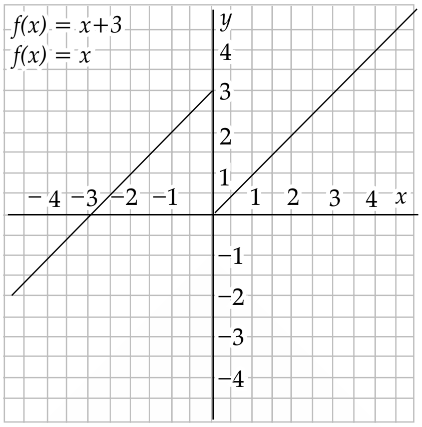
(v) In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take $X=0$ if he opposed and $X=1$ if he is in favour. Then, $E(X)$ is

(a) $\frac{7}{10}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{3}$ **(d)** $\frac{7}{11}$ **[Application]**

(vi) If $\Delta=\begin{vmatrix}1 & 2 & 3\\2 & 0 & 1\\5 & 3 & 8\end{vmatrix}$, the minor of the element a_{22} is

(a) 2 **(b)** 0 **(c)** 7 **(d)** - 7 **[Understanding]**

(vii) Consider the function, $f(x)=\begin{cases}x+3 & \text{if } x\le 0\\ x & \text{if } x>0\end{cases}$



Statement 1: $f(x)$ is continuous at $x=0$.

Statement 2: $f(x)$ is not continuous at $x=0$

Which of the following is correct?

(a) Statement 1 is true and Statement 2 is false. **(b)** Statement 2 is true and Statement 1 is false.

(c) Both the statements are true. **(d)** Both the statements are false. **[Application]**

(viii) If $y=\sin^{-1} x$, then $(1-x^2)y_2$ is equal to :

(a) xy_1 **(b)** xy **(c)** xy_2 **(d)** x^2

(ix) **Statement 1:** If a function is expressed in the form $y = f(x)$, then range of f means set of all possible real values of y corresponding to every value of x in its domain.

Statement 2: If the domain and range of a function f are subsets of R (the set of real numbers), then f is said to be a real function.

Which one of the following is correct?

(a) Statement 1 implies Statement 2. **(b)** Statement 2 implies Statement 1.

(c) Statement 1 is true only if Statement 2 is true. **(d)** Statement 1 and 2 are independent of each other.

[Understanding]

(x) Matrix P is a skew-symmetric matrix of odd order.

Assertion (A): The inverse of matrix P does not exist.

Reason (R): The determinant of matrix P is zero.

Which of the following is correct?

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true. **[Application]**

(xi) Bhuvan is preparing chapati and curry for a dinner party at his home.

Let event A = Bhuvan prepares chapati well.

Let event B = Bhuvan cooks curry well.

Given $P(A) = 00.40$, $P(B) = 0.30$ and $P(A \text{ and } B) = 0.20$

Which one of the following is true about the events A and B?

(a) They are mutually exclusive but not independent.

(b) They are independent but not mutually exclusive.

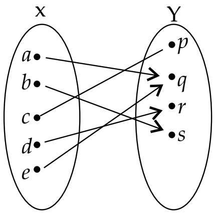
(c) They are both mutually exclusive and independent.

(d) They are neither mutually exclusive nor independent. **[Understanding]**

(xii) If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I + A)^3$, where I is an identity matrix. **[Analysis]**

(xiii) A relation R in the set of real numbers $\mathbb{R}$ defined as $\mathbb{R} - \{(a,b) : \sqrt{a} = b\}$ is a function or not. Justify. **[Understanding]**

(xiv) The function $f : X \rightarrow Y$ is not invertible. State the reason. **[Understanding]**



(xv) Evaluate $\int_0^6 |x+3|\,dx$ **[Application]**

2. (i) The table below shows some of the values of differentiable functions u and v and their derivatives.

x	$u(x)$	$u'(x)$	$v(x)$	$v'(x)$
1	0	2	–1	5
2	4	5	3	2
3	2	– 1	3	0

If $r(x) = u(v(x))$, find $r'(2)$. **[Understanding] [2]**

OR

(ii) Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x)$ is decreasing for all $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. **[Understanding]**

3. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x|\,dx$ **[Evaluate] [2]**

4. Find the point on the curve $y = x^3$ at which the slope of the tangent is equal to the y–coordinate of the point. **[Analysis] [2]**

5. (i) Evaluate: $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}}\,dx$ **[Evaluate] [2]**

OR

(ii) Evaluate: $\int \frac{5-3x^2}{6-x^3+5x}\,dx$ **[Evaluate]**

6. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \sin^{-1}\left(\frac{2b}{1+b^2}\right) = 2\tan^{-1}x$, then show that $x = \frac{a+b}{1-ab}$. **[Application] [2]**

7. Solve for x: $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$ **[Application] [4]**

8. Evaluate: $\int \frac{\sqrt{x}}{a^3 - x^3}\,dx$. **[Application] [4]**

9. (i) The length x of a rectangle is decreasing at the rate of 5 cm/min and the width y increasing at the rate of 4 cm/min, find the rate of change its area when $x = 5$ cm and $y = 8$ cm. **[Application] [4]**

OR

(ii) Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$. **[Analysis]**

10. (i) There are two boxes, Box 1 and Box 2. Each box has 8 red balls, 5 green balls and 7 blue balls. Sahil takes one ball from Box 1 and places it in Box 2. He then draws two balls from Box 2 simultaneously.

(a) What is the probability of Sahil selecting a green ball from Box 1?

(b) After Sahil transfers a ball from Box 1 to Box 2, what is the probability of drawing one red ball and one green ball simultaneously from Box 2?

(c) Given that the balls drawn from Box 2 are red and green, what is the probability that the ball transferred from Box 1 to Box 2 was green? **[Analysis] [4]**

OR

(ii) In answering a question on a multiple-choice test for Class XII, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. Let E_1, E_2, E be the events that the student knows the answer, guesses the answer and answers correctly, respectively.

(a) Find the value of $P(E \mid E_1)$.

(b) $\sum_{k=1}^{k=2} P(E \mid E_k)P(E_k)$ equals ……………………...

(c) Value of $\sum_{k=1}^{k=2} P(E_k)$ is ………………..

(d) What is the probability that the student knows the answer given that he answered it correctly? **[Analysis]**

11. A shopkeeper has 3 varieties of pen 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of ₹21. Jeevan purchased 4 pens of 'A' variety, 3 pen of 'B' variety and 2 pens of 'C' variety for ₹60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for ₹70.

(i) Form a set of simultaneous equations for the above information.

(ii) Solve the set of equations formed in **(a)** by matrix method.

(iii) Hence, find cost of each variety of pen. **[Application] [6]**

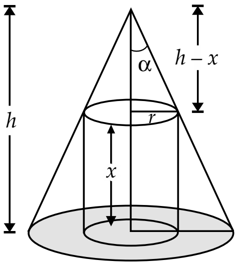
12. **(i)** Solve: $x^3 dy + (xy - 1)dx = 0$.

Given $y = 3 - e$ when $x = \frac{1}{2}$ find y when $x=1$ **[Application] [6]**

OR

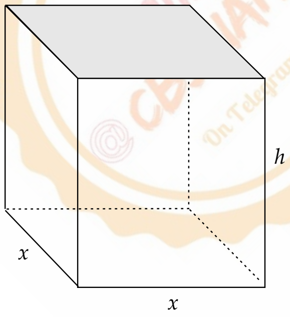
(ii) Evaluate: $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}$ **[Evaluate]**

13. **(i)** Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi-vertical angle α, is one-third that of the cone. Hence, find the greatest volume of the cylinder. **[Application] [6]**



OR

(ii) An open box with a square base is to be made out of a given quantity of cardboard of area c^2. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$.



[Application]

14. A survey says that approximately 30% of all products ordered online across various e-commerce websites are returned. Two products that are ordered online are selected at random.

If X represents the number of products that are returned,

(i) find the probability distribution of X.

(ii) find the mean or expectation of X.

Show your work.. **[Analysis] [6]**

Section-B [15 marks]

15. In subparts (i) to (iii) choose the correct options and in subparts (iv) and (v), answer the questions as instructed. **[5]**

(i) Consider the following statements and choose the correct option:

Statement 1: The area of a triangle with vertices A, B, C is given by $|\overrightarrow{AB} \times \overrightarrow{AC}|$

Statement 2: The area of the parallelogram with adjacent sides AB and AC is given by $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

Which of the following is correct?

(a) Only Statement 1
(b) Only Statement 2
(c) Both Statements 1 and 2
(d) Neither Statement 1 nor Statement 2 **[Recall]**

(ii) The coordinates of the point where the line $\frac{x-1}{3} = \frac{y+4}{7} = \frac{z+4}{2}$ cuts the XY plane is :

(a) (3,7,2) **(b)** (1,-4,-4) **(c)** (0,0,7) **(d)** (7,10,0) **[Application]**

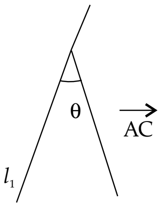
(iii) The cartesian equation of a line is given by $\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$.
The direction cosines of the line is :

(a) $\frac{3}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$ **(b)** $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

(c) $\frac{-3}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$ **(d)** $\frac{\sqrt{3}}{\sqrt{55}}, \frac{-4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$ **[Understanding]**

(iv) If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 225$ and $|\vec{a}| = 5$, then write the value of $|\vec{b}|$. **[Understanding]**

(v) The points $A(2, 4, 4)$, $B(6, 8, 4)$, $C(6, 4, 0)$, $D(2, 0, 0)$ and $M(4,4,2)$ are given.
The straight line l_1 has equation $r_1 = 6\hat{i} + 4\hat{j} + \lambda(\hat{i} + \hat{j})$, where λ is a scalar parameter. Find the acute angle between $\overrightarrow{AC}$ and l_1.
[Analysis]



16. **(i)** Given below are two vectors in their component form such that.

$$|\vec{p} \times \vec{q}| = 1$$
$$\vec{p} = x\hat{i} - \frac{y}{4}\hat{j}$$
$$\vec{q} = y\hat{i} + \frac{x}{9}\hat{j}$$

Find the relation between the variables x and y. **[Application] [2]**

OR

(ii) If the points with position vectors $10\hat{i}+3\hat{j}, 12\hat{i}-5\hat{j}$ and $\lambda\hat{i}+11\hat{j}$ are collinear, find the value of λ. **[Application]**

17. **(i)** Find the equation of the plane which is parallel to X-axis and has intercepts 5 and 7 on Y-axis and Z-axis respectively. **[Application] [4]**

OR

(ii) Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also, find their point of intersection. **[Application]**

18. **(i)** Sketch the region enclosed between the curves $y = ax^2$ and $x = ay^2$, $a > 0$.

(ii) Evaluate: $\int \left(\sqrt{\frac{x}{a}} - ax^2 \right) dx$

(iii) If the area enclosed between the curves $y = ax^2$ and $x = ay^2$, $a > 0$, is 27 sq. unit, find the value of 'a'. **[4]**

Section-C [15 marks]

19. In subparts (i) and (ii), choose the correct options and in subparts (iii) to (v), answer the questions as instructed. **[5]**

(i) If the demand function for a monopolist is given by $x = 100 - 4p$, the average revenue function is :

(a) $25 - \frac{x}{4}$ **(b)** $25 - \frac{x^2}{4}$ **(c)** $25 - \frac{x}{2}$ **(d)** none of these **[Recall]**

(ii) Read the following statements and choose the correct option:

(I) Regression measures the extent of correlation and the nature of correlation.

(II) This simple regression is called linear regression.

(III) Regression equations are algebraic formulation of regression lines.
Which of the following is correct?

(a) Only II is correct **(b)** All are correct.
(c) Only I and III are correct **(d)** Only II and IV are correct **[Recall]**

(iii) The fixed cost of a new product is ₹18,000 and the variable cost per unit is ₹550. If the demand function is $p(x) = 4000 - 150x$, find the break-even values. **[Understanding]**

(iv) If the two regression lines are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find mean of x and y. **[Understanding]**

(v) The cost function $C(x) = 3x^2 - 6x + 5$. Find the average cost when $x = 2$. **[Understanding]**

20. **(i)** A bicycle spare parts manufacturing company decided to upgrade its production unit by installing advanced machinery. As a result of this investment, the cost of producing spare parts increased. The company used to sell 1000 spare parts daily at ₹50 per unit.

The increased daily cost of production $C(x)$ for x number of spare parts is given by $C(x) = \frac{99}{2}x + 525$.

If the selling price remains the same, find how many more spare parts must the company sell daily to ensure no loss. **[Application] [2]**

OR

(ii) A company produced a commodity with ₹ 20,000 fixed costs. The variable cost is estimated to 35% of the total revenue. When it is sold at a rate of ₹ 6 per unit, find the total revenue, total cost and profit function of the revenue when it is sold. **[Application]**

21. **(i)** Shown below is a data set that includes the average temperature and the number of ice cream cones sold per day by an ice cream truck in a small town over the course of six days.

Temperature (°F) **X**	**Cones sold** **Y**
65	180
75	250
60	160
70	210
80	300
70	220

Find the line of regression using average temperature as the independent variable. **[Application] [4]**

OR

(ii) Given that the total cost function for x units of a commodity is: $C(x)=\frac{x^3}{3}+3x^2-7x+16$.

(a) Find the Marginal Cost (MC).

(b) Find the Average Cost (AC)

(c) Prove that: Marginal Average Cost $(MAC)=\frac{x(MC)-C(x)}{x^2}$. **[Application]**

22. A soap company manufactures two varieties of soap using two machines. The time taken on each machine to manufacture one batch of each variety of soap is given below.

	Machine 1	**Machine 2**
Soap A	1 hour	1 hour
Soap B	2 hours	1 hour

Machine 1 and Machine 2 cannot be operational for more than 12 and 10 hours in a day, respectively. The profit earned from one batch of Soap A and one batch of Soap B is ₹3000 and ₹4000, respectively.

(i) Formulate LPP and draw the feasible region.

(ii) Find the number of batches of each kind of soap that should be made daily to maximise the profit. **[Analysis] [4]**

■■

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Self Assessment Paper - 4

Section-A

1. (i) **Option (b) is correct.**

Explanation: Given matrix $A = \begin{pmatrix} 3 & 4 \\ -5 & 2 \end{pmatrix}$ and $B = A^{-1}$, the product AB is the identity matrix:

$$AB = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(ii) Option (d) is correct.

Explanation:

Let

$$\begin{aligned} I &= \int_0^{\pi/2} \sqrt{1-\sin 2x}\,dx \\ &= \int_0^{\pi/4} \sqrt{(\cos x - \sin x)^2}\ dx \\ &\quad + \int_{\pi/4}^{\pi/2} \sqrt{(\sin x - \cos x)^2}\ dx \\ &= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2} \\ &= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1 + \left(-0 - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) \\ &= 2\sqrt{2} - 2 = 2(\sqrt{2}-1) \end{aligned}$$

(iii) Option (b) is correct.

Explanation:

$$\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$$

$$\Rightarrow \quad \sin^{-1}\frac{1}{5} + \cos^{-1}x = \sin^{-1}(1)$$

$$\Rightarrow \quad \sin^{-1}\frac{1}{5} + \cos^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow \quad \sin^{-1}\frac{1}{5} + \cos^{-1}x = \sin^{-1}x + \cos^{-1}x$$

$$\left[\because \cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}\right]$$

$$\Rightarrow \quad x = \frac{1}{5}$$

(iv) Option (b) is correct.

Explanation:

Given,

$$\left[1 + \left(\frac{dy}{dx}\right)^2 + \cos\left(\frac{dy}{dx}\right)\right]^{3/4} = \frac{d^3y}{dx^3}$$

$$\Rightarrow \quad \left[1 + \left(\frac{dy}{dx}\right)^2 + \cos\left(\frac{dy}{dx}\right)\right]^{3} = \left(\frac{d^3y}{dx^3}\right)^4$$

Thus, of order is 3, but degree is not defined as equation is not a polynomial.

(v) Option (a) is correct.

Explanation: **Case 1:** If selected member opposed

$$P(X=0) = \frac{30}{100}$$

Case 2: If selected member favours

$$P(X=1) = \frac{70}{100}$$

Required probability distribution is

X	0	1
$p(X)$	$\frac{30}{100}$	$\frac{70}{100}$

Then,

$$\begin{aligned} E(X) &= 0 \times \frac{30}{100} + 1 \times \frac{70}{100} \\ &= \frac{70}{100} \\ &= \frac{7}{10} \end{aligned}$$

(vi) Option (d) is correct.

Explanation: Given,

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 5 & 3 & 8 \end{vmatrix}$$

$$\therefore \quad \text{Minor of } a_{22} = \begin{vmatrix} 1 & 3 \\ 5 & 8 \end{vmatrix}$$

$$\Rightarrow \quad = 8 - 15$$

$$\Rightarrow \quad = -7$$

(vii) Option (b) is correct.

Explanation: For any point on the real number line, this function is defined.

It can be seen that the value of the function $x=0$ changes suddenly. Following the concepts of limits, we can say that; Right-hand limit ≠ Left-hand limit.

It implies that this function is not continuous at $x = 0$.

(viii) Option (a) is correct.

Explanation: $y = \sin^{-1} x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2}\cdot\frac{dy}{dx} = 1$$

Again, differentiating both sides w.r. to x, we get

$$\sqrt{1-x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{-2x}{2\sqrt{1-x^2}}\right) = 0$$

Simplifying, we get $(1-x^2)y_2 = xy_1$

(ix) Option (d) is correct.

(x) Option (a) is correct.

Explanation: The determinant of a skew symmetric matrix of odd order is zero, which means skew symmetric matrix of an odd number is singular. Hence its inverse does not exist.

(xi) Option (d) is correct.

Explanation: Since, $P(A) = 0.40$, $P(B) = 0.30$ and $P(A \text{ and } B) = 0.20$

$P(A \text{ and } B) = P(A \cap B) \neq 0$

The events are not mutually exclusive.

And $P(A).P(B) = 0.40 \times 0.30 = 0.12 \neq P(A \cap B)$

The events are not independent.

(xii)
$$\begin{aligned} 7A - (I + A)^3 &= 7A - I^3 - 3A^2I - 3AI^2 - A^3 \\ &= 7A - I - 3A - 3A - A^2A \\ &\qquad [\because I^3 = I^2 = I \text{ and } A^2I = AI = A] \\ &= 7A - I - 3A - 3A - A \times A \\ &= 7A - I - 3A - 3A - A \\ &= 7A - I - 7A = -I \end{aligned}$$

(xiii) Since, $\sqrt{a}$ is not defined for $a \in (-\infty, 0)$.

$\therefore$ $\sqrt{a} = b$ is not a function.

(xiv) For a function to have an inverse, each element $b \in B$ must not have more than one pre image in $a \in A$.

But here, q has more than one pre-image in A.

Thus, function $f : X \to Y$ is not invertible.

(xv) $\int_0^6 |x+3|\,dx$

Here, the function is $f(x) = |x + 3|$

$$f(x) = x + 3, \quad x > -3$$

The integration can be done as :

$$\begin{aligned} \int_0^6 |x+3|\,dx &= \int_0^6 (x+3)dx \\ &= \left[\frac{x^2}{2} + 3x\right]_0^6 \\ &= 18 + 18 - 0 - 0 \\ &= 36 \end{aligned}$$

2. (i) If
$$\begin{aligned} r(x) &= u(v(x)) \\ \therefore \quad r(x) &= u(v(x)) \\ r'(x) &= u'\{v(x)\}\frac{d}{dx}\{v(x)\} \\ &= u'\{v(x)\}.\{v'(x)\} \\ \therefore \quad r'(2) &= u'\{v(2)\}.v'(2) \\ r'(2) &= u'(3).2 \\ r'(2) &= -1.2 \\ r'(2) &= -2 \end{aligned}$$

OR

(ii)
$$\frac{1}{1+(\sin x + \cos x)^2} > 0;\ x \in R$$

and $\frac{\pi}{4} < x < \frac{\pi}{2}$ or $\cos x < \sin x$ or $\cos x - \sin x < 0$

or $f'(x) < 0$ or $f(x)$ is decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

3.
$$|\sin x| = \begin{cases} \sin x, \text{if } \sin x \geq 0 \\ -\sin x, \text{if } \sin x < 0 \end{cases}$$

$$\begin{aligned} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} |\sin x|\,dx &= \int_{\frac{-\pi}{2}}^{0} -\sin x\,dx + \int_0^{\frac{\pi}{2}} \sin x\,dx \\ &= \cos x]_{-\frac{\pi}{2}}^{0} + (-\cos x)_0^{\frac{\pi}{2}} \\ &= \cos 0 - \cos\left(\frac{-\pi}{2}\right) + \left[-\cos\frac{\pi}{2} - (-\cos 0)\right] \\ &= (1 - 0) + (-0 + 1) \\ &= 2 \end{aligned}$$

4. Let the required point on the curve $y = x^3$ be P(x_1, y_1)

We have,

$$y = x^3 \Rightarrow \frac{dy}{dx} = 3x^2$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 3x^2$$

It is given that

Slope of the tangent at $P(x_1, y_1)$ = Ordinate of $P(x_1, y_1)$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)(x_1, y_1) = y_1$$

$$\Rightarrow \quad 3x_1^2 = y_1$$

5. (i) Let $I = \int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}}dx$ ...(i)

Using property,

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$I = \int_1^2 \frac{\sqrt{3-x}}{\sqrt{3-(3-x)}+\sqrt{3-x}}dx$$

$$I = \int_1^2 \frac{\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}}dx \qquad \text{...(ii)}$$

Adding Eqs. (i) and (ii),

$$2I = \int_1^2 \frac{\sqrt{x}+\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}}dx = \int_1^2 1.dx$$

$$= [x]_1^2 = 2 - 1 = 1$$

$$\therefore \quad I = \frac{1}{2}.$$

OR

(ii) Here, $\frac{d}{dx}(6 - x^3 + 5x) = -3x^2 + 5 = 5 - 3x^2$

Since, the numerator is the derivative of denominator

$$\Rightarrow \quad \int \frac{f'(x)}{f(x)} = \ln|f(x)| + C$$

$$\Rightarrow \quad \frac{5-3x^2}{6-x^3+5x}dx = \ln|6 - x^3 + 5x| + C$$

6. We have, $\sin^{-1}\frac{2a}{1+a^2} = 2\tan^{-1} a$

and $\sin^{-1}\frac{2b}{1+b^2} = 2\tan^{-1} b$

The equation becomes

$$2\tan^{-1} a + 2\tan^{-1} b = 2\tan^{-1} x$$

$$\left(\because 2\tan^{-1} x = \sin^{-1}\frac{2x}{1+x^2}\right)$$

$$\Rightarrow \quad \tan^{-1} a + \tan^{-1} b = \tan^{-1} x$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{a+b}{1-ab}\right) = \tan^{-1} x$$

$$\Rightarrow \quad x = \frac{a+b}{1-ab}$$ **Hence Proved.**

7. We have,

$$\begin{aligned} 2\tan^{-1}(\cos x) &= \tan^{-1}(2\operatorname{cosec} x) \\ \Rightarrow \quad \tan^{-1}\frac{2\cos x}{1-\cos^2 x} &= \tan^{-1}(2\operatorname{cosec} x) \\ \Rightarrow \quad \frac{2\cos x}{\sin^2 x} &= (2\operatorname{cosec} x) \\ \Rightarrow \quad \cos x &= \sin x \\ \Rightarrow \quad \tan x &= 1 \\ \Rightarrow \quad x &= \frac{\pi}{4} \end{aligned}$$

8. $$I = \int \frac{\sqrt{x}}{a^3 - x^3}\,dx$$

$$\Rightarrow \quad I = \int \frac{(x)^{1/2}}{(a^{3/2})^2 - (x^{3/2})^2}\,dx$$

Put $x^{3/2} = t$

$$\therefore \quad \frac{3}{2}\sqrt{x}dx = dt$$

$$\Rightarrow \quad \sqrt{x}dx = \frac{2}{3}dt$$

$$\therefore \quad I = \frac{2}{3}\int \frac{dt}{(a^{3/2})^2 - t^2}$$

$$\Rightarrow \quad I = \frac{2}{3} \times \frac{1}{2a^{3/2}} \log\left|\frac{a^{3/2}+t}{a^{3/2}-t}\right| + C$$

$$\Rightarrow \quad I = \frac{1}{3a^{3/2}} \log\left|\frac{a^{3/2}+x^{3/2}}{a^{3/2}-x^{3/2}}\right| + C$$

9. **(i)** Let A denote the area of rectangle at instant t.

$\therefore$ $A = xy$ (area of rectangle),

$$\Rightarrow \quad \left.\begin{aligned} \frac{dx}{dt} &= -5\text{cm/min} \\ \frac{dy}{dt} &= 4\,\text{cm/min} \end{aligned}\right\}\text{(given)}$$

$$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$\Rightarrow \quad \left(\frac{dA}{dt}\right)_{x=5,y=8} = 5 \times 4 + 8 \times (-5)\ \text{cm}^2/\text{min}$$

$$\Rightarrow \quad \frac{dA}{dt} = (20-40)\ \text{cm}^2/\text{min}$$

$$\Rightarrow \quad \frac{dA}{dt} = -20\ \text{cm}^2/\text{min}$$

Here, (–) ve sign shows that area is decreasing at the rate of 20 cm^2/min.

OR

(ii) Given, $y = \sin^{-1}\left(\dfrac{2^{x+1}}{1+4^x}\right)$

$$\Rightarrow \quad y = \sin^{-1}\left(\frac{2 \cdot 2^x}{1+2^{2x}}\right)$$

Let $2^x = \tan\theta, \Rightarrow \theta = \tan^{-1}2^x$

$$y = \sin^{-1}(\sin 2\theta) \quad \left[\because \frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta\right]$$

$$\Rightarrow \quad y = 2\theta$$

$$\Rightarrow \quad y = 2\tan^{-1}(2^x)$$

Differentiating w.r.t. x,

$$\frac{dy}{dx} = 2 \cdot \frac{1}{1+(2^x)^2} \cdot \frac{d}{dx}(2^x)$$

$$= \frac{2}{1+4^x} \cdot 2^x \cdot \log_e 2$$

$$= \frac{2^{x+1}}{1+4^x}\log_e 2$$

10. **(i)** **(a)** P (green ball from box 1)$= \dfrac{5}{20} = \dfrac{1}{4}$

(b) There are three cases:

Case 1: Let ball transfer from bag 1 to bag 2 was red

P(Drawing one red and one green ball)

$$= \frac{{}^9C_1 \times {}^5C_1}{{}^{21}C_1}$$

$$= \frac{9 \times 5 \times 2}{21 \times 20}$$

Case 2: Let ball transfer from box 1 to box 2 was green

P(Drawing one red and one green ball)

$$= \frac{{}^8C_1 \times {}^6C_1}{{}^{21}C_1}$$

$$= \frac{8 \times 6 \times 2}{21 \times 20}$$

Case 3: Let ball transfer from box 1 to box 2 was blue

P(Drawing one red ball and one green ball)

$$= \frac{{}^8C_1 \times {}^5C_1}{{}^{21}C_2}$$

$$= \frac{8 \times 5 \times 2}{21 \times 20}$$

P(after transfer one ball from box 1 to box 2, then drawing one red ball and one green ball from box 2)

$$= \frac{9 \times 5 \times 2}{21 \times 20} + \frac{8 \times 6 \times 2}{21 \times 20} + \frac{8 \times 5 \times 2}{21 \times 20}$$

$$= \frac{45}{210} + \frac{48}{210} + \frac{40}{210}$$

$$= \frac{133}{210} = \frac{19}{30}$$

(c) Let E_1 : Red ball transfer from box 1 to box 2

Let E_2 : Green ball transfer from box 1 to box 2

Let E_3 : Blue ball transfer from box 1 to box 2

A: After transfer one ball from box 1 to box 2, then one red ball and one green ball

$$P(E_1) = \frac{8}{20}$$

$$P(E_2) = \frac{5}{20}$$

$$P(E_3) = \frac{7}{20}$$

$$P(A/E_1) = \frac{{}^4C_1 \times {}^5C_1}{{}^{21}C_1} = \frac{9 \times 5 \times 2}{21 \times 20} = \frac{45}{210}$$

$$P(A/E_2) = \frac{{}^3C_1 \times {}^2C_1}{{}^2C_2} = \frac{8 \times 6 \times 2}{21 \times 20} = \frac{48}{210}$$

$$P(A/E_3) = \frac{{}^3C_1 \times {}^3C_1}{{}^2C_2} = \frac{8 \times 5 \times 2}{21 \times 20} = \frac{40}{210}$$

Now, $P(E_2/A)$

$$= \frac{P(E_2).P\left(\dfrac{A}{E_2}\right)}{P(E_1).P\left(\dfrac{A}{E_1}\right) + P(E_2).P\left(\dfrac{A}{E_2}\right) + P(E_1).P\left(\dfrac{A}{E_2}\right)}$$

$$= \frac{\dfrac{5}{20} \times \dfrac{48}{210}}{\dfrac{8}{20} \times \dfrac{45}{210} + \dfrac{5}{20} \times \dfrac{48}{210} + \dfrac{7}{20} \times \dfrac{40}{210}}$$

$$= \frac{240}{360} + 240 + 280$$

$$= \frac{240}{880} = \frac{6}{22} = \frac{3}{11}$$

OR

(ii) **(a)** $P(E/E_1) = 1$

(b) $$\sum_{k=1}^{k=2} P(E\mid E_k)P(E_k) = P(E\mid E_1)P(E_1) + P(E\mid E_2)P(E_2)$$

$$= 1\times\frac{3}{5}+\frac{1}{3}\times\frac{2}{5} = \frac{11}{15}$$

(c) $$\sum_{k=1}^{k=2} P(E_k) = P(E_1)+P(E_2)$$

$$= \frac{3}{5}+\frac{2}{5}$$

$$= \frac{5}{5} = 1$$

(d) $$P(E\mid E_1) = \frac{P(E_1)\cdot P(E_1\mid E)}{P(E_1)\cdot P(E_1\mid E)+P(E_2)\cdot P(E\mid E_2)}$$

$$= \frac{\frac{3}{5}\times 1}{\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{3}}$$

$$= \frac{\frac{3}{5}}{\frac{11}{15}}$$

$$= \frac{9}{11}$$

11. **(i)** Let the cost of 'A' variety pen be x, the cost of 'B' variety pen be y, the cost of 'C' variety pen be z.

According to the question,

$$x+y+z=21$$
$$4x+3y+2z=60$$
$$6x+2y+3z=70$$

(ii) It can be written as,

$$\begin{bmatrix}1&1&1\\4&3&2\\6&2&3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}21\\60\\70\end{bmatrix}$$

i.e., $AX = B$

where, $A=\begin{bmatrix}1&1&1\\4&3&2\\6&2&3\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}21\\60\\70\end{bmatrix}$

Now, $|A|=\begin{vmatrix}1&1&1\\4&3&2\\6&2&3\end{vmatrix}$

$$=1(9-4)-1(12-12)+1(8-18)$$
$$=5-10=-5\neq 0$$

$\Rightarrow A^{-1}$ exists.

Co-factors of A are :

$$A_{11}=(9-4)=5$$
$$A_{12}=-(12-12)=0$$
$$A_{13}=(8-18)=-10$$
$$A_{21}=-(3-2)=-1$$
$$A_{22}=(3-6)=-3$$
$$A_{23}=-(2-6)=4$$
$$A_{31}=(2-3)=-1$$
$$A_{32}=-(2-4)=2$$
$$A_{33}=3-4=-1$$

$$adj\ A=\begin{bmatrix}5&0&-10\\-1&-3&4\\-1&2&-1\end{bmatrix}^T=\begin{bmatrix}5&-1&-1\\0&-3&2\\-10&4&-1\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}(adj\ A)$$

Also, $AX = B$

$\Rightarrow$ $X = A^{-1}B$

$\Rightarrow$ $X = \dfrac{1}{|A|}(adj\ A)B$

$$=\frac{1}{|A|}\begin{bmatrix}5&-1&-1\\0&-3&2\\-10&4&-1\end{bmatrix}\begin{bmatrix}21\\60\\70\end{bmatrix}$$

$$=\frac{1}{-5}\begin{bmatrix}105-60-70\\0-180+140\\-210+240-70\end{bmatrix}$$

$$=\frac{1}{-5}\begin{bmatrix}-25\\-40\\-40\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}5\\8\\8\end{bmatrix}$$

$\therefore$ $x = 5, y = 8, z = 8$

(iii) $\therefore$ Cost of 'A' variety pen = ₹ 5

'B' variety pen = ₹ 8

'C' variety pen = ₹ 8

12. **(i)** $x^3dy + (xy - 1)dx = 0$

$\Rightarrow$ $x^3dy = -(xy-1)dx$

$\Rightarrow$ $x^3dy = (1-xy)dx$

$\Rightarrow$ $\dfrac{dy}{dx}=\dfrac{1-xy}{x^3}=\dfrac{1}{x^3}-\dfrac{y}{x^2}$

$\Rightarrow$ $\dfrac{dy}{dx}+\dfrac{y}{x^2}=\dfrac{1}{x^3}$

$\Rightarrow$ I.F. $=\int e^{\frac{1}{x^2}}dx = e^{\frac{-1}{x}}$

$\Rightarrow$ $y.e^{-\frac{1}{x}}=\int e^{-\frac{1}{x}}.\dfrac{1}{x^3}dx$

$\Rightarrow$ $y.e^{-\frac{1}{x}} = -\int e^t.tdt$

$\Rightarrow$ $y = \dfrac{1}{x}+1+Ce^{\frac{1}{x}}$

where C is constant $\left[\dfrac{1}{x^2}dx=dt\right]$ $\left[\text{let}\dfrac{-1}{x}=t\right]$

Now, put $x=\dfrac{1}{2}$ and $y = 3-e$

$\Rightarrow$ $3-e = 2 + 1 + Ce^2$

Now, x=1 then, C = $\left(\dfrac{-1}{e}\right)$

$$y = \frac{1}{1}+1+\left(-\frac{1}{e}\right)e^{\frac{1}{1}}$$

$$y = 1 + 1 - 1$$

$$y = 1$$

OR

(ii) $I = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$

Let $x = \cos 2\theta$

$dx = -2\sin 2\theta\, d\theta$

$$I = \int \tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}(-2\sin 2\theta)d\theta$$

$$I = \int \tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}(-2\sin 2\theta)d\theta$$

$$I = \int \tan^{-1}\tan\theta(-2\sin 2\theta)d\theta$$

$$I = -2\int \theta\sin 2\theta$$

$$= -2\left[\theta\int \sin 2\theta\, d\theta\right] - \int\left[\frac{d}{d\theta}\theta\int \sin 2d\theta\right]d\theta$$

$$= -2\left[-\theta\frac{\cos 2\theta}{2} - \int\frac{-\cos 2\theta}{2}d\theta\right]$$

$$= \theta\cos 2\theta - \int \cos 2\theta\, d\theta$$

$$= \theta\cos 2\theta - \frac{\sin 2\theta}{2} + C$$

$$= \theta\cos 2\theta - \frac{1}{2}\sqrt{1-\cos^2 2\theta} + C$$

$$= \frac{1}{2}x\cos^{-1}x - \frac{1}{2}\sqrt{1-x^2} + C$$

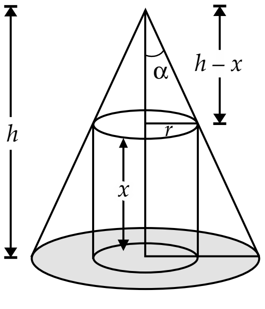
Examiner's Comment

- Some candidates made errors in substitution. In most of the cases, candidates who used appropriate substitution did not give the answer in original variables.

Answering Tip

- Thoroughly understand and do adequate practice of integration using substitution, partial fraction and special integrals. Also, do not leave your answer in terms of new variable.

13. (i)



$$\frac{r}{h-x} = \tan\alpha$$

$$r = (h-x)\tan\alpha$$

Volume of cylinder,

$$V = \pi r^2 x$$

$$V = \pi(h-x)^2 x\tan^2\alpha$$

$$\therefore \quad \frac{dV}{dx} = \pi\tan^2\alpha\{(h-x)^2 + 2x(h-x)(-1)\}$$

$$= \pi\tan^2\alpha(h-x)(h-x-2x)$$

$$\frac{dV}{dx} = \pi\tan^2\alpha(h-x)(h-3x)$$

$$\frac{dV}{dx} = 0 \text{ or } h = x \text{ or } h = 3x$$

i.e., $x = \frac{h}{3}$

$$\frac{d^2V}{dx^2} = \pi\tan^2\alpha(6x-4h)$$

$\therefore \frac{d^2V}{dx^2} < 0$ at $x = \frac{h}{3}$

$\therefore$ V is maximum, at $x = \frac{h}{3}$.

Maximum volume is $V = \frac{4}{27}\pi h^3\tan^2\alpha$

Hence Proved.

OR

(ii) Let x units be the length of a square box and h units be its height then

Total surface area of open box = $(x^2 + 4xh)$

Given: $x^2 + 4xh = c^2$

$$h = \frac{c^2-x^2}{4x}$$

Volume of box $V = x^2h = x^2\left(\frac{c^2-x^2}{4x}\right)$

$$V = \frac{c^2}{4}x - \frac{x^3}{4}$$

$$\frac{dV}{dx} = \frac{c^2}{4} - \frac{3x^2}{4}$$

$$\frac{d^2V}{dx^2} = -\frac{6x}{4}$$

for maximum/minimum

$$\frac{dV}{dx} = 0$$

$$\frac{c^2}{4} - \frac{3x^2}{4} = 0$$

$$x^2 = \frac{c^2}{3}$$

$$x = \frac{c}{\sqrt{3}}$$

$$\left(\frac{d^2V}{dx^2}\right)_{x=c} = -\frac{6}{4}$$

Hence, volume is maximum at $x = \frac{c}{\sqrt{3}}$

$$\text{Max volume of the box} = \frac{c^2}{4}x - \frac{x^3}{4}$$

$$= \frac{c^3}{4\sqrt{3}} - \frac{c^3}{3\sqrt{3}} \times \frac{1}{4}$$

$$= \frac{c^3}{4\sqrt{3}}\left(1-\frac{1}{3}\right) = \frac{c^3}{4\sqrt{3}}\left(\frac{2}{3}\right)$$

Max vol. of box = $\frac{c^3}{6\sqrt{3}}$ **Hence Proved**

14. **(i)** Writes that the possible values of X are 0,1 and 2. Takes R to be the event that the product is returned and N to be the event that the product is not returned.

Uses the given information and finds the probability that the product is returned a $P(R) = 0.3$ and the probability that the product is not returned as $P(N) = 1 - 0.3 = 0.7$.

Finds the probability of each possible values of the random variable X as:

$$P(X = 0) = P(NN)$$
$$= (0.7)(0.7) = 0.49$$
$$P(X = 1) = P(RN, NR)$$
$$= (0.3)(0.7) + (0.7)(0.3)$$
$$= 0.21 + 0.21$$
$$= 0.42$$
$$P(X = 2) = P(RR)$$
$$= (0.3)(0.3) = 0.09$$

Finds the probability distribution of X as:

X or x_i	0	1	2
P or p_i	0.49	0.42	0.09

(ii) Finds the mean or expectation of X as:

$$E(X) = \sum x_1 p_1$$
$$= (0 \times 0.49) + (1 \times 0.42) + (2 \times 0.09)$$
$$= 0.6$$

(ii)

s	0	1	2	3
P	$\frac{4}{16} = \frac{1}{4}$	$\frac{6}{16} = \frac{3}{8}$	$\frac{4}{16} = \frac{1}{4}$	$\frac{2}{16} = \frac{1}{8}$

(iii)
$$P(x \le 2) = 1 - P(x > 2)$$
$$= 1 - P(x = 3)$$
$$= 1 - \frac{2}{16}$$
$$= \frac{14}{16} = \frac{7}{8}$$

Section-B

15. (i) Option (d) is correct.

Explanation: The area of the parallelogram with adjacent sides AB and AC = $|\vec{AB} \times \vec{AC}|$. Hence, the area of the triangle with vertices A, B, C $= \frac{1}{2}|\vec{AB} \times \vec{AC}|$.

(ii) Option (d) is correct.

Explanation:

Putting $z = 0$ in given equation gives

$$\frac{x-1}{3} = \frac{y+4}{7} = 2$$

Coordinates of required point are (7, 10, 0).

(iii) Option (b) is correct.

The equation of a line can be rewritten as:

$$\frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{y+2}{2} = \frac{z-3}{3}$$

$$\frac{x - \frac{1}{2}}{\sqrt{3}} = \frac{y+2}{4} = \frac{z-3}{6}$$

Direction cosines of the line, whose direction ratios are $\sqrt{3}$, 4, 6 are:

$$\frac{\sqrt{3}}{\sqrt{\sqrt{3}^2 + 4^2 + (6)^2}}, \frac{4}{\sqrt{\sqrt{3}^2 + 4^2 + (6)^2}}, \frac{6}{\sqrt{\sqrt{3}^2 + 4^2 + (6)^2}}$$

or $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

Since, line makes obtuse angle with Y-axis, then $\cos\beta < 0$. Therefore, direction cosines are $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$.

(iv)
$$(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 225$$

or $|\vec{a}|^2 |\vec{b}|^2 (\sin^2\theta + \cos^2\theta) = 225$

or $(5)^2 |\vec{b}|^2 = 225$

or $|\vec{b}| = 3$

(v)
$$\vec{AC} = (6-2)\hat{i} + (4-4)\hat{j} + (0-4)\hat{k}$$
$$\vec{AC} = 4\hat{i} - 4\hat{k}$$
$$\cos\theta = \frac{(4\hat{i} - 4\hat{k}).(\hat{i} + \hat{j})}{\sqrt{4^2 + (-4)^2}.\sqrt{1^2 + 1^2}}$$
$$\cos\theta = \frac{4+0-0}{\sqrt{32}.\sqrt{2}}$$
$$\cos\theta = \frac{4}{8}$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = 60°$$

16. (i)
$$|\vec{p} \times \vec{q}| = 1$$
$$\Rightarrow \left|\left(x\hat{i} - \frac{y}{4}\hat{j}\right) \times \left(y\hat{i} + \frac{x}{9}\hat{j}\right)\right| = 1$$
$$\Rightarrow \left|(xy\hat{i} \times \hat{i}) + \frac{x^2}{9}(\hat{i} \times \hat{j}) - \frac{y^2}{4}(\hat{j} \times \hat{i}) - \frac{xy}{36}(\hat{j} \times \hat{j})\right| = 1$$
$$\Rightarrow \left|\left(\frac{x^2}{9}\right)(\hat{i} \times \hat{j}) - \frac{y^2}{9}(\hat{j} \times \hat{i})\right| = 1$$
$$\Rightarrow \left|\left(\frac{x^2}{9}(\hat{k}) - \frac{y^2}{4}(-\hat{k})\right)\right| = 1$$
$$\Rightarrow \left|\left(\frac{x^2}{9} + \frac{y^2}{4}\right)(\hat{k})\right| = 1$$
$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = 1$$

OR

(ii) Let A be $10\hat{i} + 3\hat{j}$, B be $12\hat{i} - 5\hat{j}$, C be $\lambda\hat{i} + 11\hat{j}$

$$\overrightarrow{AB} = 2\hat{i} - 8\hat{j}$$
$$\overrightarrow{AC} = (\lambda - 10)\hat{i} + 8\hat{j}$$

As $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear

$$\frac{2}{\lambda-10} = \frac{-8}{8}$$

So, $\lambda = 8$

17. **(i)** Direction numbers of the X-axis are <1, 0, 0>. As the plane has intercept 5 and 7 on Y-axis and Z-axis respectively it passes through (0, 5, 0), (0, 0, 7) and parallel to X-axis is

$$\begin{vmatrix} x-0 & y-5 & z-0 \\ 0-0 & 0-5 & 7-0 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow x\begin{vmatrix} -5 & 7 \\ 0 & 0 \end{vmatrix} - (y-5)\begin{vmatrix} 0 & 7 \\ 1 & 0 \end{vmatrix} + z\begin{vmatrix} 0 & -5 \\ 1 & 0 \end{vmatrix} = 0$$

$\Rightarrow \quad x \times 0 + 7y - 35 + 5z = 0$

$\Rightarrow \quad 7y + 5z - 35 = 0$

$\Rightarrow \quad 7y + 5z = 35$

OR

(ii) Let $\dfrac{x+1}{3} = \dfrac{y+3}{5} = \dfrac{z+5}{7} = u;$

and $\dfrac{x-2}{1} = \dfrac{y-4}{3} = \dfrac{z-6}{5} = v$

General points on the lines are $(3u - 1, 5u - 3, 7u - 5)$ and $(v + 2, 3v + 4, 5v + 6)$.

Lines intersect if

$3u - 1 = v + 2,$

$5u - 3 = 3v + 4,$

$7u - 5 = 5v + 6$ for some u and v.

or $\quad 3u - v = 3 \quad$ ...(i)

$5u - 3v = 7 \quad$ ...(ii)

$7u - 5v = 11 \quad$ ...(iii)

Solving equations (i) and (ii), we get

$$u = \frac{1}{2}, v = -\frac{3}{2}$$

Putting u and v in equation (iii),

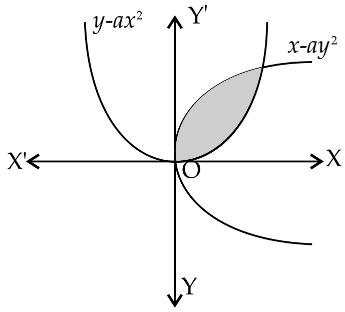
$$7\left(\frac{1}{2}\right) - 5\left(-\frac{3}{2}\right) = 11$$

∴ lines intersect :

Putting values of u and v in general points.

Point of intersection of lines is : $\left(\dfrac{1}{2}, -\dfrac{1}{2}, -\dfrac{3}{2}\right)$.

18. **(i)** The region enclosed bounded enclosed between the curves $y = ax^2$ and $x = ay^2$, $a > 0$ is given as:



(ii)
$$\int\left(\sqrt{\frac{x}{a}} - ax^2\right)dx = \frac{1}{\sqrt{a}}\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{ax^3}{3} + c$$

$$= \frac{2}{3}\cdot\sqrt{\frac{x^3}{a}} - \frac{ax^3}{3} + c$$

(iii) $\quad y = ax^2$

$x = ay^2$

$\Rightarrow \quad x = a(ax^2)^2$

$\Rightarrow x = 0, \dfrac{1}{a}$ and $y = 0, \dfrac{1}{a}$

Pt. of intersection = (0, 0) $\left(\dfrac{1}{a}, \dfrac{1}{a}\right)$

$$\Rightarrow \quad \int_0^{\frac{1}{a}}\left(\sqrt{\frac{x^3}{a}} - ax^2\right)dx = 27$$

$$\Rightarrow \quad \left[\frac{2}{3}\cdot\sqrt{\frac{x}{a}} - \frac{ax^3}{3}\right]_0^{\frac{1}{a}} = 27$$

$$\Rightarrow \quad \frac{2}{3a^2} - \frac{1}{3a^2} = 27$$

$$\Rightarrow \quad \frac{1}{3a^2} = 27$$

$$\Rightarrow \quad 3a^2 = \frac{1}{27}$$

$$\Rightarrow \quad a^2 = \frac{1}{81}$$

$$\Rightarrow \quad a = \pm\frac{1}{9}$$

Section-C

19. **(i) Option (a) is correct.**

$x = 100 - 4P$

$P = \dfrac{100-x}{4}$

$R(x) = \dfrac{(100-x)}{4}$

$= 25 - \dfrac{x}{4}$

(ii) **Option (b) is correct.**

(iii) Let x-units of product be produced and sold.

∴ Total cost $C(x)$ = Fixed cost + variable cost

$C(x) = 18000 + 550x$

Demand function $P(x) = 4000 - 150x$

∴ Revenue function $R(x) = P(x)\cdot x$

$R(x) = P(x)\cdot x = (4000 - 150x)x$

$= 4000x - 150x^2$

For break even point $C(x) - R(x) = 0$

$18000 + 550x - 4000x + 150x^2 = 0$

$150x^2 - 3450x + 18000 = 0$

$x^2 - 23x + 120 = 0$

$(x - 15)(x - 8) = 0$

$x = 15$ and $x = 8$

Hence, the break even values at $x = 8$ and $x = 15$

(iv)

$$\begin{array}{rcl} 12x + 9y + 21 &=& 0 \\ 12x + 16y + 32 &=& 0 \\ (-) \quad (-) \quad (-) && \\ \hline -7y &=& 11 \\ y &=& \dfrac{-11}{7} \end{array}$$

$$4x - \frac{33}{y} + 7 = 0$$

$$\bar{x} = -\frac{4}{7} \text{ and } \bar{y} = -\frac{11}{7}$$

(v)

$$C(2) = 3 \times 2^2 - 6 \times 2 + 5 = 5$$

$$\text{Average cost} = \frac{\text{Total cost of the units}}{\text{Number of units}}$$

$$\text{Therefore, average cost} = \frac{5}{2} = 2.5$$

20. (i)

$$C(x) = \frac{99}{2}x + 525$$

$$\text{Price} = ₹50/\text{units}$$

$$R(x) = P \times x = 50x$$

$$\text{Profit } P(x) = R(x) - C(x)$$

$$P(x) = 50x - \frac{99x}{2} - 525$$

$$= \frac{1}{2}x - 525$$

$P(x) \geq 0$ then $x = 1050$.

Company sell 50 more units to ensure no loss.

OR

(ii) Let x be the number of units produced and sold.

$\Rightarrow$ Revenue function $= R(x) = 6x$

Given that variable cost is estimated to 35% of $R(x)$

$$\Rightarrow \quad V(x) = 35\% \text{ of } R(x)$$

$$= \frac{35}{100} \times 6x$$

$$\Rightarrow \quad V(x) = \frac{21x}{10}$$

Hence, cost function $= C(x) =$ Fixed cost $+ V(x)$

$$= 20000 + \frac{21x}{10}$$

[Given, fixed cost = 20000]

Now, profit function $P(x) = R(x) - C(x)$

$$= 6x - 20000 - \frac{21x}{10}$$

$$\Rightarrow \quad P(x) = \frac{39x}{10} - 20000$$

21. (i)

Temp. (°F) X	Cones Sold Y	X²	XY
65	180	4225	11700
75	250	5625	18750
60	160	3600	9600
70	210	4900	14700
80	300	6400	24000
70	220	4900	15400
420	1320	29650	94150

$$\bar{x} = \frac{\sum x}{n} = \frac{420}{6} = 70, \ \bar{y} = \frac{\sum y}{n} = \frac{1320}{6} = 220$$

$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2} = \frac{6 \times 94150 - 420 \times 1320}{6 \times 29650 - (420)^2}$$

$$b_{yx} = \frac{564900 - 554400}{177900 - 176400}$$

$$b_{yx} = \frac{10500}{1500} = 7$$

Regression line y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 220 = 7(x - 70)$$

$$y = 7x - 490 + 220$$

$$y = 7x - 270$$

OR

(ii) (a) $C(x) = \dfrac{x^3}{3} + 3x^2 - 7x + 16$

(b) Marginal cost $(MC) = \dfrac{d}{dx}\left[\dfrac{x^3}{3} + 3x^2 - 7x + 16\right]$

$$= \frac{3x^2}{3} + 6x - 7$$

$$= x^2 + 6x - 7$$

(b) Average cost $(AC) = \dfrac{\text{Total cost}}{\text{Output}}$

$$= \frac{\dfrac{x^3}{3} + 3x^2 - 7x + 16}{x}$$

$$= \frac{x^2}{3} + 3x - 7 + \frac{16}{x}.$$

(c) Marginal Average Cost (MAC)

$$= \frac{d}{dx}\left[\frac{x^2}{3} + 3x - 7 + \frac{16}{x}\right]$$

$$= \frac{2x}{3} + 3 - \frac{16}{x^2}.$$

Again, $\dfrac{x(MC) - C(x)}{x^2}$

$$= \frac{x(x^2 + 6x - 7) - \dfrac{x^3}{3} - 3x^2 + 7x - 16}{x^2}$$

$$= \frac{1}{x^2}\left[x^3 + 6x^2 - 7x - \frac{x^3}{3} - 3x^2 + 7x - 16\right]$$

$$= \frac{1}{x^2}\left[x^3 - \frac{x^3}{3} + 3x^2 - 16\right]$$

$$= \frac{2x^3}{3x^2} + 3 - \frac{16}{x^2}$$

$$= \frac{2}{3}x + 3 - \frac{16}{x^2}$$

$\Rightarrow$ Marginal average cost $(MAC) = \dfrac{x(MC) - C(x)}{x^2}$.

Hence proved.

22. **(a)** Let the number of batches of Soap A be x and Soap B be y.

Object: To maximise the Profit

$$Z = 3000x + 4000y$$

Constraint $x + 2y \le 12$

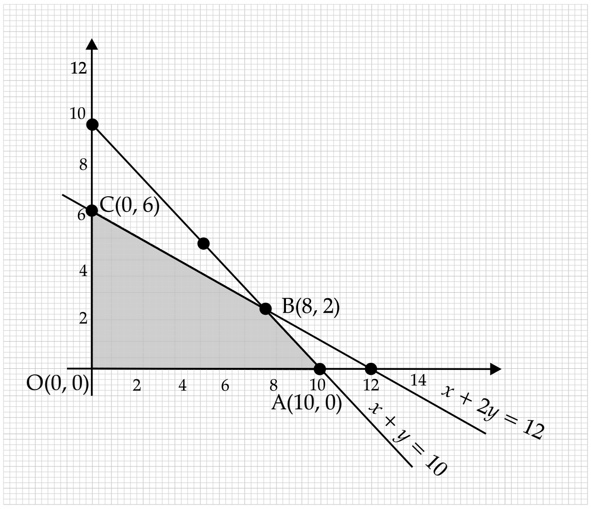
$x + y \le 10$

$x \ge 0, y \ge 0$

$x + 2y = 12$

x	0	12	6
y	6	0	3

$x + y = 10$			
x	0	10	5
y	10	0	5



Feasible region OABCO.

(b) $Z = 3000x + 4000y$

At 0(0,0) $Z = 3000 \times 0 + 4000 \times 0 = 0$

At A(10,0) $Z = 3000 \times 10 + 4000 \times 0 = 30000$

At B (8,2) $Z = 3000 \times 8 + 4000 \times 2 = 32000$ max

At C(0,6) $Z = 3000 \times 0 + 4000 \times 6 = 24000$

For maximum profit, number of batches of soap A is 8 and soap B is 2

$\therefore$ Max Profit = ₹ 32000.

❑❑

Self Assessment Paper-5

MATHEMATICS

Class-12

UNSOLVED

Time Allowed : 2.5 hours **Maximum Marks : 80**

You are allowed an additional 15 minutes for only reading the paper.
You must NOT start writing during reading time.
The question paper has 12 printed pages.
The Question Paper is divided into three sections and has 22 questions in all.

Section A is compulsory and has fourteen questions.
You are required to attempt all questions either from Section B or Section C.
Section B and Section C have four questions each.
Internal choices have been provided in two questions of 2 marks, two questions of 4 marks and two questions of 6 marks in Section A.
Internal choices have been provided in one question of 2 marks and one question of 4 marks each in Section B and Section C.
While attempting Multiple Choice Questions in Section A, B and C, you are required to write only ONE option as the answer.
The intended marks for questions or parts of questions are given in the brackets [].
All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
Mathematical tables and graph papers are provided.
Instruction to Supervising Examiner:
Kindly read aloud the instructions given above to all the candidates present in the examination hall.

Section-A [65 marks]

1. In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed. **[15]**

(i) P and Q are two matrices such that $[P+Q]^T = \begin{bmatrix} 7 & -8 \\ 3 & 0 \end{bmatrix}$. If $P = \begin{bmatrix} -1 & 1 \\ 3 & 2 \end{bmatrix}$. Which one of the following is Q ?

(a) $\begin{bmatrix} 8 & -9 \\ 0 & 2 \end{bmatrix}$ **(b)** $\begin{bmatrix} -8 & -2 \\ 11 & 2 \end{bmatrix}$ **(c)** $\begin{bmatrix} 8 & 2 \\ -11 & -2 \end{bmatrix}$ **(d)** $\begin{bmatrix} 8 & -11 \\ 2 & -2 \end{bmatrix}$

[Understanding] [1]

(ii) The order and the degree of differential equation $1+\left(\frac{dy}{dx}\right)^2 = \frac{d^2y}{dx^2}$ are:

(a) 2 and $\frac{3}{2}$ **(b)** 2 and 3 **(c)** 3 and 4 **(d)** 2 and 1 **[Recall] [1]**

(iii) The value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ is :

(a) $\frac{\pi}{6}$ **(b)** $\frac{7\pi}{6}$ **(c)** $\frac{7\pi}{2}$ **(d)** $\frac{\pi}{3}$ AI **[Application] [1]**

(iv) **Assertion:** The order of differential equation $\frac{d^3y}{dx^3} + 4 \times \sqrt{\left(\frac{dy}{dx}\right)^3 + y^2} = 0$ is 3.

Reason: The sum of order and degree of differential equation $\frac{d^3y}{dx^3} + 4 \times \sqrt{\left(\frac{dy}{dx}\right)^3 + y^2}$ is 5.

Which of the following is correct?

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true. **[Analysis] [1]**

(v) If $P(A)=\frac{4}{5}$ and $P(A\cap B)=\frac{7}{10}$, then the value of $P(B/A)$ is :

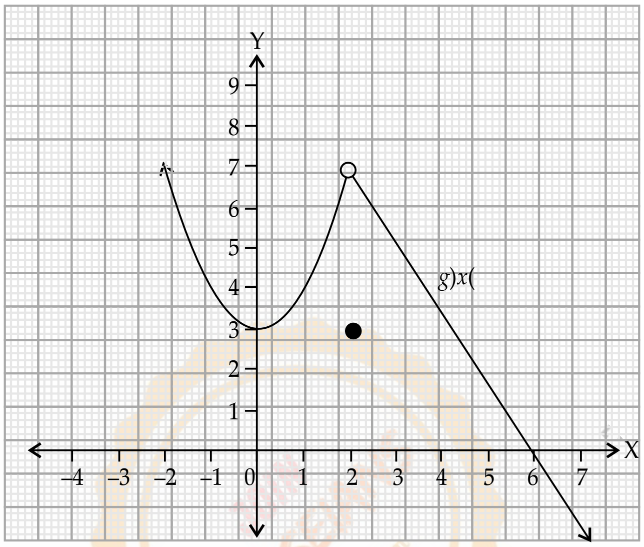
(a) $\frac{1}{8}$ **(b)** $\frac{4}{7}$ **(c)** $\frac{5}{7}$ **(d)** $\frac{7}{8}$ **[Application] [1]**

(vi) If $A=\begin{bmatrix}\alpha & 2\\ 2 & \alpha\end{bmatrix}$ and $|A^3|=27$, then the value of α is :

(a) ±1 **(b)** ±2 **(c)** $\pm\sqrt{5}$ **(d)** $\pm\sqrt{7}$

AI **[Understanding] [1]**

(vii) Shown below is the graph of a function, $g(x)$.



Statement 1: $g(x)$ is continuous and differentiable at $x = 2$.

Statement 2: $g(x)$ is neither continuous nor differentiable at $x = 2$.

Which of the following is correct?

(a) Statement 1 is true and Statement 2 is false. **(b)** Statement 2 is true and Statement 1 is false.

(c) Both the statements are true. **(d)** Both the statements are false. AI **[Application] [1]**

(viii) If $y=\log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{4x^3}{1-x^4}$ **(b)** $\frac{-4x}{1-x^4}$ **(c)** $\frac{1}{4-x^4}$ **(d)** $\frac{-4x^3}{1-x^4}$ **[Application] [1]**

[1]

(ix) **Statement 1:** A function $f : X \to Y$ is said to be bijective, if f is both one-one and onto.

Statement 2: A function $f : X \to Y$ is defined to be invertible if there exists a function $g: Y \to X$ such that $gof = I_X$ and $fog = I_Y$. The function g is called the inverse of f and is denoted by f^{-1}.

Which one of the following is correct?

(a) Statement 1 implies Statement 2.

(b) Statement 2 implies Statement 1.

(c) Statement 1 is true only if Statement 2 is true.

(d) Statement 1 and 2 are independent of each other. **[Understanding] [1]**

(x) For any square matrix A with real number entries, consider the following statements.

Assertion: $A - A'$ is a skew symmetric matrix.

Reason: If matrix A is skew symmetric, then $A = -A'$.

Which of the following is correct?

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

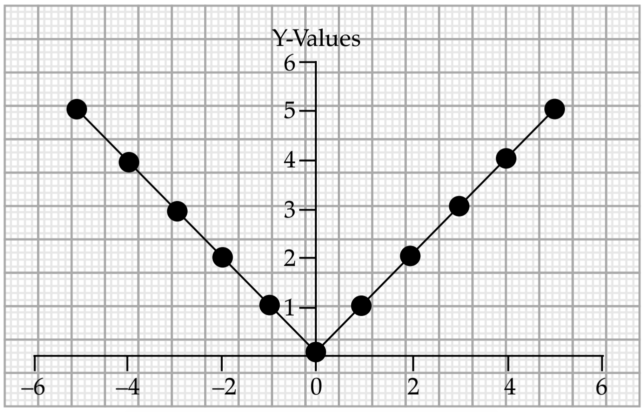
(d) Assertion is false and Reason is true. AI **[Application] [1]**

(xi) A die is thrown three times, if the first throw is a four, find the chance of getting 15 as the sum. **[Understanding] [1]**

(xii) If $A=\begin{bmatrix}3 & 1\\ 7 & 5\end{bmatrix}$, find x and y so that $A^2+xI_2=yA$. **[Analysis] [1]**

(xiii) Let $f:\mathbb{R}\rightarrow\mathbb{R}$ be defined as $f(x) = x^4$. Show that f is neither one-one nor onto. **[Understanding] [1]**

(xiv) Name the function is shown in graph?



[Understanding] [1]

(xv) A company has estimated that the probabilities of success for three products introduced in the market are $\frac{1}{3},\frac{2}{5}$ and $\frac{2}{3}$ respectively. Assuming events to be independent, find the probability that at least one product is successful. **[Application] [1]**

2. **(i)** If $ax^2 + 2hxy + by^2 = 0$ then prove that $\frac{dy}{dx}=\frac{y}{x}$. **[Understanding] [2]**

OR

(ii) If $f(x) = x^3 - 48x$, find the interval where $f : R \rightarrow R$ is decreasing. **[Understanding] [2]**

3. Evaluate: $\int_0^1 \frac{\tan^{-1}x}{1+x^2}dx$ A I **[Evaluate] [2]**

4. Find the point on the curve $y = x^2 + 2x - 1$, where the tangent is parallel to X-axis. **[Analysis] [2]**

5. **(i)** Evaluate: $\int \frac{1}{x^2}\sin^2\left(\frac{1}{x}\right)dx$ **[Evaluate] [2]**

OR

(ii) Find: $\int \frac{\left(x^4-x\right)^{\frac{1}{4}}}{x^5}dx$ **[Evaluate] [2]**

6. Prove that: $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3= \pi$ **[Application] [2]**

7. If $\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\theta$, then prove that $9x^2 - 12xy \cos\theta + 4y^2 = 36 \sin^2\theta$. **[Application] [4]**

8. Evaluate: $\int_0^{\pi/4}\log(1+\tan\theta)d\theta$ **[Application] [4]**

9. **(i)** Volume of the cube is increasing at a rate of 9 cubic inches per second. What is the rate at which surface area is increasing when the length of the edge of the cube is 12 inches? **[Application] [4]**

OR

(ii) If $x^y - y^x = a^b$ then find $\frac{dy}{dx}$. **[Analysis] [4]**

10. **(i)** In an office, three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay processes 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

(a) Find the probability that Sonia processed the form and committed an error.

(b) Find the probability of committing an error in processing the form.

(c) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, find the probability that the form is not processed by Vinay. **[Analysis] [4]**

OR

(ii) In a play zone, Aastha is playing crane game. It has 12 blue balls, 8 red balls, 10 yellow balls and 5 green balls. If Aastha draws two balls one after the other without replacement, then answer the following questions.

(a) What is the probability that the first ball is blue and the second ball is green?
(b) What is the probability that the first ball is yellow and the second ball is red?
(c) What is the probability that both the balls are red?
(d) What is the probability that the first ball is green and the second ball is not yellow? A I **[Analysis] [4]**

11. A mixture is to be made of three foods A, B and C. The three foods A, B and C contain nutrients P, Q and R as shown below.

	Ounces per pound of Nutrient		
Food	**P**	**Q**	**R**
A	1	2	5
B	3	1	1
C	4	2	1

The mixture which will have 8 ounces of P, 5 ounces of Q and 7 ounces of R.

(a) Form a set of simultaneous equations for the above information.
(b) Solve the set of equations formed in (a) by matrix method.
(c) How to form a mixture which will have 8 ounces of P, 5 ounces of Q and 7 ounces of R. **[Application] [6]**

12. **(i)** Find the particular solution, satisfying the given conditions, for the following differential equation:

$$\frac{dy}{dx}-\frac{y}{x}+\operatorname{cosec}\left(\frac{y}{x}\right)=0\,;\ y=0 \text{ when } x=1.$$ A I **[Application] [6]**

OR

(ii) Evaluate: $\displaystyle\int\frac{(x-4)e^x}{(x-2)^3}dx$ **[Evaluate] [6]**

13. **(i)** Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius 10 cm is a square of side $10\sqrt{2}$ cm.

OR

(ii) A rectangle is inscribed in a semicircle of radius r with one of its sides on the diameter of the semicircle. Find the dimensions of the rectangle to get maximum area. Also, find the maximum area. **[Application] [6]**

14. Let X denote the number of colleges where you will apply after your results and $P(X = x)$ denotes your probability of getting admissions in x number of colleges. It is given that:

$$P(X=x)=\begin{cases}kx, & \text{if } x=0 \text{ or } 1\\ 2kx, & \text{if } x=2\\ k(5-x), & \text{if } x=3 \text{ or } 4\\ 0, & \text{if } x>4\end{cases}$$

where k is a positive constant. Find the value of k. Also find the probability that you will get admission in :

(i) exactly one college.
(ii) atmost 2 colleges.
(iii) atleast 2 colleges. **[Analysis] [6]**

Section-B [15 marks]

15. In subparts (i) to (iii) choose the correct options and in subparts (iv) and (v), answer the questions as instructed. **[5]**

(i) Consider the following statements and choose the correct option:

Statement 1: Two vectors $\vec{a}$ and $\vec{b}$ are collinear or parallel if there exists a non-zero scalar λ such that $\vec{a}=\lambda\vec{b}$.

Statement 2: The vectors which can undergo parallel displacement without changing its magnitude and direction are called free vectors.

Which of the following is correct? **[Recall]**

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(a) Only Statement 1 **(b)** Only Statement 2

(c) Both Statements 1 and 2 (d) Neither Statement 1 nor Statement 2

(ii) Find the intercepts on the coordinate axis cut off by the plane.

$\vec{r}\cdot\left(2\hat{i}-4j+3k\right)=12$

(a) $a = 6, b = -3, c = 4$ **(b)** $a = 6, b = 3, c = 4$

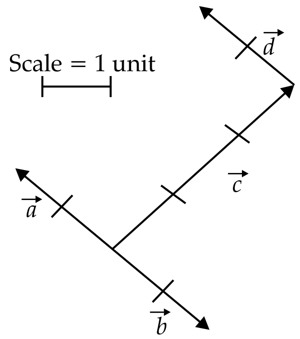
(c) $a = -6, b = -3, c = 4$ **(d)** $a = 6, b = -3, c = -4$ **[Application]**

(iii) The vector equation of XY-plane is

(a) $\vec{r}\cdot\hat{k}=0$ (b) $\vec{r}\cdot\hat{j}=0$ (c) $\vec{r}\cdot\hat{i}=0$ (d) $\vec{r}\cdot\hat{n}=0$ **[Understanding]**

(iv) Find the position vector of the point which divides the join of points $2\vec{a}-3\vec{b}$ and $\vec{a}+\vec{b}$ in the ratio 3:1. **[Understanding]**

(v) Which of the following vectors are collinear in the figure given below? **[Analysis]**



16. (i) If $\vec{r}=x\hat{i}+y\hat{k}+z\hat{k}$ find $\left(\vec{r}\times\hat{i}\right)\left(\vec{r}\times\hat{j}\right)+xy$. **[Application] [2]**

OR

(ii) $\vec{r}\cdot\vec{a}=0, \vec{r}\cdot\vec{b}=0$ and $\vec{r}\cdot\vec{c}=0$ for some non-zero vector $\vec{r}$, then find the value of $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$. **[Application]**

17. (i) Find the equation of the plane passing through the point (2, –3, 1) and perpendicular to the line joining the points (4, 5, 0) and (1, –2, 4). **[Application] [4]**

OR

(ii) Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $2x + 3y - 5z = 4$. **[Application]**

18. (i) Draw a sketch of the region enclosed by curve $y^2 = 8x$ and the line $x = 2$.

(ii) Evaluate: $\int_0^2 \sqrt{x}dx$

(iii) Find the area of the region bounded by curve $y^2 = 8x$ and the line $x = 2$. **[4]**

Section-C [15 marks]

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed **[5]**

(i) If $C(x) = 3x^2 - 2x + 3$, the marginal cost when $x = 3$ is **[1]**

(a) 24 **(b)** 16 **(c)** 11 **(d)** none of the above **[Recall]**

(ii) Read the following statements and choose the correct option:

(I) Regression equation of x on y is used to estimate the probable values of x on the basis of the given values of y.

(II) Regression equation of y on x is used to estimate the probable values of y on the basis of the given values of x. **[Recall]**

(III) The two regression lines coincide if and only if there is perfect linear relation between x and y.

Which of the following is correct?

(a) Only III is correct **(b)** Only I and II are correct

(c) Only I and III are correct **(d)** All are correct

(iii) If $\Sigma x = 30$, $\Sigma y = 15$, $\Sigma xy = 94$ and $\Sigma x^2 = 190$, then find b_{yx}. **[Understanding]**

(iv) The marginal revenue of a manufacturing facility is given by the function $MR(x) = 408 - 17x$. The manufacturer should limit the production to 24 units. Why? **[Understanding]**

(v) Vishwanath runs a company that produces and sells a particular model of laptop. Its revenue function is given by $R(x)=4x^3+\frac{1}{5}x^2+x$, where x is the number of laptops demanded and $R(x)$ is in rupees.

What is the ideal unit price (in rupees) for Vishwanath to charge for each laptop if there is a demand for 100 laptops? **[Understanding]**

20. **(i)** A manufacture's marginal cost function is $\frac{500}{\sqrt{2x+25}}$. Find the cost involved to increase production from 100 units to 300 units. **[Application] [2]**

OR

(ii) The marginal cost function of x units of a product is given by $MC = 3x^2 - 10x + 3$. The cost of producing one unit is ₹7. Find the total cost function and average cost function. **[Application]**

21. **(i)** The selling price of a commodity is fixed at ₹60 and its cost function is $C(x) = 35x + 250$. **[4]**

(a) Determine the profit function

(b) Find the break-even points. **[Application]**

OR

(ii) For the given regressions lines $4x + 2y - 3 = 0$ and $3x + 6y + 5 = 0$, find :

(a) regression coefficients b_{yx} and b_{xy}.

(b) coefficient of correlation $r(x, y)$. **[Application]**

22. A manufacturer wishes to produce two commodities A and B. The number of units of material, labour and equipment needed to produce one unit of each commodity is shown in the table given below. Also shown is the available number of units of each item, material, labour and equipment. **[4]**

Items	Commodity A	Commodity B	Availability
Material	1	2	8
Labour	3	2	12
Equipment	1	1	10

[Analysis]

Given that each unit of commodity A earns a profit of ₹2 and each unit of B earns a profit of ₹3.

(a) Formulate the LPP for the above situation.

(b) Solve it graphically to find the maximum profit.

❑❑

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Self Assessment Paper - 5

Section-A

1. **(i) Option (c) is correct.**

Explanation:

$$[P+Q]^T = \begin{pmatrix} 7 & -8 \\ 3 & 0 \end{pmatrix}$$

$$\Rightarrow \quad P^T + Q^T = \begin{pmatrix} 7 & -8 \\ 3 & 0 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix}^T + Q^T = \begin{pmatrix} 7 & -8 \\ 3 & 0 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} -1 & 3 \\ 1 & 2 \end{pmatrix} + Q^T = \begin{pmatrix} 7 & -8 \\ 3 & 0 \end{pmatrix}$$

$$\Rightarrow \quad Q^T = \begin{pmatrix} 7+1 & -8-3 \\ 3-1 & 0-2 \end{pmatrix}$$

$$\Rightarrow \quad Q^T = \begin{pmatrix} 8 & -11 \\ 2 & -2 \end{pmatrix}$$

$$\Rightarrow \quad Q = \begin{pmatrix} 8 & 2 \\ -11 & -2 \end{pmatrix}$$

(ii) Option (d) is correct.

Explanation: Degree of differential equation is the power of highest order derivative and the order is the highest order of derivative.

Here, the degree is 1 and order is 2.

(iii) Option (a) is correct.

Explanation:

Let $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = x$, where $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Then,

$$\tan x = \tan\frac{7\pi}{6} = \tan\left(\pi + \frac{\pi}{6}\right) = \tan\left(\frac{\pi}{6}\right)$$

$$x = \frac{\pi}{6}$$

(iv) Option (b) is correct.

Explanation: We have,

$$\frac{d^3y}{dx^3} + 4\sqrt{\left(\frac{dy}{dx}\right)^3 + y^2} = 0$$

$$\Rightarrow \quad \frac{d^3y}{dx^3} = -4\sqrt{\left(\frac{dy}{dx}\right)^3 + y^2}$$

$$\Rightarrow \quad \left(\frac{d^3y}{dx^3}\right)^2 = 16\left[\left(\frac{dy}{dx}\right)^3 + y^2\right]$$

Thus, order = 3 and degree = 2

Hence, sum of order and degree is 5.

(v) Option (d) is correct.

Explanation: Given that $P(A) = \frac{4}{5}$ and

$$P(A \cap B) = \frac{7}{10}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{7}{10}}{\frac{4}{5}} = \frac{7}{8}$$

(vi) Option (d) is correct.

Explanation: Given,

$$A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix} \quad \text{....(i)}$$

$$|A^3| = 27$$

$$|A| = 3 \quad [\because |A^n| = |A|^n] \text{ ....(ii)}$$

From (i) and (ii), we get

$$\alpha^2 - 4 = 3$$

$$\alpha^2 = 7$$

$$\alpha = \pm\sqrt{7}$$

(vii) Option (b) is correct.

(viii) Option (b) is correct.

Explanation: Given that,

$$y = \log\left(\frac{1-x^2}{1+x^2}\right)$$

$$\Rightarrow \quad y = \log(1-x^2) - \log(1+x^2)$$

Differentiate with respect to x, we have

$$\frac{dy}{dx} = \frac{d}{dx}[\log(1-x^2)] - \frac{d}{dx}[\log(1+x^2)]$$

$$= \frac{-2x}{1-x^2} - \frac{2x}{1+x^2}$$

$$= -2x\left(\frac{2}{(1-x^2)(1+x^2)}\right)$$

$$= -\frac{4x}{1-x^4}$$

(ix) Option (d) is correct.

(x) Option (a) is correct.

Explanation:

Let $C = A - A'$

$C' = (A - A')' = A' - (A')'$

$= A' - A = -(A - A') = -C$

Therefore, $C = A - A'$ is a skew symmetric matrix.

Hence, both A and R are true and R is the correct explanation of A.

(xi) Consider the following events:

A = Getting 15 as the sum in a throw of three times

B = Getting 4 on the first throw.

$\therefore$ Required Probability = $P\left(\frac{A}{B}\right)$

= Probability of getting 15 as the sum of the numbers if there is 4 on the first die

$$= \frac{2}{36}$$

[Total number of ways = 1 × 6 × 6 = 36. There are two favourable elementary events viz. (4, 6, 5), (4, 5, 6)].

$$= \frac{1}{18}$$

(xii)
$$\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} + x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = y\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

$\therefore$ Comparing the equal matrices,

$$x = y = 8$$

(xiii) We know that $f : \mathbb{R} \to \mathbb{R}$ is defined as $f(x) = x^4$.

Let $x, y \in \mathbb{R}$ such that $f(x) = f(y)$

$$\Rightarrow \quad x^4 = y^4$$

$$\Rightarrow \quad x = \pm y$$

$$\therefore \quad f(x) = f(y)$$

does not imply that $x = y$.

For example, $f(1) = f(-1) = 1$

$\therefore$ f is not one-one.

Consider an element 2 in co-domain $\mathbb{R}$. It is clear that there does not exist any x in domain $\mathbb{R}$ such that, $f(x) = 2$.

$\therefore$ f is not onto.

Hence, function f is neither one-one nor onto.

(xiv) {(–1, 1) (1, 1), (–2, 2), (2, 2), (–3, 3), (3, 3),.... }

This function involves relation $\{(x, y) : y = |x|\}$ which is involved in modulus function. So, above function is modulus function.

(xv) Let $P(A)$, $P(B)$ and $P(C)$ be the probabilities of success of each product.

$$P(A) = \frac{1}{3}, P(B) = \frac{2}{5} \text{ and } P(C) = \frac{2}{3}$$

$$P(\bar{A}) = \frac{2}{3}, P(\bar{B}) = \frac{3}{5} \text{ and } P(\bar{C}) = \frac{1}{3}$$

Since the events are independent, the probability of occurrence of at least one event = $P(A \cup B \cup C)$.

$$= 1 - P(\bar{A})P(\bar{B})P(\bar{C})$$

$$= 1 - \frac{2}{3} \times \frac{3}{5} \times \frac{1}{3}$$

$$= 1 - \frac{2}{15}$$

$$= \frac{13}{15}$$

Therefore, the probability that at least one product is successful is $\frac{13}{15}$.

2. **(i)** Given: $ax^2 + 2hxy + by^2 = 0$

$$\frac{d}{dx}(ax^2) + \frac{d}{dx}(2hxy) + \frac{d}{dx}(by^2) = 0$$

$$\Rightarrow 2ax + 2h\left(x\frac{dy}{dx} + y\right) + 2by\frac{dy}{dx} = 0$$

$$\Rightarrow ax + hy + (hx + by)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{ax+hy}{hx+by}\right) \quad \text{....(i)}$$

$$\Rightarrow ax^2 + 2hxy + by^2 = 0$$

$$\Rightarrow ax^2 + hxy + hxy + by^2 = 0$$

$$\Rightarrow x(ax + hy) + y(hx + by) = 0$$

$$\Rightarrow x(ax + hy) = -y(hx + by)$$

$$\Rightarrow \frac{y}{x} = -\left(\frac{ax+hy}{hx+by}\right) \quad \text{....(ii)}$$

$$\Rightarrow \frac{x}{y} = -\left(\frac{hx+by}{ax+hy}\right)$$

From (i) and (ii) $\frac{dy}{dx} = \frac{y}{x}$

Hence Proved.

OR

(ii)
$$f(x) = x^3 - 48x$$

$$\Rightarrow f'(x) = 3x^2 - 48$$

$$\Rightarrow 3x^2 - 48 = 0$$

$$\Rightarrow x = 4, -4$$

The values which make the derivative equal to 0 are 4, – 4.

The values that make derivative 0 or undefined are $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

Since, $f(0) = 3(0) - 48 = -48$

$$\Rightarrow f'(x) < 0$$

$f : R \to R$ is is decreasing at (– 4, 4).

3. Let
$$I = \int_0^1 \frac{\tan^{-1} x}{1+x^2}$$

Let
$$\tan^{-1} x = t \Rightarrow \frac{1}{1+x^2}dx = dt$$

When $x = 0$, $t = 0$;

When $x = 1$, $t = \frac{\pi}{4}$.

$$\therefore \qquad I = \int_0^{\pi/4} t\,dt = \left[\frac{t^2}{2}\right]_0^{\pi/4}$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{4}\right)^2 - 0\right]$$

$$= \frac{1}{2}\left(\frac{\pi^2}{16}\right) = \frac{\pi^2}{32}$$

4. $y = x^2 + 2x - 1$

Differentiating w.r.t. x

$$\frac{dy}{dx} = 2x + 2 = \text{slope of tangent}$$

Since, tangent is parallel to X-axis

Slope of tangent = 0

$\Rightarrow \quad 2x + 2 = 0$

$\Rightarrow \quad 2x = -2$

$\Rightarrow \quad x = -1$

$\Rightarrow \quad y = (-1)^2 + 2(-1) - 1$

$\Rightarrow \quad y = 1 - 2 - 1$

$y = -2$

$\therefore$ $(-1, -2)$ is the required point.

5. **(i)** Let $\quad I = \int \frac{1}{x^2}\sin^2\left(\frac{1}{x}\right)dx$

Let $\quad t = \frac{1}{x}$

$$\therefore \qquad dt = \frac{-1}{x^2}dx$$

$$\therefore \qquad I = -\int \sin^2 t\,dt$$

$$= -\int \frac{1-\cos 2t}{2}dt$$

$[\because \cos 2\theta = 1 - 2\sin^2\theta]$

$$= -\frac{1}{2}\left[\int 1\,dt - \int \cos 2t\,dt\right]$$

$$= -\frac{1}{2}\left[\frac{1}{2}t - \frac{\sin 2t}{2}\right] + C$$

$$= \frac{1}{4}\sin 2t - \frac{1}{2}t + C$$

$$I = \frac{1}{4}\sin\left(\frac{2}{x}\right) - \frac{1}{2x} + C$$

Examiner's Comment

- Candidates made errors by making inappropriate substitution. Some candidates applied correct substitution $\left(\frac{1}{x} = t\right)$ but made errors in further simplification. Many attempted it by incorrect method of integration by parts so could not get the correct solution.

Answering Tip

- Practice should be needed for the questions of integration by substitution.

OR

(ii) Let, $\quad I = \int \frac{(x^4 - x)^{\frac{1}{4}}}{x^5}dx$

$$= \int \frac{x\left(1 - \frac{1}{x^3}\right)^{\frac{1}{4}}}{x^5}dx$$

$$= \int \frac{\left(1 - \frac{1}{x^3}\right)^{\frac{1}{4}}}{x^4}dx$$

Put $1 - \frac{1}{x^3} = t$

So that, $\quad \frac{3}{x^4}dx = dt$

$$\Rightarrow \qquad dt = \frac{x^4}{3}dt$$

$$= \frac{1}{3}\int t^{\frac{1}{4}}dt = \frac{1}{3}\cdot\frac{4}{5}t^{\frac{5}{4}}$$

$$= \frac{4}{15}\left(1 - \frac{1}{x^3}\right)^{\frac{5}{4}} + C$$

6. We have

L.H.S. $= \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$

$$= \pi + \tan^{-1}\left(\frac{1+2}{1-1\times 2}\right) + \tan^{-1} 3$$

$\left[\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right),\right.$ if $x > 0$, $y > 0$ and $\left. xy > 1\right]$

$$= \pi + \tan^{-1}\left(\frac{3}{1-2}\right) + \tan^{-1} 3$$

$= \pi + \tan^{-1}(-3) + \tan^{-1} 3$

$= \pi - \tan^{-1}(3) + \tan^{-1} 3$

$[\tan^{-1}(-x) = -\tan^{-1}(x)$ for all $x \in R]$

π = R.H.S.

7. $$\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$$

$$\cos^{-1}\left(\frac{x}{2}\times\frac{y}{3} - \sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}}\right)$$

$\left[\cos^{-1} a + \cos^{-1} b = \cos^{-1}\left(ab - \sqrt{1-a^2}\sqrt{1-b^2}\right)\right]$

$$\frac{xy}{6} - \sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}} = \cos\theta$$

$$\frac{xy}{6} - \cos\theta = \sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}}$$

Squaring both sides

$$\frac{x^2y^2}{36} + \cos^2\theta - \frac{xy\cos\theta}{3} = \left(1-\frac{x^2}{4}\right)\left(1-\frac{y^2}{9}\right)$$

$$\frac{x^2y^2}{36}+\cos^2\theta-\frac{xy\cos\theta}{3} = 1-\frac{x^2}{4}-\frac{y^2}{9}+\frac{x^2y^2}{36}$$

$$\frac{x^2}{4}+\frac{y^2}{9}-\frac{xy\cos\theta}{3} = 1-\cos^2\theta$$

$$\frac{x^2}{4}+\frac{y^2}{9}-\frac{xy\cos\theta}{3} = \sin^2\theta$$

Multiply by 36

$$9x^2 + 4y^2 - 12xy\cos\theta = 36\sin^2\theta$$

$$9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta$$ **Hence Proved.**

Examiner's Comment

- Many candidates fail to prove the required equation. Some of them made errors in applying formula of $\cos^{-1}x + \cos^{-1}y$.

Answering Tip

- Learn all the formula, laws and inter conversions of inverse circular functions.

8. Let $$I = \int_0^{\pi/4}\log(1+\tan\theta)\,d\theta \quad ...(i)$$

$$= \int_0^{\frac{\pi}{4}}\log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\frac{\pi}{4}}\log\left[1+\left\{\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right\}\right]d\theta$$

$$\left[\because \tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}\right]$$

$$\therefore \quad I = \int_0^{\frac{\pi}{4}}[\log 2-\log(1+\tan\theta)]d\theta \quad ...(ii)$$

Adding eqs. (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{4}}\log 2\,d\theta$$

$$= \log 2[\theta]_0^{\frac{\pi}{4}} = \log 2.\frac{\pi}{4}$$

$$\therefore \quad I = \frac{\pi}{8}.\log 2$$

Examiner's Comment

- Many candidates made errors in applying the properties of definite integrals correctly and some candidates attempted it by integration by parts.

Answering Tip

- Learn the properties of definite integrals well and do adequate practice to apply them appropriately.

9. **(i)** Given, increasing rate of volume of cube,

$$\frac{dV}{dt} = 9 \text{ cubic inches/s}$$

Since, Volume of a cube, $V = a^3$

Differentiating both sides w.r.t 't', we get

$$\frac{dV}{dt} = 3a^2\frac{da}{dt}$$

$$9 = 3a^2\frac{da}{dt}$$

$$\Rightarrow \quad \frac{da}{dt} = \frac{3}{a^2} \quad ...(i)$$

Now, surface area of cube, $S = 6a^2$

$\therefore$ Rate of change of surface area, $\frac{dS}{dt} = 12a\frac{da}{dt}$

$$\therefore \quad \frac{dS}{dt} = 12.a.\frac{3}{a^2} \quad \text{[from (i)]}$$

$$\Rightarrow \quad \frac{dS}{dt} = \frac{36}{12} = 3 \text{ inches}^2\text{/s}$$

[Given a = 12 inches]

OR

(ii) Given,

$$x^y - y^x = a^b$$

$$e^{y\log x} - e^{x\log y} = e^{b\log a}$$

On differentiating both sides w.r.t. x, we get

$$\frac{d}{dx}\left(e^{y\log x}\right)-\frac{d}{dx}\left(e^{x\log y}\right)=\frac{d}{dx}\left(e^{b\log a}\right)$$

$$\Rightarrow e^{y\log x}\frac{d}{dx}(y\log x)-e^{x\log y}\frac{d}{dx}(x\log y)=0$$

$$\Rightarrow x^y\left\{\frac{dy}{dx}\times\log x+y\times\frac{1}{x}\right\}$$

$$-y^x\left\{1\times\log y+x\times\frac{1}{y}\times\frac{dy}{dx}\right\}=0$$

$$\Rightarrow \left\{x^y\times\log x-y^x\times\frac{x}{y}\right\}\frac{dy}{dx}+\left\{x^y\times\frac{y}{x}-y^x\times\log y\right\}=0$$

$$\Rightarrow \left\{x^y\log x-xy^{x-1}\right\}\frac{dy}{dx}+\left\{yx^{y-1}-y^x\log y\right\}=0$$

$$\Rightarrow \frac{dy}{dx}=\left\{\frac{y^x\log y-x^{y-1}y}{x^y\log x-y^{x-1}x}\right\}$$

Examiner's Comment

- Some students take logarithm directly without separating the equation.

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Answering Tip

- log $(a + b) \neq$ log a + log b. So, separate the function and find the derivatives.

10. (i) (a) P (Sonia processed the form and committed an error) $= 20\% \times 0.04$

$$= \frac{20}{100} \times 0.04$$

$$= \frac{1}{5} \times 0.04$$

$$= 0.008$$

(b) Required Probability = P (Vinay processes the form) P (Vinay's error rate) + P (Sonia processes the form) P (Sonia's error rate) + P (Iqbal processes the rate) P (Iqbal's error rate)

$$= \frac{50}{100} \times 0.06 + \frac{20}{100} \times 0.04 + \frac{30}{100} \times 0.03$$

$$= \frac{470}{10000} = 0.047$$

(c) P (Error is committed in processing given that Vinay processed the form)

$$= \frac{\frac{5}{10} \times 0.06}{\frac{5}{10} \times 0.06 + \frac{2}{10} \times 0.04 + \frac{3}{10} \times 0.03}$$

$$= \frac{30}{47}$$

P(Form has an error and it was not processed by Vinay)

$$= 1 - \frac{30}{47}$$

$$= \frac{17}{47}$$

OR

(ii) Let B, R, Y and G denote the events that ball drawn is blue, red, yellow and green respectively.

$$\therefore P(B) = \frac{12}{35}, P(R) = \frac{8}{35}, P(Y) = \frac{10}{35} \text{ and } P(G) = \frac{5}{35}$$

(a) $P(G \cap B) = P(B) \cdot P(G \mid B) = \frac{12}{35} \cdot \frac{5}{34} = \frac{6}{119}$

(b) $P(R \cap Y) = P(Y) \cdot P(R \mid Y) = \frac{10}{35} \cdot \frac{8}{34} = \frac{8}{119}$

(c) Let E = event of drawing a first red ball and F = event of drawing a second red ball

Here, $P(E) = \frac{8}{35}$ and $P(E) = \frac{7}{34}$

$$\therefore P(F \cap E) = P(E) \cdot P(F \mid E) = \frac{8}{35} \cdot \frac{7}{34} = \frac{4}{85}$$

(d) $P(Y' \cap G) = P(G) \cdot (Y' \mid G) = \frac{5}{35} \cdot \frac{24}{34} = \frac{12}{119}$

11. (a) Let x pounds of food A, y pounds of food B and z pounds of food C be needed to form the mixture.

Since, one pound of food A contains 1 ounce of nutrient P. So, x pounds of food A will contain x ounces of nutrient P.

Similarly, the amount of nutrient P in y pounds of food B and z pounds of food C are $3y$ and $4z$ ounces respectively. Therefore,

Total quantity of nutrient P in x pounds of food A, y pounds of food B and z pounds of food C is $x + 3y + 4z$ ounces.

$\therefore \quad x + 3y + 4z = 8$

Similarly, $\quad 2x + y + 2z = 5 \quad$ [For nutrient Q]

and $\quad 5x + y + z = 7 \quad$ [For nutrient R]

(b) The above system of simultaneous linear equations can be written in matrix form as

$$\begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$$

Or, $\quad AX = B$

where $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$

(c) Now, $|A| = \begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{vmatrix} = 1(1-2) - 3(2-10) + 4(2-5)$

$$= -1 + 24 - 12 = 11 \neq 0$$

So, A^{-1} exists.

Let C_{ij} be the cofactor of a_{ij} in $A = [a_{ij}]$. Then,

$C_{11} = -1, \quad C_{12} = 8, \quad C_{13} = -3,$

$C_{21} = 1, \quad C_{22} = -19, \quad C_{23} = 14$ and

$C_{31} = 2, \quad C_{32} = 6$ and $\quad C_{33} = -5$

$\therefore$ adj A = Transpose of

$$\begin{bmatrix} -1 & 8 & -3 \\ 1 & -19 & 14 \\ 2 & 6 & -5 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{ . adj } A = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$$

Thus, the solution of the system of equations is given by

$$X = A^{-1} \cdot B = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -8+5+14 \\ 64-95+42 \\ -24+70-35 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 11 \\ 11 \\ 11 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow x = 1, y = 1$ and $z = 1$

Hence, the mixture is formed by mixing one pound of each of the foods A, B and C.

12. (i) Given differential equation can be written as

$$\frac{dy}{dx} = \frac{y}{x} - \text{cosec}\left(\frac{y}{x}\right) \qquad \text{...(i)}$$

It is a homogeneous differential equation.

Put $y = vx$, then $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Now (i) reduces to

$$v + x\frac{dv}{dx} = v - \text{cosec } v$$

$$x\frac{dv}{dx} = -\text{cosec } v$$

$$\frac{dv}{\text{cosec } v} = -\frac{dx}{x}$$

$$\sin v\, dv = -\frac{dx}{x}$$

Integrating both sides, we get

$$\int \sin v\, dv = -\int \frac{dx}{x} - C$$

$$-\cos v = -\log|x| - C$$

$$\cos v = \log|x| + C$$

$$\cos\left(\frac{y}{x}\right) = \log|x| + C \qquad \text{...(ii)}$$

It is given that : $y = 0$ when $x = 1$

$$\cos\left(\frac{0}{1}\right) = \log|1| + C$$

$$\cos 0 = 0 + C$$

$$C = 1$$

Substituting $C = 1$ in (ii), we get

$\cos\left(\frac{y}{x}\right) = \log|x| + 1$, as the required solution.

OR

(ii) We have,

$$\int \frac{(x-4)e^x}{(x-2)^3}dx = \int e^x\left[\frac{(x-2)-2}{(x-2)^3}\right]dx$$

$$= \int e^x\left[\frac{1}{(x-2)^2} - \frac{2}{(x-2)^3}\right]dx \qquad \text{...(i)}$$

Take $f(x) = \frac{1}{(x-2)^2}$, then $f'(x) = -\frac{2}{(x-2)^3}$

Now (i) reduces to

$$\int \frac{(x-4)e^x}{(x-2)^3}dx = \int e^x[f(x) + f'(x)]dx$$

$$= e^x f(x) + C$$

$$= e^x \frac{1}{(x-2)^2} + C$$

13. (i)

$AB = 2x; \quad BC = 2y$

In ΔABC,

$$(2x)^2 + (2y)^2 = (20)^2$$

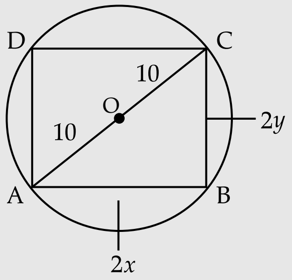
$$\Rightarrow \quad 4x^2 + 4y^2 = 400$$

$$\Rightarrow \quad x^2 + y^2 = 100 \qquad \text{...(i)}$$

$$P = 4x + 4y = 4x + 4\cdot\sqrt{100 - x^2}$$

$$\therefore \quad \frac{dP}{dx} = 4 - \frac{4x}{\sqrt{100-x^2}} = 0 \qquad \text{[from (i)]}$$

$$\Rightarrow \quad x = 5\sqrt{2} \text{ cm}$$



$$\frac{d^2P}{dx^2} = -4\left\{\frac{\sqrt{100-x^2} - \frac{x(-x)}{\sqrt{100-x^2}}}{100-x^2}\right\}$$

$$= \frac{-4\times 100}{(100-x^2)^{\frac{3}{2}}} < 0$$

Hence, perimeter is maximum, when $x = 5\sqrt{2}$.

$\therefore \quad y = 5\sqrt{2}$ [from (i)]

$\Rightarrow \quad x = y$

$\therefore$ $ABCD$ is square of side $10\sqrt{2}$ cm

[ISC Marking Scheme, 2015]

OR

(ii)

$OQ = r\cos\theta$

$QR = r\sin\theta$

$A(\theta) =$ Area of the rectangle

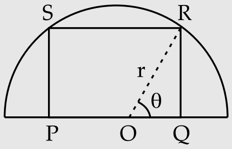
$= 2OQ \times QR$

$= 2r\cos\theta . r\sin\theta$

$= r^2 \sin 2\theta$

For maximum and minimum area

$$\frac{dA}{d\theta} = 2r^2\cos 2\theta = 0$$



$$\cos 2\theta = 0$$

$$\Rightarrow \quad 2\theta = \frac{\pi}{2}$$

$\Rightarrow \quad \theta = \frac{\pi}{4}$ is the critical point

$$\frac{d^2A}{d\theta^2} = -4r^2\sin 2\theta$$

$$\left(\frac{d^2A}{d\theta^2}\right)_{\theta=\frac{\pi}{4}} = -4r^2 < 0$$

$\Rightarrow$ Area is maximum at $\theta = \frac{\pi}{4}$

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So, sides of rectangle are

$$QR = r\sin\theta = r.\sin\frac{\pi}{4} = \frac{r}{\sqrt{2}} = \frac{\sqrt{2}r}{2}$$

and $PQ = 2OQ = 2\,r\cos\theta = 2\,r\cos\left(\frac{\pi}{4}\right)$

$$= 2r\frac{1}{\sqrt{2}} = \sqrt{2}r$$

Maximum area $= PQ \times RQ$

$$= \sqrt{2}r \times \frac{\sqrt{2}r}{2}$$

$$= r^2$$

[ISC Marking Scheme, 2016]

14. The probability distribution of x is:

X	0	1	2	3	4
$P(X)$	0	k	$4k$	$2k$	k

The given distribution is a probability.

$$\therefore \quad \sum_{i=0}^{4} p_i = 1$$

$\Rightarrow \quad 0 + k + 4k + 2k + k + 1 = 1$

$\Rightarrow \quad 8k = 1$

$\Rightarrow \quad k = \frac{1}{8} = 0.125$

(i) P (getting admission exactly one college)
$= P(X = 1) = k = -0.125$

(ii) P (getting admission in atmost 2 colleges)
$= P(X \le 2) = 0 + k + 4k = 5k = 0.625$

(iii) P (getting admission in atleast 2 colleges)
$= P(X \ge 2) = 4k + 2k + k = 7k = 0.875$

Section-B

15. (i) Option (c) is correct.

Explanation: Two vectors are parallel if they are in the same direction. And the statement 2 is the definition of the free vector. So , its also true.

(ii) Option (a) is correct.

Explanation:

$$\vec{r}(2\hat{i} - 4\hat{j} + 3\hat{k}) = 12$$

$$\frac{x}{6} + \frac{y}{(-3)} + \frac{z}{4} = 1$$

Intercepts = 6, –3, 4

$\Rightarrow a = 6,\ b = -3$ and $c = 4$

(iii) Option (a) is correct.

Explanation: The equation of XY plane is $z = 0$ or $\vec{r}.k = 0$.

(iv) Let the position vector of the R divides the join of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$.

$$\therefore \text{ Position vector, } R = \frac{3(\vec{a} + \vec{b}) + 1(2\vec{a} - 3\vec{b})}{3 + 1}$$

Since, the position vector of a point R dividing the line segments joining the points P and Q, whose position vectors are $\vec{p}$ and $\vec{q}$ in the ratio $m : n$ internally, is given by $\frac{m\vec{q} + n\vec{p}}{m + n}$.

$$\therefore \quad R = \frac{5\vec{a}}{4}$$

(v) Collinear vectors are those vectors which are parallel to the same line irrespective of the magnitude and direction. In the given figure, $\vec{a}, \vec{b}$ and $\vec{d}$ are collinear vectors.

16. (i) $(\vec{r} \times \hat{i}) = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{i} = -y\hat{k} + z\hat{j}$

$(\vec{r} \times \hat{j}) = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{j} = x\hat{k} - z\hat{i}$

$\therefore (\vec{r} \times \hat{i}) \times (\vec{r} \times \hat{j}) = (0\hat{i} + z\hat{j} - y\hat{k}) \times (-z\hat{i} + 0\hat{j} + x\hat{k}) = -xy$

$\therefore (\vec{r} \times \hat{i}) \times (\vec{r} \times \hat{j}) + xy = 0$

OR

(ii) Since, $\vec{r}$ is a non-zero vector. So, we can say that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are in a same plane.

$\therefore \vec{a}.(\vec{b} \times \vec{c}) = 0$

[Since, angles between $\vec{a}$, $\vec{b}$ and $\vec{c}$ are zero i.e., $\theta = 0$]

17. (i) Plane passing through the point (2, –3, 1) and perpendicular to the line joining the point (4, 5, 0) and (1, –2, 4) direction ratio of the line is

{(1 – 4), (– 2 – 5), (4 – 0)}

= (– 3, – 7, 4)

Direction ratio of the plane

= Direction ratio of the line

= (– 3, – 7, 4)

Equation of the plane

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$
$$-3(x - 2) - 7(y + 3) + 4(z - 1) = 0$$
$$-3x + 6 - 7y - 21 + 4z - 4 = 0$$
$$-3x - 7y + 4z - 19 = 0$$
$$3x + 7y - 4z + 19 = 0$$

Required equation of the plane is

$$3x + 7y - 4z + 19 = 0$$

Examiner's Comment

- Most of the candidate didn't apply the concept of perpendicular lines and equation of plane correctly.

Answering Tip

- Understand the different situation of finding the equation of the plane by applying different condition.

OR

(ii) Equation of line is $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$

and plane is $2x + 3y - 5z = 4$

Here, $\quad \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

and $\quad \vec{n} = 2\hat{i} + 3\hat{j} - 5\hat{k}$

$$\therefore \quad \sin\theta = \frac{|\vec{b}.\vec{n}|}{|\vec{b}||\vec{n}|}$$

$$= \frac{|(2\hat{i}+3\hat{j}+6\hat{k}).(2\hat{i}+3\hat{j}-5\hat{k})|}{\sqrt{2^2+3^2+6^2}\,\sqrt{2^2+3^2+(-5)^2}}$$

$$= \frac{|(2)(2)+(3)(3)+(6)(-5)|}{\sqrt{49}\,\sqrt{38}}$$

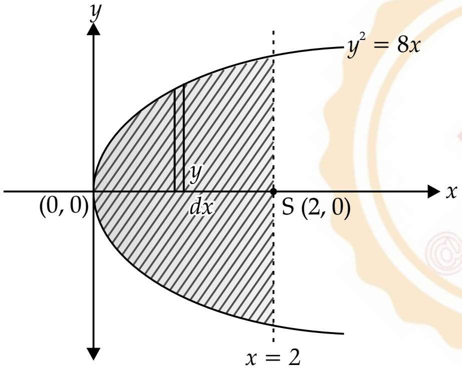
$$= \frac{|4+9-30|}{7\sqrt{38}}$$

$$= \frac{17}{7\sqrt{38}}$$

$$\Rightarrow \quad \sin\theta = \frac{17}{7\sqrt{38}}$$

$$\therefore \quad \theta = \sin^{-1}\left(\frac{17}{7\sqrt{38}}\right)$$

18. **(i)** The region enclosed by curve $y^2 = 8x$ and line $x = 2$ is shown in below.



(ii)

$$\int_0^2 \sqrt{x}\,dx = \frac{[x^{3/2}]_0^2}{\frac{3}{2}} = \frac{2}{3}[2^{3/2}-0]$$

$$= \frac{4\sqrt{2}}{3}$$

(iii) Given curve $y^2 = 8x$ and line $x = 2$

$$\Rightarrow \quad y^2 = 4 \times 2x$$

$y = \sqrt{8}.\sqrt{x}$ is in I quadrant

We have, $$\text{Area} = 2\int_0^2 y\,dx$$

$$= 2\sqrt{8}\int_0^2 \sqrt{x}\,dx$$

$$= 2\times\sqrt{8}\times\frac{4\sqrt{2}}{3}$$

[From part (ii)]

$$= \frac{32}{3} \text{ sq. units}$$

Examiner's Comment

- Many candidates identified the symmetry of the curve about x-axis incorrectly and either did not apply correct limits or considered area twice the result. In most of the cases, rough sketch of the curve was not drawn correctly.

Answering Tip

- Sufficient practice needed in solving problems based on area under a given curve and straight line, etc.

Section-C

19. **(i) Option (b) is correct.**

Explanation: M.C. $= \frac{d}{dx}C(x) = 6x-2$

M.C. $= 6x - 2$

At $x = 3$, M.C. $= 18 - 2 = 16$

(ii) Option (d) is correct.

(iii)

$$byx = \frac{\Sigma xy - \frac{\Sigma x.\Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$$

$$= \frac{94 - \frac{30\times 15}{5}}{190 - \frac{(30)^2}{5}}$$

$$= \frac{470-450}{950-900} = 0.4$$

(iv)

$$MR(x) = 408 - 17x$$

$$R(x) = \int MR(x) = \int(408-17x)dx$$

$$= 408x - 17\frac{x^2}{2} + C$$

$$R(x) = 408x - \frac{17}{2}x^2 + C$$

When $x = 0$, $R(x) = 0$

$$\therefore \quad C = 0$$

$$R(x) = 408x - \frac{17}{2}x^2$$

$$R'(x) = 408 - 17x$$

For Maximum/Minimum

$$R'(x) = 0$$

$$408x - 17x = 0$$

$$x = \frac{408}{17} = 24$$

$$R''(x) = -17 < 0$$

Hence, maximum revenue obtained at $x = 24$. Manufacturer should limit the production to 24 units.

(v)

$$R(x) = 4x^3 + \frac{1}{5}x^2 + x$$

$$\text{Price} = \frac{R(x)}{x} = \frac{4x^3+\frac{1}{5}x^2+x}{x}$$

$$P = 4x^2 + \frac{1}{5}x + 1$$

Price for demand of 100 laptops

$$\therefore \quad x = 100$$

$$P = 4(100)^2 + \frac{1}{5}(100) + 1$$

$$P = 40{,}000 + 20 + 1$$

$$= ₹\ 40{,}021$$

20. **(i)** Given, $MC = \dfrac{500}{\sqrt{2x+25}}$

Total increased cost when x increases from 100 to 300 units is

$$C(300) - C(100) = \int_{100}^{300} MC(x)dx$$

$$= \int_{100}^{300} \frac{500}{\sqrt{2x+25}}dx$$

$$= 500[\sqrt{2x+25}]_{100}^{300}$$

$$= 500[\sqrt{625} - \sqrt{225}]$$

$$= 500(25 - 15) = 5000$$

Required cost increase = ₹ 5000

OR

(ii) $MC = 3x^2 - 10x + 3$

$$C(x) = \int MC\ dx + c$$

$$C(x) = x^3 - 5x^2 + 3x + c$$

Cost of producing 1 unit is ₹ 7

$$\therefore \quad 7 = 1 - 5 + 3 + c$$

$$\Rightarrow \quad c = 8$$

∴ Total cost function = $C(x) = x^3 - 5x^2 + 3x + 8$

Average cost function = $\dfrac{C(x)}{x} = x^2 - 5x + 3 + \dfrac{8}{x}$

21. **(i)** Let the number of articles be x

Selling price of 1 article = ₹ 60

Selling price of x articles, $R(x)$ =₹ 60 x

Total cost price, $C(x) = 35x + 250$

(a) Profit function, $P(x) = R(x) - C(x)$

$$= 60x - 35x - 250$$

$$= 25x - 250$$

Hence, profit function is $25x - 250$

(b) For break-even point,

$$P(x) = 0$$

$$\Rightarrow \quad 25x - 250 = 0$$

$$\Rightarrow \quad x = 10 \text{ units.}$$

OR

(ii) **(a)** Let the line of regression of x on y be

$$4x + 2y - 3 = 0$$

$$\Rightarrow \quad x = -\frac{1}{2}y + \frac{3}{4}$$

$$\Rightarrow \quad b_{xy} = -\frac{1}{2}$$

(b) Let the line of regression of y on x be

$$3x + 6y + 5 = 0$$

$$\Rightarrow \quad y = -\frac{1}{2}x - \frac{5}{6}$$

$$\Rightarrow \quad b_{xy} = -\frac{1}{2}$$

$$\therefore \quad r^2 = b_{yx} \times b_{xy}$$

$$= \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) = \frac{1}{4}$$

$$\therefore \quad r(x, y) = -\frac{1}{2}$$

[Since b_{xy} and b_{yx} are negative.]

22. Let x be the no. of units of Commodity A produced and y be the no. of units of Commodity B produced, then $x > 0, y > 0$

A.T.Q.,

$$x + 2y \le 8$$

$$3x + 2y \le 12$$

and $\quad x + y \le 10$

Let Z be profit, then

$$\text{Max. } Z = 2x + 3y$$

Thus, LPP is

$$\text{Max. } Z = 2x + 3y$$

Subject to constraint

$$x + 2y \le 8 \quad \text{...(i)}$$

$$3x + 2y \le 12 \quad \text{...(ii)}$$

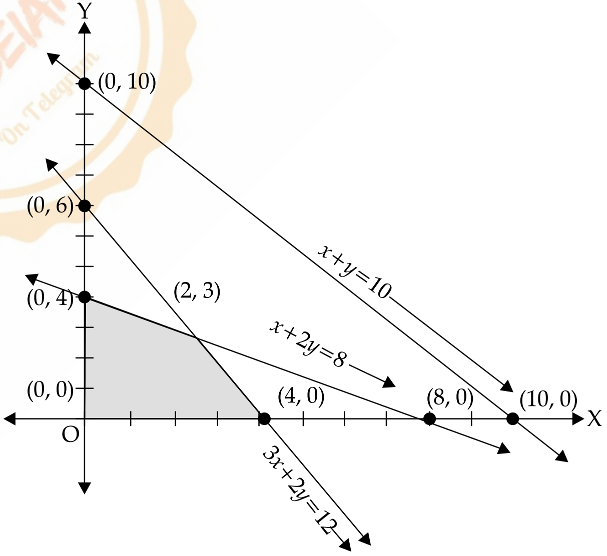
and $\quad x + y \le 10 \quad$ ...(iii)

$x > 0, y > 0$

(b) The solution set of above three equations are:

(0, 4), (4, 0) and (2, 3)

Draw the diagram with these points:



Now, we can determine the maximum value of Z by evaluating the value of Z at the vertices.

Points	Z = 2x + 3y
(0,0)	0
(4,0)	8
(2,3)	13 (maximum)
(0,4)	12

Hence, the maximum profit is ₹13.

❑❑

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Sample Question Paper-1

(Specimen Paper 2025, Issued by CISCE)

MATHEMATICS

Class-12

SOLVED

Time Allowed : 3 hours **Maximum Marks : 80**

You are allowed an additional 15 minutes for only reading the paper.
You must NOT start writing during reading time.
The question paper has 12 printed pages.
The Question Paper is divided into three sections and has 22 questions in all.

Section A is compulsory and has fourteen questions.
You are required to attempt all questions either from Section B or Section C.
Section B and Section C have four questions each.
Internal choices have been provided in two questions of 2 marks, two questions of 4 marks and two questions of 6 marks in Section A.
Internal choices have been provided in one question of 2 marks and one question of 4 marks each in Section B and Section C.
While attempting Multiple Choice Questions in Section A, B and C, you are required to write only ONE option as the answer.
The intended marks for questions or parts of questions are given in the brackets [].
All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
Mathematical tables and graph papers are provided.
Instruction to Supervising Examiner:
Kindly read aloud the instructions given above to all the candidates present in the examination hall.

Section-A [65 marks]

1. In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed. **[15]**

(i) A matrix which is both symmetric and skew symmetric matrix is a / an

(a) triangular **(b)** identity matrix **(c)** diagonal matrix **(d)** null matrix

[Understanding]

(ii) The value of $\int a^x.e^x\ dx$ equals

(a) $(a^x . \log_e a)\, e^x + \text{c}$ **(b)** $\dfrac{a^x.e^x}{\log_e(ae)}+c$ **(c)** $\dfrac{a^x.e^x}{\log_{ae}{}^{\text{e}}}+c$ **(d)** $\log_e (ae)\, (\boldsymbol{ae})^x + c$

[Recall]

(iii) The trigonometric equation $\tan^{-1} x = 3 \tan^{-1} a$ has solution for

(a) $|a| \leq \dfrac{1}{\sqrt{3}}$ **(b)** $|a| > \dfrac{1}{\sqrt{3}}$ **(c)** $|a| < \dfrac{1}{\sqrt{3}}$ **(d)** all real value of a

[Application]

(iv) **Assertion (A):** Degree of the differential equation: $a\left(\dfrac{dy}{dx}\right)^2 + b\dfrac{dx}{dy} = c$, is 3

Reason (R): If each term involving derivatives of a differential equation is a polynomial (or can be expressed as polynomial) then highest exponent of the highest order derivative is called the degree of the differential equation. **[Analysis]**

Which of the following is correct?

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true.

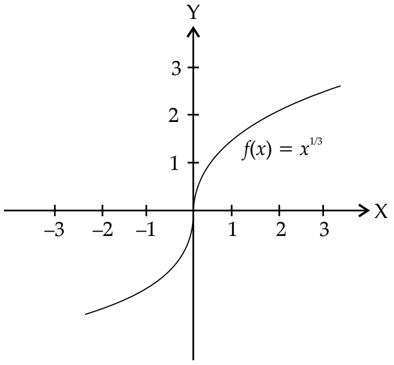
(v) Five numbers x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3, ……., 18 and are arranged in the increasing order such that $x_1 < x_2 < x_3 < x_4 < x_5$. What is the probability that $x_2 = 7$ and $x_4 = 11$?

(a) $\dfrac{26}{51}$ **(b)** $\dfrac{3}{104}$ **(c)** $\dfrac{1}{68}$ **(d)** $\dfrac{1}{34}$ **[Application]**

(vi) If $\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix} = k$, then what is the value of $\begin{vmatrix} 6a & 2b & 2c \\ 3m & n & p \\ 3x & y & z \end{vmatrix}$?

(a) $\frac{k}{6}$ **(b)** $2k$ **(c)** $3k$ **(d)** $6k$ **[Understanding]**

(vii) Consider the graph $y = x^{\frac{1}{3}}$ **[1]**



Statement 1: The above graph is continuous at $x = 0$

Statement 2: The above graph is differentiable at $x = 0$

Which of the following is correct?

(a) Statement 1 is true, and Statement 2 is false.

(b) Statement 2 is true, and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false. **[Application]**

(viii) The value of $\frac{dy}{dx}$ if $y = |x-1| + |x-4|$ at $x = 3$ is

(a) –2 **(b)** 0 **(c)** 2 **(d)** 4

(ix) **Statement 1:** The intersection of two equivalence relations is always an equivalence relation.

Statement 2: The Union of two equivalence relations is always an equivalence relation.

Which of the following is correct?

(a) Statement 1 implies Statement 2. **(b)** Statement 2 implies Statement 1.

(c) Statement 1 is true only if Statement 2 is true. **(d)** Statements 1 and 2 are independent of each other.

[Understanding]

(x) In a third order matrix a_{ij} denotes the element of the i^{th} row and the j^{th} column.

$$A = a_{ij} = \begin{cases} 0, \text{for } i = j \\ 1, \text{for } i > j \\ -1, \text{for } i < j \end{cases}$$

Assertion (A): Matrix ‘A’ is not invertible.

Reason (R): Determinant $A = 0$

Which of the following is correct?

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true. **[Application]**

(xi) Given two events A and B such that $(A/B) = 0.25$ and $P(A \cap B) = 0.12$.

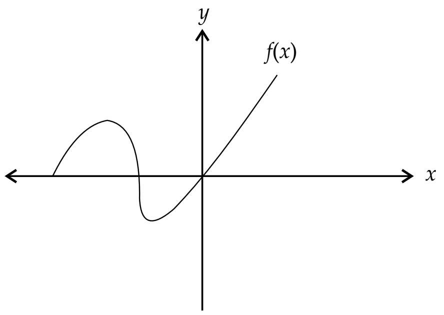
The value $P(A \cap B')$ is **[Understanding]**

(a) 0.36 **(b)** 0.48 **(c)** 0.88 **(d)** 0.036

(xii) The value of the determinant of a matrix A of order 3 is 3. If C is the matrix of cofactors of the matrix A, then what is the value of determinant of C^2? **[Analysis]**

(xiii) If a relation R on the set $\{a, b, c\}$ defined by $R = \{(b, b)\}$, then classify the relation. **[Understanding]**

(xiv) The given function $f : R \to R$ is not ‘onto’ function. Give reason. **[Understanding]**



(xv) There are three machines and 2 of them are faulty. They are tested one by one in a random order till both the faulty machines are identified. What is the probability that only two tests are needed to identify the faulty machines? **[Application]**

2. **(i)** If $x^y = y^x$, then find $\frac{dy}{dx}$ **[Understanding] [2]**

OR

(ii) Find the interval in which the function $f(x) = x^2e^{-x}$ is strictly increasing or decreasing. **[Understanding]**

3. Evaluate : $\int_0^{\sqrt{2}} [x^2]\,dx$ **[Evaluate] [2]**

4. Find the equation to the tangent at (0,0) on the curve $y = 4x^2 - 2x^3$ **[Analysis] [2]**

5. **(i)** Evaluate : $\int \frac{2x^3 - 1}{x^4 + x}\,dx$ **[Evaluate] [2]**

OR

(ii) Evaluate : $\int e^x \operatorname{cosec} x\,(1 - \cot x)\,dx$ **[Evaluate]**

6. Find the value of : $\tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y-x}{y+x}\right)$ **[Application] [2]**

7. Solve : $\sin^{-1}(x) + \sin^{-1}(1 - x) = \cos^{-1} x$. **[Application] [4]**

8. Evaluate : $\int \sqrt{\sec\frac{x}{2} - 1}\,dx$ **[Application] [4]**

9. **(i)** A kite is being pulled down by a string that goes through a ring on the ground 8 metres away from the person pulling it. If the string is pulled in at 1 metre per second, how fast is the kite coming down when it is 15 metres high? **[Application] [4]**

OR

(ii) If $y = \left(x + \sqrt{a^2 + x^2}\right)^m$, prove tthat $(a^2 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$ **[Analysis]**

10. **(i)** Three friends go to a restaurant to have pizza. They decide who will pay for the pizza by tossing a coin. It is decided that each one of them toss a coin and if one person gets a different result (heads or tails) than the other two, that person will pay. If all three get the same result (all heads or all tails), they will toss again until they get a different result.

(a) What is the probability that all three friends will get the same result (all heads or all tails) in one round of tossing?

(b) What is the probability that they will get a different result in one round of tossing? **[Analysis] [4]**

(c) What is the probability that they will need exactly four rounds of tossing to determine who would pay?

OR

(ii) Students of under graduation submitted a case study on

"Understanding the Probability of Left-Handedness in Children Based on Parental Handedness". Following Recent studies suggest that roughly 12% of the world population is left-handed. Depending on the parents' handedness, the chances of having a left-handed child vary as follows:

Scenario A:

Both parents are left-handed, with a 24% chance of the child being left-handed.

Scenario B:

The father is right-handed and the mother is left-handed, with a 22% chance of child being left-handed.

Scenario C:

The father is left-handed and the mother is right-handed, with a 17% chance of child being left-handed.

Scenario D:

Both parents are right-handed, with a 9% chance of having a left-handed child.

Assuming that scenarios A, B, C and D are equally likely, and L denotes the event that the child is left-handed, answer the following questions.

(a) What is the overall probability that a randomly selected child is left-handed?

(b) Given that exactly one parent is left-handed, what is the probability that a randomly selected child is left-handed?

(c) If a child is left-handed, what is the probability that both the parents are left- handed?

11. To raise money for an orphanage, students of three schools A, B and C organised an exhibition in their residential colony, where they sold paper bags, scrap books and pastel sheets made by using recycled paper. Student of School A sold 30 paper bags, 20 scrap books and 10 pastel sheets and raised ₹ 410. Student of School B sold 20 paper bags, 10 scrap books and 20 pastel sheets and raised ₹ 290. Student of School C sold 20 paper bags, 20 scrap books and 20 pastel sheets and raised ₹ 440.

Answer the following questions:

(i) Translate the problem into a system of equations.

(ii) Solve the system of equation by using matrix method.

(iii) Hence, find the cost of one paper bag, one scrap book and one pastel sheet. **[Application] [6]**

12. **(i)** Solve the differential equation:

$$(xdy - ydx)\, y \sin\left(\frac{y}{x}\right) = (ydx + xdy)\, x \cos\left(\frac{y}{x}\right).$$

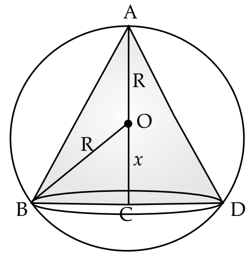
Find the particular solution satisfying the condition that $y = \pi$ when $x = 1$.

[Application] [6]

OR

(ii) Evaluate: $\int_0^{\pi}(\sin^4 x + \cos^4 x)dx$

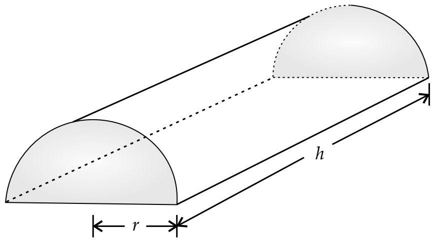
Hence evaluate: $\int_{-2\pi}^{2\pi} \frac{\sin^4 x + \cos^4 x}{1+e^x}dx$ **[Evaluate]**



13. **(i)** A cone of maximum volume is inscribed in a given sphere. Then prove that ratio of the height of the cone to the diameter of the sphere is equal to $\frac{2}{3}$. **[Application] [6]**

OR

(ii) A given quantity of metal is to be cast into a solid half circular cylinder with a rectangular base and semi-circular ends. If the total surface is minimum then prove that the ratio of the length of cylinder to the diameter of semi-circular ends is $\pi : \pi+2$ **[Application]**

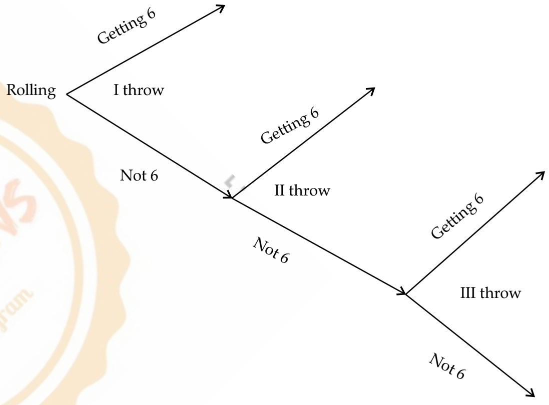


14. Kiran plays a game of throwing a fair die 3 times but to quit as and when she gets a six. Kiran gets +1 point for a six and −1 for any other number. **[Analysis] [6]**

(i) If X denotes the random variable "points earned" then what are the possible values X can take?

(ii) Find the probability distribution of this random variable X.

(iii) Find the expected value of the points she gets.



Section-B [15 marks]

15. In subparts (i) and (iii) choose the correct options and in subparts (iv) to (v), answer the questions as instructed. **[5]**

(i) Consider the following statements and choose the correct option:

Statement 1: If $\vec{a}$ and $\vec{b}$ represents two adjacent sides of a parallelogram then the diagonals are represented by $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

Statement 2: If $\vec{a}$ and $\vec{b}$ represents two diagonals of a parallelogram then the adjacent sides are represented by $2(\vec{a} + \vec{b})$ and $2(\vec{a} - \vec{b})$.

Which of the following is correct? **[Recall]**

(a) Only Statement 1 **(b)** Only Statement 2

(c) Both Statements 1 and 2 **(d)** Neither Statement 1 nor Statement 2

(ii) The distance of the plane through (1, 1, 1) and perpendicular to the line $\frac{x-1}{3} = \frac{y-1}{0} = \frac{z-1}{4}$ from the origin is **[Application]**

(a) $\frac{3}{4}$ **(b)** $\frac{4}{3}$ **(c)** $\frac{7}{5}$ **(d)** 1

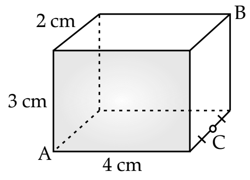
(iii) If the direction cosines of a line are $<\frac{1}{c},\frac{1}{c},\frac{1}{c}>$ then: **[Understanding]**

(a) $c > 0$ **(b)** $0 < c < 1$

(c) $c = \pm\sqrt{3}$ **(d)** $c > 2$

(iv) If $\vec{a}$ is a unit vector perpendicular to $\vec{b}$ and $(\vec{a} + 2\vec{b}).(3\vec{a} - \vec{b}) = -5$, find $|\vec{b}|$. **[Understanding]**

(v) Shown below is a cuboid. Find $\overrightarrow{BA}.\overrightarrow{BC}$. **[Analysis]**



16. (i) Find a vector of magnitude 9 units and perpendicular to the vectors.

$\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$ **[Application] [2]**

OR

(ii) What are the values of x for which the angle between the vectors?

$2x^2\hat{i} + 3x\hat{j} + \hat{k}$ and $\hat{i} - 2\hat{j} + x^2\hat{k}$ is obtuse? **[Application]**

17. (i) Show that the line whose vector equation is $\vec{r} = (2\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ is parallel to the plane whose vector equation is $\vec{r}\,.\,(\hat{i} + 5\hat{j} + \hat{k}) = 5$. Also, find the distance between them. **[Application] [4]**

OR

(ii) Find the equation of the plane containing the line $\frac{x}{-2} = \frac{y-1}{3} = \frac{1-z}{1}$ and the point$(-1, 0, 2)$. **[Application]**

18. (i) Sketch the region enclosed bounded by the curve, $y = x\,|x|$ and the ordinates $x = -1$ and $x = 1$

(ii) Evaluate : $\int_0^1 x^2 dx$.

(iii) Hence, find the area bounded by the curve, $y = x|x|$ and the ordinates $x = -1$ and $x = 1$. **[4]**

Section-C [15 marks]

19. In subparts **(i)** and **(ii)** choose the correct options and in subparts **(iii)** to **(v)**, answer the questions as instructed. **[5]**

(i) Which condition is true if Average Cost (AC) is constant at all levels of output **[Recall]**

(a) $MC > AC$ **(b)** $MC = AC$ **(c)** $MC < AC$ **(d)** $MC = \frac{1}{2}AC$

(ii) Read the following statements and choose the correct option:

(I) If $r = 0$, then regression lines are not defined.
(II) If $r = 0$, then regression lines are parallel.
(III) If $r = 0$, then regression lines are perpendicular.
(IV) If $r = \pm 1$, then regression lines coincide. **[Recall]**

Which of the following is correct?

(a) Only IV is correct **(b)** Only I and II are correct
(c) Only I and IV are correct **(d)** Only III and IV are correct

(iii) Mean of x=53, mean of y=28 regression co-efficient y on $x = -1{\cdot}2$, regression co-efficient x on y=$-0{\cdot}3$. Find the coefficient of correlation **(*r*)**. **[Understanding]**

(iv) The total revenue received from the sale of x unit of a product is given by $R(x)= 3x^2+36x++5$. Find the marginal revenue when x=5. **[Understanding]**

(v) A manufacturing company finds that the daily cost of producing x item of product is given by $C(x)= 210x+7000$. Find the minimum number that must be produced and sold daily, if each item is sold for ₹280. **[Understanding]**

20. (i) The cost function of a commodity is $C(x)=200+20x-\frac{1}{2}x^2$ (in rupees). Find the range where AC falls. **[Application] [2]**

OR

(ii) The demand function of a monopoly is given by $x=100-4p$. Find the quantity at which the MR will be zero. **[Application]**

21. (i) A survey of 50 families to study the relationships between expenditure on accommodation in (₹ x) and expenditure on food and entertainment (₹ y) gave the following results:

$\Sigma x=8500$, $\Sigma y=9600$, $\sigma_x=60$, $\sigma_y=20$, $r=0.6$

Estimate the expenditure on food and entertainment when expenditure on accommodation is ₹200. **[Application] [4]**

OR

(ii) The random variables have regression lines $3x+2y-26=0$ and $6x+y-31=0$

Calculate

(a) Mean value of x and y.
(b) Co-efficient of correlations. **[Application]**

22. A linear programming problem is given by $Z=px+qy$ where $p, q > 0$ subject to the constraints:

$x+y \le 60$, $5x + y \le 100$, $x \ge 0$ and $y \ge 0$ **[Analysis] [4]**

(i) Solve graphically to find the corner points of the feasible region.

(ii) If $Z = px+qy$ is maximum at (0, 60) and (10, 50), find the relation of p and q. Also, mention the number of optimal solution(s) in this case.

■■

SOLUTIONS

Sample Question Paper - 1

MATHEMATICS

(With Board Marking Scheme 2025)

Section-A

1. (i) **Option (d) is correct.**

Explanation:

The transpose of matrix is equal to itself is called Symmetric Matrix.

$$A = [a_{ij}]_{m\times n} \text{ then } A' = [a_{ij}]_{n\times m}$$

$$\Rightarrow \quad A = A'$$

If the transpose of a matrix is equal to the negative of itself is called Skew Symmetric Matrix.

$$A' = -A$$

Only null matrix follow both the conditions.

(ii) **Option (b) is correct.**

Explanation:

Let, $\quad I = \int a^x e^x dx$

$$I = \int \underset{\text{I}}{a^x}\, \underset{\text{II}}{e^x}\, dx \quad \text{....(i)}$$

On integrating by parts,

$$I = a^x \int e^x dx - \int\left(\frac{d}{dx}a^x \int e^x dx\right)$$

$$\Rightarrow \quad I = a^x e^x - \int a^x \log a e^x$$

$$\Rightarrow \quad I = a^x e^x - \log a \int a^x e^x \quad \text{....(ii)}$$

Substitute (i) from (ii),

$$I = a^x e^x - I \log a + C$$

$$\Rightarrow \quad I + \log aI = a^x e^x + C$$

$$\Rightarrow \quad I(1 + \log a) = a^x e^x + C \qquad [\because \log_e e = 1]$$

$$\Rightarrow \quad I(\log e + \log a) = a^x e^x + C$$

$$[\because \log a + \log b = \log(ab)]$$

$$\Rightarrow \quad I[\log_e(ae)] = a^x e^x + C$$

$$\Rightarrow \quad I = \frac{a^x e^x}{\log_e(ae)} + C'$$

(iii) **Option (c) is correct.**

Explanation: The trigonometric equations $\tan^{-1}x = 3 \tan^{-1} a$ will have a solution if :

$$3 \tan^{-1} a \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\because \tan^{-1} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\Rightarrow \quad \tan^{-1} a \in \left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$$

$$\Rightarrow \quad a \in \left(-\tan\frac{\pi}{6}, \tan\frac{\pi}{6}\right)$$

$$\Rightarrow \quad a \in \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \quad |\,a\,| < \left(\frac{1}{\sqrt{3}}\right)$$

(iv) **Option (d) is correct.**

Explanation: Degree of the differential equation is '2'. If each term involving derivatives of a differential equation is a polynomial then highest exponent of the highest order derivative is called the degree of the differential equation.

(v) **Option (c) is correct.**

Explanation: No. of ways to select and arrange x_1, x_2, x_3, x_4, x_5 from 1, 2, 3, 4,............, 18.

$$n(S) = {}^{18}C_5$$

$$x_1 < x_2 < x_3 < x_4 < x_5$$

Given, $x_2 = 7$ and $x_4 = 11$

So, the number before x_2 is 6

Number between x_2 and x_4 is 3

Number after x_4 is 7.

$$n(E) = {}^6C_1 \times {}^3C_1 \times {}^7C_1$$

$$P(E) = \frac{6\times3\times7}{{}^{18}C_5}$$

$$= \frac{1}{17\times4} = \frac{1}{68}$$

(vi) **Option (d) is correct.**

Explanation:

$$\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix} = k$$

By the properties of multiplying Row (or Column) By scalar, $\quad C_1 \to 3C_1, R_1 \to 2R_1$

$$\begin{vmatrix} 6a & 2b & 2c \\ 3m & n & p \\ 3x & y & z \end{vmatrix} = 2 \times 3 \times k = 6k$$

(vii) **Option (a) is correct.**

Explanation: The cube root of 'x' can be uniformly continuous on subset of R.

The cube root function has a vertical tangent at $x = 0$, function are not differentiable where they have vertical tangents.

(viii) **Option (b) is correct.**

Explanation: $\quad y = |x - 1| + |x - 4|$

Value of y at $x = 3$ is :

$$y = (x - 1) + (4 - x)$$

$$y = x - 1 + 4 - x$$
$$y = 3$$
$$\frac{dy}{dx} = 0$$

(ix) Option (c) is correct.

(x) Option (a) is correct.

Explanation: Given condition three order matrix.

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\Rightarrow \quad \begin{vmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix} = A$$

Matrix 'A' is not invertible because $|A| = 0$.

(xi) Option (a) is correct.

*Explanation:*We have, $P\left(\frac{A}{B}\right) = 0.25$, $P(A \cap B) = 0.12$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

So, $$0.25 = \frac{0.12}{P(B)}$$

$$P(B) = \frac{12}{25}$$

$$P(A \cap B') = P(B) - P(A \cap B)$$
$$= \frac{12}{25} - \frac{12}{100}$$
$$P(A \cap B') = \frac{36}{100} = 0.36$$

(xii) Option (a) is correct.

$$n = 3$$
$$|A| = 3.$$

As per question,

$$C = (adj\ A)^T$$
$$\therefore \quad |C| = |(adj\ A)^T| = |adj\ A|$$
$$\therefore \quad |C| = |A|^{n-1} = 3^{3-1} = 3^2 = 9$$
$$|C^2| = |C|.|C| = 9 \times 9 = 81$$

[ISC Marking Scheme 2025]

Detailed Answer:

Explanation: We have,

Order of matrix 'n' $= 3$

$$|A| = 3$$

We know that,

$$\therefore \quad |C| = |(adj\ A)^T| = |adj\ A|$$
$$|C| = |A|^{n-1} = 3^{3-1} = 3^2 = 9$$
$$|C^2| = |C|.|C|$$
$$= 9 \times 9$$
$$= 81 \quad [\because |adj\ A| = |A|^{n-1}]$$

(xiii) The relation is symmetric and transitive but not reflexive.

[ISC Marking Scheme 2025]

Detailed Answer:

Explanation: If, $A = \{a, b, c\}$, $R = \{(b, b)\}$

For Reflexive: $(a, a) \notin R$ So, this relation is not reflexive.

For Symmetric:

$$b\ R\ b \Rightarrow b\ R\ b,\ b \in A$$

So, the relation is symmetric and transitive also.

(xiv) Since, each line in co-domain of the function parallel to x–axis, doesn't cut the graph of function at least one point, therefore the $f(x)$ is not an onto function.

(xv) Two tests will be required, if first machine is faulty and second is good or both the machines are faulty.

Probability that only two tests are needed

$$= \frac{2}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{3} = \frac{2}{3}$$

[ISC Marking Scheme 2025]

Total number of machines is '3'.

Number of faulty machines is '2'

$$\Rightarrow \quad \text{Both are working} = \frac{2}{3} \times \frac{1}{2}$$
$$\Rightarrow \text{Both are not working} = \frac{2}{3} \times \frac{1}{2}$$
$$\text{Total probability} = \frac{2}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2}$$
$$= \frac{1}{3} + \frac{1}{3}$$
$$\text{Required probability} = \frac{2}{3}$$

2. (i) $$x^y = y^x$$
$$y \log x = x \log y$$

By differentiating on both sides,

$$y.\frac{1}{x} + \log x \frac{dy}{dx} = x.\frac{1}{y}\frac{dy}{dx} + \log y$$

$$\frac{dy}{dx}\left(\log x - \frac{x}{y}\right) = \log y - \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{\log y - \frac{y}{x}}{\log x - \frac{x}{y}}$$

OR

(ii) $$f(x) = x^2e^{-x}$$
$$\frac{dy}{dx} = -x^2e^{-x} + e^{-x}.\,2x$$
$$= \frac{1}{e^x}\left(2x - x^2\right) = \frac{1}{e^x}(2 - x)x$$
$$= -e^{-x}\,[x(x - 2)]$$

$\because$ e^x is always positive

$\therefore$ Setting $x\,(x - 2) = 0 \Rightarrow x = 0$ or $x = 2$

For strictly decreasing function

$$\frac{dy}{dx} < 0 \Rightarrow x < 0 \text{ or } x > 2$$

$$\Rightarrow x \in (-\infty, 0) \cup (2, \infty)$$

For strictly increasing function

$$\frac{dy}{dx} > 0 \Rightarrow 0 < x < 2 \Rightarrow x \in (0, 2)$$

$\therefore$ f **(**x**)** is increasing in (0, 2) and decreasing in $(-\infty, 0) \cup (2, \infty)$

[ISC Marking Scheme 2025]

Detailed Solution:

(i) $x^y = y^x$

Taking log on both sides,

$$\log (x)^y = \log (y)^x$$

$$\Rightarrow \quad y \log x = x \log y$$

By differentiating on both the sides,

$$\Rightarrow \quad y\frac{1}{x} + \log x\frac{dy}{dx} = x\frac{1}{y}\frac{dy}{dx} + \log y$$

$$\Rightarrow \quad \frac{dy}{dx}\left(\log x - \frac{x}{y}\right) = \log y - \frac{y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\log y - \frac{y}{x}}{\log x - \frac{x}{y}}$$

$$= \frac{\frac{x\log y - y}{x}}{\frac{y\log x - x}{y}}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y[x\log y - y]}{x[y\log x - x]}$$

3. $\int_0^{\sqrt{2}} \left[x^2\right] dx$

We know greatest integer function is discontinuous when x is an integer.

$$\therefore \quad \int_0^{\sqrt{2}} \left[x^2\right] dx = \int_0^1 0\, dx + \int_1^{\sqrt{2}} 1\, dx$$

$$= x\Big|_1^{\sqrt{2}} = \sqrt{2} - 1.$$

4. $y = 4x^2 - 2x^3$

$$\frac{dy}{dx} = 8x - 6x^2$$

Let P(x_1, y_1) be the point of contact.

$\therefore$ Slope of the tangent $= 8x_1 - 6x_1^2$

Tangent equation: $y - y_1 = (8x_1 - 6x_1^2)(x - x_1)$

$\because$ the tangent passes through the origin

$$y_1 = (8x_1 - 6x_1^2)x_1$$

Now, $y_1 = 4x_1^2 - 2x_1^3 = 8x_1^2 - 6x_1^3$

$\Rightarrow \quad 4 - 2x_1 = 8 - 6x_1$

$\Rightarrow \quad 4x_1 = 4$

$x_1 = 1 \Rightarrow y_1 = 2$

The point on the curve is (1, 2).

5. (i) $\int \frac{2x^3 - 1}{x^4 + x} dx$

Dividing both nominator and denominator by 'x^2' we get

$$= \int \frac{2x - \frac{1}{x^2}}{x^2 + \frac{1}{x}} dx$$

Let $x^2 + \frac{1}{x} = t \Rightarrow \left(2x - \frac{1}{x^2}\right)dx = dt$

$$= \int \frac{dt}{t}$$

$$= \log_e |t| + c$$

$$= \log_e \left|x^2 + \frac{1}{x}\right| + c$$

OR

(ii) $\int e^x \operatorname{cosec} x\,(1 - \cot x)dx$

$$= \int e^x (\operatorname{cosec} x - \operatorname{cosec} x . \cot x)dx$$

$$= \int e^x (\operatorname{cosec} x + (-\operatorname{cosec} x . \cot x))dx$$

$$\left[\because \int e^x (f(x) + f'(x))dx = e^x f(x) + c\right]$$

$$= e^x \operatorname{cosec} x + c$$

6. (i) $\tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y - x}{y + x}\right)$

$$= \tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{1 - \frac{x}{y}}{1 + \frac{x}{y}}\right)$$

$$= \tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}(1) - \tan^{-1}\left(\frac{x}{y}\right)$$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$

7. $\sin^{-1}(x) + \sin^{-1}(1 - x) = \cos^{-1} x$

$$\Rightarrow \quad \sin^{-1}(x) + \sin^{-1}(1 - x) = \frac{\pi}{2} - \sin^{-1} x$$

$$\Rightarrow \quad \sin^{-1}(1 - x) = \frac{\pi}{2} - 2\sin^{-1} x$$

$$\Rightarrow \quad (1 - x) = \sin\left(\frac{\pi}{2} - 2\sin^{-1} x\right)$$

$$\Rightarrow \quad (1 - x) = \cos(2\sin^{-1} x)$$

$$\Rightarrow \quad (1 - x) = \cos(\cos^{-1}(1 - 2x^2))$$

$$\Rightarrow \quad (1 - x) = 1 - 2x^2$$

$$\Rightarrow \quad 2x^2 - x = 0$$

$$\therefore \quad x = 0, \frac{1}{2}$$

8. $$I = \int\sqrt{\sec\frac{x}{2}-1}\,dx = \int\sqrt{\frac{1-\cos\frac{x}{2}}{\cos\frac{x}{2}}}\,dx$$

$$= \int\sqrt{\frac{\left(1-\cos\frac{x}{2}\right)\left(1+\cos\frac{x}{2}\right)}{\cos\frac{x}{2}\left(1+\cos\frac{x}{2}\right)}}\,dx$$

$$\int\frac{\sin\frac{x}{2}}{\sqrt{\cos^2\frac{x}{2}+\cos\frac{x}{2}}}\,dx$$

Let $\cos\frac{x}{2} = t \Rightarrow -\sin\frac{x}{2}\cdot\frac{1}{2}dx = dt$

$$\Rightarrow \sin\frac{x}{2}dx = -2dt$$

$$\therefore I = -2\int\frac{dt}{\sqrt{t^2+t}} = -2\int\frac{dt}{\sqrt{\left(t+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$$

$$= -2\log_e\left|\left(t+\frac{1}{2}\right)+\sqrt{t^2+t}\right|+c$$

$$= -2\log_e\left|\left(\cos\frac{x}{2}+\frac{1}{2}\right)+\sqrt{\cos^2\frac{x}{2}+\cos\frac{x}{2}}\right|+c$$

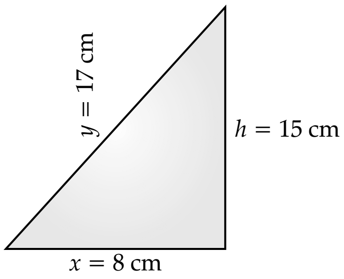
9. **(i)** $x^2 + h^2 = y^2 \Rightarrow x^2 + 225 = y^2$

Given, $\frac{dy}{dt} = 1$ m/sec

Differentiating w.r.t 't'

$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

$$8\times\frac{dx}{dt} = 17\times 1$$



Then, $\frac{dx}{dt} = \frac{17}{8}$

OR

(ii) **Method I** $y = \left[x+\sqrt{a^2+x^2}\right]^m$

$$\frac{dy}{dx} = \frac{m\left[x+\sqrt{a^2+x}^{2}\right]^{m}}{x+\sqrt{a^2+x^2}}\cdot\left[1+\frac{1.2x}{2\sqrt{a^2+x^2}}\right]$$

$$= my\frac{1}{\sqrt{a^2+x^2}}$$

$$\sqrt{a^2+x^2}\frac{dy}{dx} = my$$

$$\sqrt{a^2+x^2}\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot\frac{1}{2}\cdot\frac{2x}{\sqrt{a^2+x^2}} = m\frac{dy}{dx}$$

$$(a^2+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-m\sqrt{a^2+x^2}\frac{dy}{dx} = 0$$

$$(a^2+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-m^2y = 0$$

Method II $y = \left[x+\sqrt{a^2+x^2}\right]^m$

$$\frac{dy}{dx} = \frac{m\left[x+\sqrt{a^2+x}^{2}\right]^{m}}{x+\sqrt{a^2+x^2}}\cdot\left[1+\frac{1.2x}{2\sqrt{a^2+x^2}}\right]$$

$$= my\frac{1}{\sqrt{a^2+x^2}}$$

$$\sqrt{a^2+x^2}\frac{dy}{dx} = my$$

Squaring on both sides we get,

$$(a^2+x^2)\left(\frac{dy}{dx}\right)^2 = m^2y^2$$

Differentiating w.r.t 'x' ,

$$2x\left(\frac{dy}{dx}\right)^2+(a^2+x^2)2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} = 2m^2y\frac{dy}{dx}$$

$$\Rightarrow (a^2+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-m^2y = 0$$

10. **(i)** **(a)** P (no odd person) $= P(\text{HHH}) + P(\text{TTT})$

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$$

(b) P (odd person) $= 1-\frac{1}{4} = \frac{3}{4}$

(c) P (odd person in 4th round) $= \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4} = \frac{3}{256}$

OR

(ii) **(a)** Since, events A, B, C, D are equally likely

$$\Rightarrow \quad P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$$

As per question, $P(L/A) = \frac{24}{100}$, $P(L/B) = \frac{22}{100}$,

$P(L/C) = \frac{17}{100}$, $P(L/D) = \frac{9}{100}$

The probability that a randomly selected child is left-handed $= P(A)\times P(L/A) + P(B)\times P(L/B) + P(C)\times P(L/C) + P(D)\times P(L/D)$ $P(A/L)$

$$= \frac{1}{4}\times\frac{24}{100}+\frac{1}{4}\times\frac{22}{100}+\frac{1}{4}\times\frac{17}{100}+\frac{1}{4}\times\frac{9}{100}$$

(b) The probability that a randomly selected child is left-handed, given that exactly one of the parents is left-handed

$$= P(L/B) + P(L/C)$$

$$= \frac{22}{100} + \frac{17}{100} = \frac{39}{100}$$

(c) $P(A/L)$

$$= \frac{P(A)\times P(L/A)}{PA\times P(L/A) + P(B)\times P(L/B) + P(C)\times P(L/C) + P(D)\times P(L/D)\, P(A/L)}$$

$$= \frac{\frac{1}{4}\times\frac{24}{100}}{\frac{1}{4}\times\frac{24}{100} + \frac{1}{4}\times\frac{22}{100} + \frac{1}{4}\times\frac{17}{100} + \frac{1}{4}\times\frac{9}{100}} = \frac{1}{3}$$

11. **(i)** Let the cost of one paper bag, one scrap book and one pastel sheet be x, y and z respectively.

$$30x + 20y + 10z = 410 \Rightarrow 3x + 2y + z = 41$$

$$20x + 10y + 20z = 290 \Rightarrow 2x + y + 2z = 29$$

$$20x + 20y + 20z = 440 \Rightarrow x + y + z = 22$$

(ii) Given system of equations is equivalent to $AX = B$

Where $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 41 \\ 29 \\ 22 \end{bmatrix}$

$$|A| = -2 \neq 0 \Rightarrow A^{-1} \text{ exists.}$$

$$\text{adj } A = \begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}$$

Thus $A^{-1} = \frac{1}{|A|}adj\ A = -\frac{1}{2}\begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}$

$$AX = B \Rightarrow X = A^{-1}B = -\frac{1}{2}\begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 41 \\ 29 \\ 22 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 15 \\ 5 \end{bmatrix}$$

$\therefore \quad x = 2, y = 15, z = 5$

(iii) The cost of one paper bag, one scrap book and one pastel sheet be ₹ 2, ₹ 15 and ₹ 5 respectively.

12. **(i) Method-I**

$$(xdy - ydx)\, y \sin\left(\frac{y}{x}\right) = (ydx + xdy)\, x \cos\left(\frac{y}{x}\right).$$

$$\Rightarrow \quad (xdy - ydx)\frac{\sin\left(\frac{y}{x}\right)}{\cos\left(\frac{y}{x}\right)} = \frac{(ydx + xdy)x}{y}$$

$$\Rightarrow \quad \frac{(xdy - ydx)}{x^2}\tan\left(\frac{y}{x}\right) = \frac{(ydx + xdy)x}{x^2 y}$$

$$\Rightarrow \quad \frac{(xdy - ydx)}{x^2}\tan\left(\frac{y}{x}\right) = \frac{(ydx + xdy)}{xy}$$

$$\Rightarrow \quad \tan\left(\frac{y}{x}\right).d\left(\frac{y}{x}\right) = \frac{d(xy)}{xy},$$

which is variable separable form where variables are $\frac{y}{x}$ and xy. Integrating both sides,

$$\Rightarrow \quad \int \tan\frac{y}{x}.d\left(\frac{y}{x}\right) = \int\frac{d(xy)}{xy}$$

$$\Rightarrow \quad \log\left|\sec\left(\frac{y}{x}\right)\right| = \log(xy) + \log c$$

$$\Rightarrow \quad \sec\left(\frac{y}{x}\right) = c\,(xy).$$

Given, when $x = 1, y = \pi \Rightarrow c = -\frac{1}{\pi}$.

Hence, required particular solution is:

$$\sec\left(\frac{y}{x}\right) = -\frac{1}{\pi}(xy).$$

Method-II

$$(xdy - ydx)\, y \sin\left(\frac{y}{x}\right) = (ydx + xdy)\, x \cos\left(\frac{y}{x}\right).$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(xy)\cos\left(\frac{y}{x}\right) + y^2\sin\left(\frac{y}{x}\right)}{(xy)\sin\left(\frac{y}{x}\right) - x^2\cos\left(\frac{y}{x}\right)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1 + \left(\frac{y}{x}\right)\tan\left(\frac{y}{x}\right)}{\tan\left(\frac{y}{x}\right) - \frac{1}{\left(\frac{y}{x}\right)}},$$

which is homogenous differential equation of type

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right).$$

Let $y = vx = \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{1 + v\tan v}{\tan v - \frac{1}{v}}$$

$$\Rightarrow \quad \left(\tan v - \frac{1}{v}\right)dv = 2\frac{dx}{x},$$

which is variable separable form where variables are v and x. Integrating both sides,

$$\Rightarrow \quad \int\left(\tan v - \frac{1}{v}\right)dv = 2\int\frac{dx}{x}$$

$$\Rightarrow \log|\sec v| - \log|v| = 2\log|x| + \log c$$

$$\log|\sec v| = \log|v.\, x^2.\, c\}$$

$$\Rightarrow \qquad \sec v = c\,(xy)$$

$$\Rightarrow \qquad \sec\left(\frac{y}{x}\right) = c\,(xy).$$

Given, when $x = 1, y = \pi \Rightarrow c = -\frac{1}{\pi}$.

Hence, required particular solution is:

$$\sec\left(\frac{y}{x}\right) = -\frac{1}{\pi}(xy).$$

OR

(ii) $\int_0^{\pi}\left(\sin^4 x + \cos^4 x\right)dx \left[\because \sin^4 x + \cos^4 x = 1 - \frac{1}{2}\sin^2 2x = \frac{1}{4}(3 + \cos 4x)\right]$

$$= \frac{1}{4}\int_0^{\pi}(3 + \cos 4x)dx$$

$$= \frac{1}{4}\left[3x + \frac{\sin 4x}{4}\right]_0^{\pi} = \frac{3\pi}{4}$$

$$I = \int_{-2\pi}^{2\pi}\frac{\sin^4 x + \cos^4 x}{1 + e^x}dx$$

$$= \int_{-2\pi}^{2\pi}\frac{\sin^4(-x) + \cos^4(-x)}{1 + e^{-x}}dx$$

$$\left[\because \int_a^b f(x)\,dx = \int_a^b f(a + b - x)dx\right]$$

$$= \int_{-2\pi}^{2\pi}\frac{\sin^4 x + \cos^4 x}{1 + e^{-x}}dx$$

$$= \int_{-2\pi}^{2\pi}\frac{e^x\left(\sin^4 x + \cos^4 x\right)}{1 + e^x}dx$$

$$I + I = \int_{-2\pi}^{2\pi}\frac{\sin^4 x + \cos^4 x}{1 + e^x}dx + \int_{-2\pi}^{2\pi}\frac{e^x\left(\sin^4 x + \cos^4 x\right)}{1 + e^x}dx$$

$$\therefore\ 2I = \int_{-2\pi}^{2\pi}\left(\sin^4 x + \cos^4 x\right)dx$$

$$= 2\int_0^{2\pi}\left(\sin^4 x + \cos^4 x\right)dx$$

$$\left[\because \int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx \text{ if } f(-x) = f(x)\right]$$

$$\therefore\quad I = \int_0^{2\pi}\left(\sin^4 x + \cos^4 x\right)dx$$

$$= 2\int_0^{\pi}\left(\sin^4 x + \cos^4 x\right)dx$$

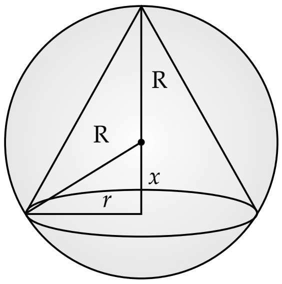
$$\left[\because \int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx \text{ if } f(2a - x) = f(x)\right]$$

$$= 2.\frac{3\pi}{4} = \frac{3\pi}{2}$$

13. (i) Let the given radius of sphere be R.

Consider radius of cone be r and height be

$= h = R + x$



Method-I

From the diagram $r^2 + x^2 = R^2$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(R + x) = \frac{1}{3}\pi(R + x)(R^2 - x^2)$$

$$\frac{dV}{dx} = \frac{1}{3}\pi\left[(R^2 - x^2)(1) + (R + x)(-2x)\right]$$

Equating, $\frac{dV}{dx} = 0$

$$\Rightarrow \frac{1}{3}\pi\left[(R^2 - x^2)(1) + (R + x)(-2x)\right] = 0$$

$$\Rightarrow \qquad (R + x)(R - 3x) = 0$$

$$\Rightarrow \qquad R = 3x \text{ or } x = \frac{R}{3}$$

$$\frac{d^2V}{dx^2} = \frac{1}{3}\pi\left[-2R - 6x\right] < 0 \text{ at } x = \frac{R}{3}$$

Volume is maximum at $x = \frac{R}{3}$

Height of cone $= R + x = R + \frac{R}{3} = \frac{4R}{3}$

Ratio of height of cone to diameter of sphere

$$= \frac{h}{2R} = \frac{\frac{4R}{3}}{2R} = \frac{2}{3}$$

Method-II

From the diagram $R^2 = r^2 + x^2 = r^2 + (h - R)^2$

$$\therefore \qquad R^2 = r^2 + h^2 + R^2 - 2Rh$$

$$\therefore \qquad r^2 = 2Rh - h^2$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2Rh - h^2)h$$

$$= \frac{1}{3}\pi(2Rh^2 - h^3)$$

$$\therefore \qquad \frac{dV}{dh} = \frac{1}{3}\pi(4Rh - 3h^2)$$

Setting $\frac{dV}{dh} = 0$, we get $h = \frac{4R}{3}$

$$\frac{d^2V}{dh^2} = \frac{1}{3}\pi(4R - 6h) = \frac{1}{3}\pi(4R - 8R) < 0$$

$\therefore$ Volume is maximum at $h = \frac{4R}{3}$

Ratio of height of cone to diameter of sphere $=$

$\frac{h}{2R}$

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$$= \frac{\frac{4R}{3}}{2R} = \frac{2}{3}, \text{ put } \frac{dV}{dh}=0, \text{ we get } h = \frac{4R}{3}$$

$$\frac{d^2V}{dh^2} = \frac{1}{3}\pi(4R-6h) = \frac{1}{3}\pi(4R-8R)<0$$

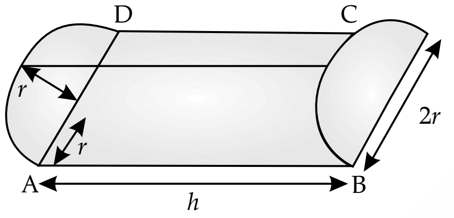
∴ Volume is maximum at $h = \frac{4R}{3}$

Ratio of height of cone to diameter of sphere

$$= \frac{h}{2R} = \frac{\frac{4R}{3}}{2R} = \frac{2}{3}$$

OR

(ii)



Volume of half cylinder $= \frac{1}{2}\pi r^2 h = V \Rightarrow h = \frac{2V}{\pi r^2}$

$$S = \pi r^2 + \pi r h + 2rh = \pi r^2 + \pi r \,.\, \frac{2V}{\pi r^2} + 2r.\frac{2V}{\pi r^2}$$

$$= \pi r^2 + \frac{1}{r}\left(2V + \frac{4V}{\pi}\right)$$

$$\frac{dS}{dr} = 2\pi r - \frac{1}{r^2}\left(2V + \frac{4V}{\pi}\right)$$

Putting $\frac{dS}{dr} = 0 \Rightarrow 2\pi r - \frac{1}{r^2}\left(2V + \frac{4V}{\pi}\right) = 0$

$$\Rightarrow 2\pi r = \frac{1}{r^2}\left(2V + \frac{4V}{\pi}\right) \Rightarrow \pi r^3 = V + \frac{2V}{\pi}$$

$$\Rightarrow \pi r^3 = V\left(\frac{\pi+2}{\pi}\right)$$

$$\pi r^3 = \frac{1}{2}\pi r^2 h.\frac{\pi+2}{\pi}$$

$$\Rightarrow \frac{2r}{h} = \frac{\pi+2}{\pi}$$

∴ Ratio of the length of cylinder to the diameter of semi-circular ends

$$= \frac{\pi}{\pi+2}$$

$$\frac{d^2S}{dr^2} = 2\pi + \frac{2}{r^3}\left(2V + \frac{4V}{\pi}\right) > 0$$

∴ Surface area is minimum.

14. (i) $X = -3, -1, 0, 1$

(ii) $P(X = -3) = \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6} = \frac{125}{216}$

$P(X = -1) = \frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} = \frac{25}{216}$

$P(X = 0) = \frac{5}{6}\times\frac{1}{6} = \frac{5}{36}$

$P(X = 1) = \frac{1}{6}$

Required probability distribution :

X	-3	-1	0	1
$P(X)$	$\frac{125}{216}$	$\frac{25}{216}$	$\frac{5}{36}$	$\frac{1}{6}$

(iii) Expected value $= X.P(X)$

$$= (-3)\left(\frac{125}{216}\right) + (-1)\left(\frac{25}{216}\right) + 0 + \left(\frac{1}{6}\right)$$

$$= -\frac{91}{54} = -1\frac{37}{54} \approx -1.69$$

Section-B

15. (i) Option (a) is correct.

(ii) Option (c) is correct.

Explanation:

Let, 'K' be the distance of the plane through (1, 1, 1) and perpendicular to the line $\frac{x-1}{3} = \frac{y-1}{0} = \frac{z-1}{4}$ from the origin.

Equation of plane perpendicular to a line $\frac{x-1}{3} = \frac{y-1}{0} = \frac{z-1}{4}$ is $3x + 0(y) + 4z = P$

Given that plane passes through (1, 1, 1) $3(1)+4(1)=P$

$\Rightarrow P=7$

∴ equation of the plane $\Rightarrow 3x + 4z = 7$

The perpendicular distance from the origin to plane is K

$$\Rightarrow K = \frac{|3(0)+4(0)-7|}{\sqrt{3^2+4^2}} = \frac{7}{\sqrt{25}} = \frac{7}{5}$$

$$\Rightarrow K = \frac{7}{5}$$

(iii) Option (c) is correct.

Explanation:

Given that the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}\frac{1}{c},$

We know the sum of the squares of the direction cosines is one.

i.e., $\left(\frac{1}{c}\right)^2 + \left(\frac{1}{c}\right)^2 + \left(\frac{1}{c}\right)^2 = 1$

$\Rightarrow \quad \frac{3}{c^2} = 1$

$\Rightarrow \quad c^3 = 3$

$\Rightarrow \quad c = \pm\sqrt{3}$

(iv) As per question $|\vec{a}| = 1, \vec{a}.\vec{b} = 0$

∴ $(\vec{a} + 2\vec{b}).(3\vec{a} - \vec{b}) = -5$

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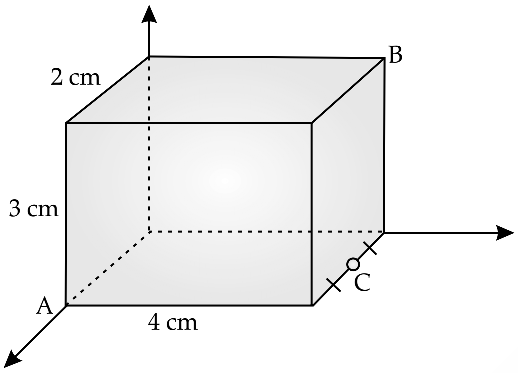
⇒ $3|\vec{a}|^2 + 5(\vec{a}.\vec{b}) - 2|\vec{b}|^2 = -5$

⇒ $2|\vec{a}|^2 = 8 \Rightarrow |\vec{b}| = 2$

(v) Placing the coordinate axes as illustrated , coordinates of A is (2,0,0) , B (0,4,3) and C (1,4,0).



∴ $\overrightarrow{BA} = 2\hat{i} - 4\hat{j} - 3\hat{k}$ and $\overrightarrow{BC} = \hat{i} - 3\hat{k}$

∴ $\overrightarrow{BA}.\overrightarrow{BC} = 2 + 9 = 11$

16. **(i)** A vector of magnitude 9 units and perpendicular to the vectors $\vec{a}$ and $\vec{b}$ is $9\left(\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}\right)$

Given, = $4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$

$$\vec{a}\times\vec{b} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 1 \\ -2 & 1 & -2 \end{bmatrix} = \hat{i} + 6\hat{j} + 2\hat{k} \text{ and } |\vec{a}\times\vec{b}|$$

$$= \sqrt{41}$$

Hence required vector is $\dfrac{9}{\sqrt{41}}\left(\hat{i} + 6\hat{j} + 2\hat{k}\right)$

OR

(ii) Let $\vec{a} = 2x^2\hat{i} + 3x\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + x^2\hat{k}$

∵ angle between the vectors is obtuse

⇒ $\cos\theta < 0$

⇒ $\dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} < 0$

⇒ $\vec{a}.\vec{b} < 0$

⇒ $2x^2 - 6x + x^2 < 0$

⇒ $3x(x - 2) < 0$

⇒ $0 < x < 2$. i.e., $x \in (0, 2)$

17. **(i)** P.V. of the point on the line $2\hat{i} - 2\hat{j} + 3\hat{k}$

Direction vector $\hat{i} - \hat{j} + 4\hat{k}$

Given plane $\vec{r}.\left(\hat{i} + 5\hat{j} + \hat{k}\right) - 5 = 0$

∴ normal vector of the plane $\hat{i} + 5\hat{j} + \hat{k}$

If straight line and plane are parallel to each other

⇒ Direction vector of line and normal vector of the plane are perpendicular to each other

Now we get $(\hat{i} - \hat{j} + 4\hat{k})(\hat{i} + 5\hat{j} + \hat{k}) = 0$

∵ Since, the dot product is zero, they are perpendicular to each other.

⇒ straight line and plane are parallel to each other.

Required distance $= \dfrac{(2\hat{i} - 2\hat{j} + 3\hat{k}).(i + 5\hat{j} + k) - 5|}{\sqrt{1 + 25 + 1}}$

$= \dfrac{10}{\sqrt{27}} = \dfrac{10}{3\sqrt{3}}$ units.

OR

(ii) **Method-I**

Let the equation of the plane be $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ is passing through (–1, 0, 2)

⇒ $a(x + 1) + b(y - 0) + c(z - 2) = 0$ ...**(i)**

Given line $\dfrac{x-0}{-2} = \dfrac{y-1}{3} = \dfrac{z-1}{-1}$ passing through (0, 1, 1) and having d.r. {–2, 3, –1}

Since, the plane contains the line and the point

⇒ $a(1) + b(1) + c(1 - 2) = 0 \Rightarrow a + b - c = 0$ ...**(ii)**

Also the line and normal to the plane are perpendicular

⇒ $-2a + 3b - c = 0$ ...**(iii)**

Solving (ii) and (iii) $\dfrac{a}{2} = \dfrac{b}{3} = \dfrac{c}{5} = k$

Hence, required equation of the plane is

⇒ $2(x + 1) + 3(y - 0) + 5(z - 2) = 0$.

⇒ $2x + 3y + 5z - 8 = 0$

Method-II

Let the equation of the plane be $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ is passing through (–1, 0, 2)

⇒ $a(x + 1) + b(y - 0) + c(z - 2) = 0$ ...**(i)**

Given line $\dfrac{x-0}{-2} = \dfrac{y-1}{3} = \dfrac{z-1}{-1}$ passing through (0, 1, 1) and having d.r. {–2, 3, –1}

Since, the plane contains the line and the point

⇒ $a(1) + b(1) + c(1 - 2) = 0 \Rightarrow a + b - c = 0$ ...**(ii)**

Also the line and normal to the plane are perpendicular

⇒ $-2a + 3b - c = 0$ ...**(iii)**

Hence required equation of the plane is

$$\begin{bmatrix} x+1 & y & z-2 \\ 1 & 1 & -1 \\ -2 & 3 & -1 \end{bmatrix} = 0$$

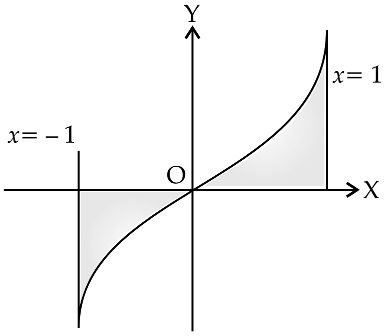
⇒ $2(x + 1) + 3(y - 0) + 5(z - 2) = 0$

⇒ $2x + 3y + 5z - 8 = 0$

18. **(i)** Given, the equation of the curve: $y = x|x|$

$$\therefore\ y = \begin{cases} -x^2, x < 0 \\ x^2, x \geq 0 \end{cases}$$

Hence, sketch of the region enclosed bounded by the curve, $y = x|x|$ and the ordinates $x = -1$ and $x = 1$ is as follows.



(ii) $\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$

(iii) $\therefore$ Area of the shaded region $= 2\int_0^1 x^2\,dx = \frac{2}{3}$ sq. units.

Section-C

19. (i) Option (b) is correct.

(ii) Option (d) is correct.

(iii) Given: $\bar{x} = 53$, $\bar{y} = 28$, $b_{yx} = -1.2$, $b_{xy} = -0.3$

Using, $r^2 = b_{yx} \times b_{xy}$

$= -1.2 \times -0.3 = 0.36$

$\Rightarrow \quad r = -0.6$

(iv) Given: $R(x) = 36x + 3x^2 + 5$

$$\Rightarrow \quad MR = \frac{d}{dy}\left(36x + 3x^2 + 5\right)$$

$= 36 + 6x$

$\therefore \quad MR\ (x = 5) = 66$

(v) Given, $C(x) = 210x + 7000$, $R(x) = 280x$

Minimum number must be sold daily when

$R(x) = C(x)$

$\Rightarrow \quad 210x + 7000 = 280x$

$\Rightarrow \quad 70x = 7000$

$\Rightarrow \quad x = 100.$

$\therefore$ Minimum number that must be sold is 100.

20. (i) $C(x) = 200 + 20x - \frac{1}{2}x^2$

$$\Rightarrow \quad AC = \frac{200}{x} + 20 - \frac{1}{2}x$$

To find AC falls :

$$\frac{d}{dx}(AC) = \frac{d}{dx}\left(\frac{200}{x} + 20 - \frac{1}{2}x\right)$$

$$= -\frac{200}{x^2} - \frac{1}{2} = -\left(\frac{200}{x^2} + \frac{1}{2}\right) < 0, \forall\ x > 0.$$

Hence, AC falls continuously.

OR

(ii) Given, $x = 100 - 4p$

$$\Rightarrow \quad p = \frac{100 - x}{4}$$

$$\therefore \quad R(x) = px = \frac{100x - x^2}{4}$$

$$\Rightarrow \quad MR = \frac{d}{dx}\left(\frac{100x - x^2}{4}\right) = \frac{100 - 2x}{4}$$

$\therefore \quad MR = 0$

$\therefore \quad x = 50$

21. (i) $\bar{x} = \frac{\Sigma y}{n} = 170$, $\quad \bar{y} = \frac{\Sigma y}{n} = 192$,

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = 0.6 \times \frac{20}{60} = 0.2$$

Regression equation y on x is

$y - 192 = 0.2\,(x - 170)$

$\Rightarrow y - 192 = 0.2x - 34$

$\Rightarrow \quad y = 0.2x + 158$

Put $\quad x = 200$

$\Rightarrow \quad y = 0.2(200) + 158 = 198$

$\therefore$ Expenditure on food and entertainment

= ₹ 198.

OR

(ii) (a) Regression lines are $3x + 2y - 26 = 0 \quad$ ...(i)

$6x + y - 31 = 0 \quad$ ...(ii)

$(\bar{x}, \bar{y})$ satisfies the equation (i) and (ii),

$\therefore$ Solving (i) and (ii) we get, $\bar{x} = 4$ and $\bar{y} = 7$.

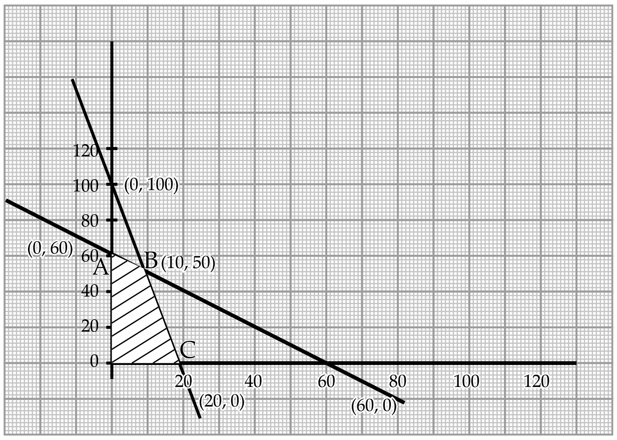
(b) On finding we get, $b_{xy} = -\frac{1}{6}, b_{yx} = -\frac{3}{2}$

$$\therefore \quad r^2 = b_{yx} \times b_{xy} = -\frac{1}{6} \times -\frac{3}{2} = \frac{1}{4}$$

$$\therefore \quad r = -\frac{1}{2}.$$

$\left[\because b_{yx}, b_{xy} \text{ and } r \text{ must have same sign}\right]$

22. (i) From graph, corner points are: A (0, 60) , B (10, 50), C (20, 0) , D (0, 0)



(ii) $Z = px + qy$

Given , Z is maximum at (0, 60) and (10, 50)

$\therefore\ 0.p + 60.\,q = 10.\,p + 50.q$

$\Rightarrow \quad 10p = 10q$

$\Rightarrow \quad p = q$

So, there can be infinite number of optimal solutions on the line joining BC.

Sample Question Paper-2

MATHEMATICS

Class-12

SOLVED

Time Allowed : 3 hours **Maximum Marks : 80**

You are allowed an additional 15 minutes for only reading the paper.
You must NOT start writing during reading time.
The question paper has 12 printed pages.
The Question Paper is divided into three sections and has 22 questions in all.

Section A is compulsory and has fourteen questions.
You are required to attempt all questions either from Section B or Section C.
Section B and Section C have four questions each.
Internal choices have been provided in two questions of 2 marks, two questions of 4 marks and two questions of 6 marks in Section A.
Internal choices have been provided in one question of 2 marks and one question of 4 marks each in Section B and Section C.
While attempting Multiple Choice Questions in Section A, B and C, you are required to write only ONE option as the answer.
The intended marks for questions or parts of questions are given in the brackets [].
All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
Mathematical tables and graph papers are provided.
Instruction to Supervising Examiner:
Kindly read aloud the instructions given above to all the candidates present in the examination hall.

Section-A [65 marks]

1. In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed. **[15]**

(i) If A and B are symmetric matrices of the same order, then AB – BA is

(a) Skew-symmetric matrix **(b)** Symmetric matrix
(c) Diagonal matrix **(d)** Identity matrix **[Understanding]**

(ii) The value of $\int x^{10}\ x\,dx$ equals

(a) $\frac{x^{12}}{12}+C$ **(b)** $\frac{x^{11}}{11}+C$ **(c)** $12x^{12}+C$ **(d)** None of these **[Recall]**

(iii) The value of $\operatorname{cosec}\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)-\sec\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$ equal to

(a) –4 **(b)** 0 **(c)** –1 **(d)** 4 **[Application]**

(iv) **Assertion (A):** The numbers of arbitrary constants in the particular solution of a differential equation of second order are 0.

Reason (R): The numbers of arbitrary constants in the general solution of a differential equation of fourth order are 4.

Which of the following is correct?

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true. **[Analysis]**

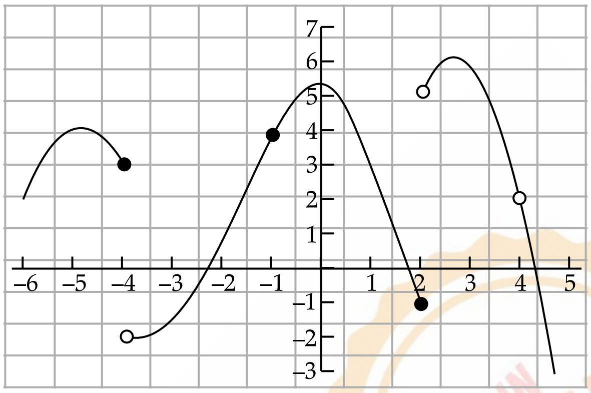
(v) Bhuvan is preparing chapati and curry for a dinner party at his home.
Let event A = Bhuvan prepares chapati well.

Let event B = Bhuvan cooks curry well.
Given $P(A) = 0.40$, $P(B) = 0.30$ and $P(A \text{ and } B) = 0.20$
Which one of the following is true about the events A and B?
(a) They are mutually exclusive but not independent.
(b) They are independent but not mutually exclusive.
(c) They are both mutually exclusive and independent.
(d) They are neither mutually exclusive nor independent. **[Application]**

(vi) If $\begin{vmatrix} p & q & r \\ u & v & w \\ l & m & n \end{vmatrix} = k$, then what is the value of $\begin{vmatrix} 12p & 3q & 3r \\ 4u & v & w \\ 4l & m & n \end{vmatrix}$?

(a) k **(b)** $12k$ **(c)** $4k$ **(d)** $3k$ **[Understanding]**

(vii) Shown below is the graph of a function, $f(x)$.



Statement (A): Points of discontinuity are (1, 3, – 6).

Statement (B): Points of discontinuity are (– 4, 2, 4).

Which of the following is correct?

(a) Statement (A) is true, and Statement (B) is false. **(b)** Statement (B) is true, and Statement (A) is false.

(c) Both the statements are true. **(d)** Both the statements are false. **[Application]**

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(viii) The derivative of log x with respect to $1/x$ is

(a) $\frac{1}{x}$ **(b)** $-\frac{1}{x^3}$ **(c)** $-\frac{1}{x}$ **(d)** $-x$

(ix) **Statement 1:** Let $A = \{1, 3, 7, 8\}$, $B = \{2, 6, 9\}$ and $f = \{(1, 2), (3, 6), (7, 2), (8, 6)\}$, then f is not a bijective function.
Statement 2: Let $A = \{1, 3, 7, 8\}$, $B = \{2, 6, 9\}$ and $f = \{(1, 2), (3, 6), (7, 2), (8, 6)\}$, then f is not a surjective function.
Which one of the following is correct?
(a) Statement 1 implies Statement 2. **(b)** Statement 2 implies Statement 1.
(c) Statement 1 is true only if Statement 2 is true. **(d)** Statements 1 and 2 are independent of each other.
[Understanding & Evaluation]

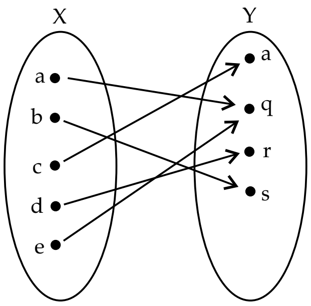
(x) **Assertion (A):** If $[a\ \ 2]\begin{vmatrix} 3 & 0 \\ -6 & 0 \end{vmatrix} = 0$, then $a = 2$.

Reason (R): If $[a\ \ 2]\begin{vmatrix} 3 & 0 \\ -6 & 0 \end{vmatrix} = 0$, then $a = 4$.

Which of the following is correct?
(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true. **[Application]**

(xi) If two balls are drawn from a bag containing 3 white, 4 black and 5 red balls. Then, the probability that the drawn balls are of different colours is:

(a) $\frac{1}{66}$ **(b)** $\frac{3}{66}$ **(c)** $\frac{19}{66}$ **(d)** $\frac{47}{66}$ **[Understanding]**



(xii) If $A = \begin{vmatrix} 2 & 5 & 8 \\ 3 & -4 & 0 \\ 8 & -1 & 7 \end{vmatrix}$, write the minor of the element a_{22}. **[Analysis]**

(xiii) Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then, justify why R is transitive but not symmetric. **[Understanding]**

(xiv) The function $f : X \to Y$ is not invertible. State the reason. **[Understanding]**

(xv) A and B appeared for an interview for two vacancies. The probability of A's selection is $\frac{4}{5}$ and B's selection is $\frac{1}{3}$. Find the probability that none of them will be selected. **[Application]**

2. **(i)** Find $\frac{dy}{dx}$, if $x = at^2$ and $y = 2at$. **A I** **[Understanding] [2]**

OR

(ii) The volume of a cube is increasing at the rate of 9 cm^3/s. How fast is the surface area increasing when the length of an edge is 10 cm. **[Understanding]**

3. Evaluate: $\int \cos^{-1}(\sin x)dx$ **[Evaluate] [2]**

4. Find the slope of tangent and normal to the curve $x^2 + 2y + y^2 = 0$ at $(-1, 2)$. **[Analysis] [2]**

5. Evaluate: $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}dx$ **A I** **[2]**

OR

Evaluate: $\int_1^3 \frac{dx}{9+x^2}$ **[Evaluate]**

6. If $\left(\tan^{-1}x\right)^2 + \left(\cot^{-1}x\right)^2 = \frac{5\pi^2}{8}$, find x. **[Application] [2]**

7. Evaluate: $\cos\left(2\cos^{-1} x + \sin^{-1} x\right)$ at $x = \frac{1}{5}$. **A I** **[Application] [4]**

8. Evaluate: $\int \frac{x(1+x^2)}{1+x^4}dx$ **A I** **[Evaluate] [4]**

9. **(i)** Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x(2\log x+1)}{\sin y + y\cos y}$ given that $y = \frac{\pi}{2}$ where $x = 1$. **[4]**

OR

(ii) If $y = 3\cos(\log x) + 4\sin(\log x)$, show that

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

10. **(i)** Two horses are considered for a race. The probability of selection of the first horse is $\frac{1}{4}$ and that of the second is $\frac{1}{3}$. What is the probability that:

(a) both of them will be selected.

(b) only one of them will be selected.

(c) none of them will be selected. **A I** **[4]**

OR

(ii) During a festival, two friends, Shanti and Diya, received identical gift bag containing 7 dark chocolates, 8 milk chocolates and 5 white chocolates each.

(a) What is the probability of Diya drawing a dark chocolate from Shanti's bag?

(b) While Shanti was away, Diya transferred one chocolate from Shanti's bag to hers without looking. What is the probability of drawing a dark chocolate from Diya's bag after the transfer?

(c) Which one of the following probabilities is higher? Why?

The probability of drawing a dark chocolate from Shanti's bag before the transfer.

11. A furniture factory uses three types of wood namely, teakwood, rosewood and satinwood for manufacturing three types of furniture, that are, table, chair and cot. The wood requirements (in tonnes) for each type of furniture are given below: **[6]**

	Table	**Chair**	**Cot**
Teakwood	2	3	4
Rosewood	1	1	2
Satinwood	3	2	1

It is found that 29 tonnes of teakwood, 13 tonnes of rosewood and 16 tonnes of satinwood are available to make all three types of furniture.

Using the above information, answer the following questions:

(i) Express the data given in the table above in the form of a set of simultaneous equations.
(ii) Solve the set of simultaneous equations formed in subpart (i) by matrix method.
(iii) Hence, find the number of table(s), chair(s) and cot(s) produced.

12. (i) Solve the following differential equation:

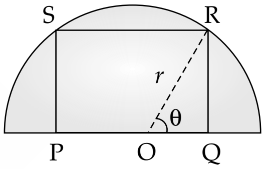
$ye^{y}dx = (y^3 + 2xe^{y})dy$, given that $x = 0$, $y = 1$ **AI** **[6]**

OR

(ii) Evaluate : $\int_0^{\frac{3}{2}} |x\cos \pi x|\, dx$. **AI**

13. (i) A given quantity of metal is to be cast into a solid half circular cylinder with a rectangular base and semi-circular ends. Show that in order that total surface area is minimum, the ratio of length of cylinder to the diamter of semi-circular ends is π : π : 2. **[6]**

OR



(ii) A rectangle is inscribed in a semicircle of radius ***r*** with one of its sides on the diameter of the semicircle. Find the dimensions of the rectangle to get maximum area. Also, find the maximum area.

14. A primary school teacher wants to teach the concept of 'larger number' to the students of Class II.

To teach this concept, he conducts an activity in his class. He asks the children to select two numbers from a set of numbers given as 2, 3, 4, 5 one after the other without replacement. All the outcomes of this activity are tabulated in the form of ordered pairs given below: **[6]**

	2	**3**	**4**	**5**
2	(2, 2)	(2, 3)	(2, 4)	
3	(3, 2)	(3, 3)		(3, 5)
4	(4, 2)		(4, 4)	(4, 5)
5		(5, 3)	(5, 4)	(5, 5)

(i) Complete the table given above.
(ii) Find the total number of ordered pairs having one larger number.
(iii) Let the random variable X denote the larger of two numbers in the ordered pair. Now, complete the probability distribution table for X given below.

X	3	4	5
$P(X = x)$			

(iv) Find the value of P(X < 5)
(v) Calculate the expected value of the probability distribution.

Section-B [15 marks]

15. In subparts (i) to (iii) choose the correct options and in subparts (iii) and (v), answer the questions as instructed.**[5]**

(i) Consider the following statements and choose the correct option:

Statement 1: If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ a non-zero scalar, then $\lambda\vec{a}$ is unit vector if $a = |\lambda|$.

Statement 2: If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ a non-zero scalar, then $\lambda\vec{a}$ is unit vector if $a = \frac{1}{|\lambda|}$.

Which of the following is correct?

(a) Only Statement 1
(b) Only Statement 2
(c) Both Statements 1 and 2
(d) Neither Statement 1 nor Statement 2 AI

(ii) If a line makes an angle α, β, γ with X-axis, Y-axis and Z-axis respectively, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is:

(a) -1 **(b)** 1 **(c)** 0 **(d)** 2

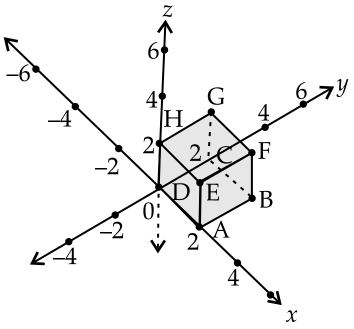
(iii) The equation of the plane which is parallel to $2x - 3y + z = 0$ and which passes through (1, –1, 2) is:

(a) $2x - 3y + z - 7 = 0$ **(b)** $2x - 3y + z + 7 = 0$ **(c)** $2x - 3y + z - 8 = 0$ **(d)** $2x - 3y + z + 6 = 0$ AI

(iv) Find the angle between two vectors $\vec{a}$ and $\vec{b}$ having the same length $\sqrt{2}$ and their vector product is $-\hat{i} - \hat{j} + \hat{k}$. AI

(v) A cube is shown alongside:

Find the equation of its diagonal which passes through the centre of the cube and also the origin.



16. **(i)** Write a vector of magnitude of 27 units in the direction of the vector $\hat{i} + 2\hat{j} + 2k$. AI [2]

(ii) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $(\vec{r} \times \hat{i}).(\vec{r} \times \hat{j}) + xy$. AI

17. **(i)** Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ are coplanar. AI [4]

OR

(ii) Find the equation of a line passing through the points P(–1, 3, 2) and Q(–4, 2, –2). Also, if the point R(5, 5, λ) is collinear with the points P and Q, then find the value of λ. AI

18. **(i)** Sketch the region enclosed bounded by the curve, $y^2 = 4ax$ and the line $x = a$.

(ii) Evaluate: $\int_0^a y\,dx$

(iii) Hence, find the area bounded by the curve, $y^2 = 4ax$ and the line $x = a$. AI [4]

Section-C [15 marks]

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) Which condition is true if AC decreases with x and AC curve is falling.

(a) $MC < AC$ **(b)** $MC = AC$ **(c)** $MC > AC$ **(d)** None of these **[5]**

(ii) Read the following statements and choose the correct option:

(I) The purpose of regression is to maximise the r^2.
(II) The purpose of regression is to explain the mean of the independent variable.
(III) The purpose of regression is to explain the variation of the dependent variable.
(IV) The purpose of regression is to explain the variation of the independent variable.

Which of the following is correct?

(a) Only III is correct
(b) Only III and II are correct
(c) Only I and IV are correct
(d) Only III and IV are correct

(iii) Out of the two regression lines $x + 2y - 5 = 0$ and $2x + 3y = 8$, find the line of regression of y on x.

(iv) The cost function $C(x) = 3x^2 - 6x + 5$. Find the average cost when $x = 2$. AI

(v) The cost of overhaul of an engine is ₹10,000. The operating cost per hour is at the rate of $2x - 240$ where the engine has run x km. Find out the total cost if the engine run for 300 hours after overhaul.

20. **(i)** The selling price of a commodity is fixed at ₹60 and its cost function is: $C(x) = 35x + 250$

(a) Determine the profit function.

(b) Find the break-even points. AI [2]

OR

(ii) The average revenue function is given by $AR = 25 - \frac{x}{4}$.

Find total revenue function and marginal revenue function. A I

21. **(i)** An analyst analysed 102 trips of a travel company. He studied the relation between travel expenses **(*y*)** and the duration **(*x*)** of these trips. He found that the relation between ***x*** and ***y*** was linear. Given the following data, find the regression equation of y on x.

$\Sigma x = 510, \Sigma y = 7140, \Sigma x^2 = 4150, \Sigma y^2 = 7{,}40{,}200, \Sigma xy = 54{,}900$ **[4]**

OR

(ii) The correlation coefficient between x and y is 0.6. If the variance of x is 225, the variance of y is 400, mean of x is 10 and mean of y is 20, find

(a) the equations of two regression lines,

(b) the expected value of y when $x = 2$.

22. A dietician has to develop a special diet using two foods P and Q. Each packet of food P contains 12 units of calcium, 4 units of Iron, 6 units of cholesterol and 6 units of Vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of Iron, 4 units of cholesterol and 3 units of Vitamin A. The diet requires atleast 240 units of calcium, at least 460 units of iron and atmost 300 units of cholesterol.

(i) How many packets of each food should be used to minimise the amount of Vitamin A? What is the minimum amount of Vitamin A?

(ii) How many packets of each food should be used to maximise the amount of Vitamin A in the diet ? What is the maximum amount of Vitamin A. **[4]**

■■

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SOLUTIONS

Sample Question Paper - 2

Section-A

(i) Option (a) is correct.

Let A and B be symmetric matrices then $A = A^T$ and $B = B^T$

$$\begin{aligned}(AB-BA)^T &= (AB)^T-(BA)^T\\ &= (B^TA^T)-(A^TB^T)\\ &= BA-AB\\ &= -(AB-BA)\end{aligned}$$

Hence, it is skew-symmetric.

(ii) Option (a) is correct.

Explanation: $\int x^{10}\ \ x\,dx$

$$= \int x^{10+1}dx$$

$$= \int x^{11}\,dx$$

$$= \frac{x^{11+1}}{11+1}+C$$

$$= \frac{x^{12}}{12}+C$$

(iii) Option (b) is correct.

Explanation: $\operatorname{cosec}\left(\sin^{-1}\left(-\frac{1}{2}\right)-\sec\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)\right)$

$$= \operatorname{cosec}\left(\sin^{-1}\left(-\sin\frac{\pi}{6}\right)\right)-\sec\left(\cos^{-1}\left(\cos\frac{2\pi}{3}\right)\right)$$

$$= \operatorname{cosec}\left(-\frac{\pi}{6}\right)-\sec\left(\frac{2\pi}{3}\right)$$

$$= -\operatorname{cosec}\frac{\pi}{6}+\sec\frac{\pi}{3}$$

$$= -2+2=0$$

(iv) Option (b) is correct.

***Explanation*: For Assertion:** The number of constants in the particular solution of a differential equation of order n is zero.

For Reason: We know that the number of constants in the general solution of a differential equation of order n is equal to its order.

Therefore, the number of constants in the general solution of fourth-order differential equation is four.

(v) Option (d) is correct.

Explanation: Since, $P(A) = 0.40$, $P(B) = 0.30$ and $P(A \text{ and } B) = 0.20$

$P(A \text{ and } B) = P(A\cap B) \neq 0$

The events are not mutually exclusive.

And $P(A).P(B) = 0.40 \times 0.30 = 0.12 \neq P(A\cap B)$

The events are not independent.

Examiner's Comment

- Due to the lack of concepts, few students are not able to differentiate between mutually exclusive and independent events.

Answering Tip

- To score well in the Topic of Probability, the students must have clear concepts of sets, operations on sets and better understanding of the terms: random experiment, outcomes, sample space, trials, events, types of events, probability, conditional probability, independent events, etc.

(vi) Option (b) is correct.

Given, $\begin{vmatrix} p & q & r\\ u & v & w\\ l & m & n\end{vmatrix} = k$ ...(i)

Now $\begin{vmatrix} 12p & 3q & 3r\\ 4u & v & w\\ 4l & m & n\end{vmatrix}$

$$= 3\begin{vmatrix} 4p & q & r\\ 4u & v & w\\ 4l & m & n\end{vmatrix}$$

(By taking 3 common from 1st row)

$$= 3\times 4\begin{vmatrix} p & q & r\\ u & v & w\\ l & m & n\end{vmatrix}$$

(By taking 4 common from 1st column)

$= 3\times4\times k$ (Using (i))

$=12k$

(vii) Option (b) is correct.

Explanation: Before starting the solution recall that in order for a function to be continuous at $x=a$ both $f(a)$ and $\lim f(x)$ must exist and we must have,

$$\underset{x\to a}{\log} f(x)=f(a)$$

Using this idea it should be fairly clear where the function is not continuous.

First, at $x = -4$ we have,

$\log_{x\to -4^-} - f(x) = 3 \neq -2 = \log_{x\to -4^+} f(x)$ and therefore, we also know that $\log_{x\to -4} f(x)$ doesn't exist. We can therefore conclude that $f(x)$ is discontinuous at $x=-4$ because the limit does not exist.

Likewise, at $x=2$ we have,

$\log_{x\to -2^-} - f(x) = -1 \neq 5 = \log_{x\to 2^+} f(x)$

and therefore, we also know that $\log_{x\to 2} f(x)$ doesn't exist. So again, because the limit does not exist, we can see that $f(x)$ is discontinuous at $x=2$.

Finally let's take a look at $x=4$. Here we can see that,

$\log_{x\to -4^-} - f(x) = 2 = \log_{x\to -4^+} f(x)$ and therefore, we also know that $\log_{x\to -4^-} f(x) = 2$. However, we can also see that $f(4)$ doesn't exist and so once again $f(x)$ is discontinuous at $x=4$ because this time the function does not exist at $x=4$.

All other points on this graph will have both the function and limit exist and we'll have $\log_{x\to a} f(x) = f(a)$ and so will be continuous.

In summary then the points of discontinuity for this graph are : $x=-4$, $x=2$ and $x=4$.

Hence, Statement 2 is true, and Statement 1 is false

(viii) Option **(d)** is correct.

Explanation : Let $u = \log x$ and $v = \dfrac{1}{x}$

$$\therefore \quad \frac{du}{dx} = \frac{1}{x}$$

$$\textbf{and} \quad \frac{dv}{dx} = -\frac{1}{x^2}$$

$$\textbf{Thus,} \quad \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$$

$$= \frac{\frac{1}{x}}{-\frac{1}{x^2}} = -x$$

(ix) Option (c) is correct.

Explanation: We have, $A = \{1, 3, 7, 8\}$, $B = \{2, 6, 9\}$ and $f = \{(1, 2), (3, 6), (7, 2), (8, 6)\}$

So, element 9 of B does not have a domain element on A

Therefore, $f : A \to B$ is not a surjective function.

Also, for a bijective function, f must be both one-to-one and onto.

Thus, Statement 1 is true only if Statement 2 is true.

(x) Option (d) is correct.

Explanation: On solving we get,

$$3a - 12 = 0$$

$$a = 4$$

So, assertion is false and reason is true.

(xi) Option (d) is correct.

Explanation: Required probability that the drawn balls are of different colour

$$= \frac{{}^3C_1 \times {}^4C_1}{{}^{12}C_2} + \frac{{}^3C_1 \times {}^5C_1}{{}^{12}C_2} + \frac{{}^4C_1 \times {}^5C_1}{{}^{12}C_2}$$

$$= \frac{3\times 4}{\frac{12\times 11}{2}} + \frac{3\times 5}{\frac{12\times 11}{2}} + \frac{4\times 5}{\frac{12\times 11}{2}}$$

$$= \frac{12}{66} + \frac{15}{66} + \frac{20}{66} = \frac{47}{66}$$

(xii) Given

$$A = \begin{vmatrix} 2 & 5 & 8 \\ 3 & -4 & 0 \\ 8 & -1 & 7 \end{vmatrix},$$

$$\therefore \quad \text{Minor of } a_{22} = \begin{vmatrix} 2 & 8 \\ 8 & 7 \end{vmatrix}$$

$$= 14 - 64 = -50$$

(xiii) $aRb \Rightarrow a$ is brother of b.

This does not mean b is also a brother of a as b can be a sister of a.

Hence, R is not symmetric.

$aRb \to a$ is brother of b

and $bRc \to b$ is a brother of c.

So, a is brother of c.

Hence, R is transitive.

(xiv) For a function to have an inverse, each element $b \in B$ must not have more than one pre-image in $a \in A$.

But here, Y has more than one pre-image in X. Thus, function $f : X \to Y$ is not invertible.

(xv) Event $A \to A$ will be selected.

Event $B \to B$ will be selected.

$$\therefore \quad P(A) = \frac{4}{5} \;\&\; P(B) = \frac{1}{3}$$

$\therefore$ Probability that none of them will be selected

$$= P(A').P(B')$$

$$= [1 - P(A)][1 - P(B)]$$

$$= \left(1 - \frac{4}{5}\right)\left(1 - \frac{1}{3}\right) = \frac{2}{15}$$

2. (i) $x = at^2$

$$\therefore \quad \frac{dx}{dt} = 2at$$

$$y = 2at$$

$$\therefore \quad \frac{dy}{dt} = 2a$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

OR

(ii) Let x denote the edge of cube, V denote the volume and s denotes the surface area of cube at instant t.

Given $\frac{dV}{dt} = 9\text{ cm}^3/\text{s}$

$\because \quad V = x^3$

$\Rightarrow \quad \frac{dV}{dt} = 3x^2\frac{dx}{dt}$

$\Rightarrow \quad \frac{9}{3x^2} = \frac{dx}{dt}$

Now, $\quad S = 6x^2$

$\therefore \quad \frac{dS}{dt} = 12x \times \frac{dx}{dt}$

$$\Rightarrow \left(\frac{dS}{dt}\right)_{x=10} = 12\times10\times\frac{9}{3\times10\times10}$$

$$= \frac{12\times9}{3\times10} = \frac{36}{10}$$

$$= 3.6\text{ cm}^2/\text{s}$$

3. Let $\cos^{-1}(\sin x) = \theta$

$\Rightarrow \quad \sin x = \cos\theta$

$\Rightarrow \quad \sin x = \sin\left(\frac{\pi}{2}-\theta\right)$

$\Rightarrow \quad x = \frac{\pi}{2}-\theta$

$\Rightarrow \quad \theta = \frac{\pi}{2}-x$

$\therefore \quad I = \int\theta\,dx$

$$= \int\left(\frac{\pi}{2}-x\right)dx$$

$$= \frac{\pi}{2}x-\frac{x^2}{2}+C$$

[ISC Marking Scheme, 2023]

4. Given,

$$x^2 + 2y + y^2 = 0$$

$\Rightarrow \quad 2x+2\frac{dy}{dx}+2y\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx}(2+2y) = -2x$

$\Rightarrow \quad \frac{dy}{dx} = \frac{-2x}{2(1+y)} = -\frac{x}{1+y}$

Slope of tangent at (– 1, 2)

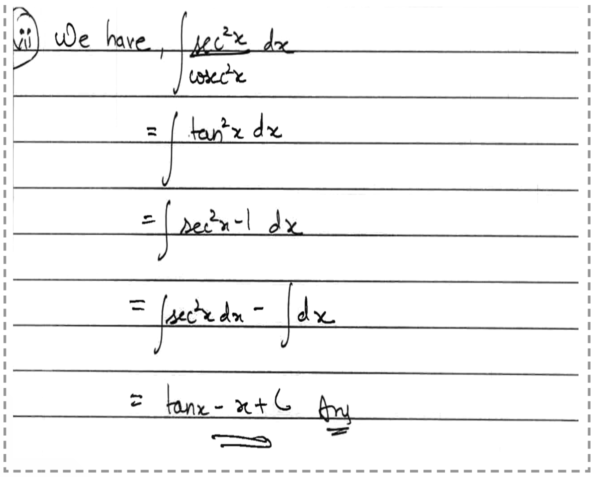
$$\left[\frac{dy}{dx}\right]_{(-1,\,2)} = \frac{-(-1)}{1+2} = \frac{1}{3}$$

Slope of normal at (– 1, 2)

$$\frac{-1}{\left[\frac{dy}{dx}\right]_{(-1,\,2)}} = -\frac{3}{1} = -3.$$

5. (i)

Topper Answer, 2019



(ii) We have, $\int\frac{\sec^2x}{\operatorname{cosec}^2x}\,dx$

$= \int\tan^2x\,dx$

$= \int\sec^2x-1\,dx$

$= \int\sec^2x\,dx - \int dx$

$= \tan x - x + C$ Ans

OR

(ii) $I = \int_1^3\frac{dx}{9+x^2}$

$$= \int_1^3\frac{dx}{3^2+x^2}$$

$$= \frac{1}{3}\left[\tan^{-1}\left(\frac{x}{3}\right)\right]_1^3$$

$$= \frac{1}{3}\left[\tan^{-1}(1)-\tan^{-1}\frac{1}{3}\right]$$

$$= \frac{1}{3}\left[\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{3}\right)\right]$$

Examiner's Comment

- Sometimes students attempt it by other methods of integration and could not reach the proper result.

Answering Tip

- Learn all the special integrals thoroughly.

6. $$(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$$

or $$(\tan^{-1}x)^2 + \left(\frac{\pi}{2}-\tan^{-1}x\right)^2 = \frac{5\pi^2}{8}$$

or $$2(\tan^{-1}x)^2 - \pi\tan^{-1}x + \frac{\pi^2}{4}-\frac{5\pi^2}{8} = 0$$

or $$2(\tan^{-1}x)^2 - \pi\tan^{-1}x - \frac{3\pi^2}{8} = 0$$

or $$\tan^{-1}x = \frac{\pi\pm\sqrt{\pi^2+3\pi^2}}{4}$$

or $$\tan^{-1}x = \frac{3\pi}{4}, \frac{-\pi}{4}$$

$\therefore$ $$x = -1$$

7. $\cos(2\cos^{-1}x + \sin^{-1}x)$ at $x = \frac{1}{5}$

$$= \cos(\cos^{-1}x + \sin^{-1}x + \cos^{-1}x)$$

$$= \cos\left(\frac{\pi}{2} + \cos^{-1}x\right)$$

$$= -\sin(\cos^{-1}x)$$

$$= -\sin\left[\sin^{-1}\left(\sqrt{1-x^2}\right)\right]$$

$$= -\left(\sqrt{1-x^2}\right)$$

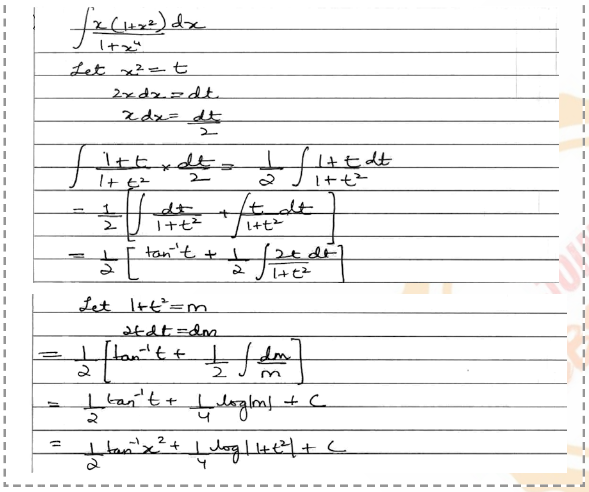
$$= -\left(\sqrt{1-\frac{1}{25}}\right)$$

$$= -\sqrt{\frac{24}{25}} = -\frac{2\sqrt{6}}{5}$$

[ISC Marking Scheme, 2020]

8.

Topper Answer's, 2019



$\int \frac{x(1+x^2)}{1+x^4}dx$

Let $x^2 = t$

$2x\,dx = dt$

$x\,dx = \frac{dt}{2}$

$\int \frac{1+t}{1+t^2} \times \frac{dt}{2} = \frac{1}{2}\int \frac{1+t}{1+t^2}dt$

$= \frac{1}{2}\left[\int \frac{dt}{1+t^2} + \int \frac{t\,dt}{1+t^2}\right]$

$= \frac{1}{2}\left[\tan^{-1}t + \frac{1}{2}\int \frac{2t\,dt}{1+t^2}\right]$

Let $1+t^2 = m$

$2t\,dt = dm$

$= \frac{1}{2}\left[\tan^{-1}t + \frac{1}{2}\int \frac{dm}{m}\right]$

$= \frac{1}{2}\tan^{-1}t + \frac{1}{4}\log|m| + C$

$= \frac{1}{2}\tan^{-1}x^2 + \frac{1}{4}\log|1+t^2| + C$

9. (i) The given differential equation is :

$$\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$$

$$\Rightarrow \quad (\sin y + y\cos y)\,dy = x(2\log x + 1)\,dx$$

$$\Rightarrow \quad \int(\sin y + y\cos y)\,dy = \int x(2\log x + 1)\,dx$$

On integrating both sides,

$$\Rightarrow \quad -\cos y + y\sin y + \cos y = x^2\log x - \frac{x^2}{2} + \frac{x^2}{2} + c$$

$$\Rightarrow \quad y\sin y = x^2\log x + c \qquad \text{...(i)}$$

It is given that when $y = \frac{\pi}{2}, x = 1$

So, putting $y = \frac{\pi}{2}, x = 1$ in (i), we get

$$\Rightarrow \quad \frac{\pi}{2}\sin\frac{\pi}{2} = 1.\log 1 + \frac{1}{2} + c$$

$$\Rightarrow \quad c = \frac{\pi - 5/2}{2}$$

Putting $c = \frac{\pi}{2}$ in (i), we obtain

$$\Rightarrow \quad y\sin y = x^2\log x + \frac{\pi - 5/2}{2}$$

which is the required solution.

OR

(ii) $y = 3\cos(\log x) + 4\sin(\log x)$

$$\frac{dy}{dx} = -3\sin(\log x)\,\frac{d}{dx}\log x + 4\cos(\log x)\,\frac{d}{dx}\log x$$

$$\frac{dy}{dx} = \frac{-3\sin(\log x)}{x} + \frac{4\cos(\log x)}{x}$$

$$\frac{xdy}{dx} = -3\sin(\log x) + 4\cos(\log x)$$

$$\frac{xd^2y}{dx^2} + \frac{dy}{dx} = -\frac{3\cos(\log x)}{x} - \frac{4\sin(\log x)}{x}$$

$$\frac{x^2d^2y}{dx^2} + \frac{xdy}{dx} = (3\cos(\log x) + 4\sin(\log x))$$

$$\frac{x^2d^2y}{dx^2} + \frac{xdy}{dx} + y = 0$$

Hence Proved

10. Let the probability of selecting first horse

$$= P(A) = \frac{1}{4}$$

and the probability of selecting second horse

$$= P(B) = \frac{1}{3}$$

then, $P(\overline{A}) = 1 - P(A) = 1 - \frac{1}{4} = \frac{3}{4}$

and, $P(\overline{B}) = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$

(a) P(Both of them selected) $= P(A).P(B)$

$$= \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

(b) P(Only one selected) $= P(A)P(\overline{B}) + P(\overline{A})P(B)$

$$= \frac{1}{4} \times \frac{2}{3} + \frac{1}{3} \times \frac{3}{4} = \frac{5}{12}$$

(c) P(None of them selected) $= P(\overline{A})P(\overline{B})$

$$= \frac{3}{4} \times \frac{2}{3}$$

$$= \frac{1}{2}$$

OR

(ii) (a) P(Diya drawing dark chocolate from Shanti's box)

$$= \frac{7}{20}$$

(b) E_1 = drawing dark chocolate from Shanti's bag

E_2 = drawing milk chocolate from Shanti's bag

E_3 = drawing white chocolate from Shanti's bag

A = after transfer chocolate from Shanti's bag to Diya's bag then draw 1 dark chocolate from Diya's bag

$$P(E_1) = \frac{7}{20}$$

$$P(E_2) = \frac{8}{20}$$

$$P(E_3) = \frac{5}{20}$$

$$P(A/E_1) = \frac{8}{21}$$

$$P(A/E_2) = \frac{7}{21}$$

$$P(A/E_3) = \frac{7}{21}$$

$$P(A) = P(E_1).P(A/E_1)+P(E_2).P(A/E_2)+P(E_3).P(A/E_3)$$

$$P(A) = \frac{7}{20}\times\frac{8}{21}+\frac{8}{20}\times\frac{7}{21}+\frac{5}{20}\times\frac{7}{21}$$

$$= \frac{56+56+35}{20\times 21} = \frac{147}{420} = \frac{7}{20}$$

(c) P(drawing a dark chocolate from Diya's bag after transfer) $= \frac{7}{20}$

OR

P(drawing a dark chocolate from Shanti's bag before transfer) $= \frac{7}{20}$

Both are same.

[ISC Specimen Paper Marking Scheme 2024]

Examiner's Comment

- The concept of Bayes' theorem was clear to most candidates but some candidates found in correct probability. While some candidates found conditional probability for the happening of an event incorrectly, even probability of a specific known event was found wrongly by a few candidates.

Answering Tip

- Learn Bayes' theorem with proper explanation and illustration. Pay heed to the laws of total probability. Do adequate practice of Bayes theorem.

11. **(i)** Let the number of table, chair and cot be x, y and z respectively

$$2x + 3y + 4z = 29$$
$$x + y + 2z = 13$$
$$3x + 2y + z = 16$$

(ii) Equations can be rearranged in matrix form

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 29 \\ 13 \\ 16 \end{bmatrix}$$

$$AX = B \qquad \text{Where } A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A^{-1}AX = A^{-1}B \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 29 \\ 13 \\ 16 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} + 4\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}$$

$$= -6 + 15 - 4 = 5$$

$C_{11} = (1-4) = -3$

$C_{12} = -(1-6) = 5$

$C_{13} = (2-3) = -1$

$C_{21} = -(3-8) = 5$

$C_{22} = (2-12) = -10$

$C_{23} = -(4-9) = 5$

$C_{31} = (6-4) = 2$, $C_{32} = -(4-4) = 0$, $C_{33} = 2-3 = -1$

$$\text{Adj } A = \begin{bmatrix} -3 & 5 & -1 \\ 5 & -10 & 5 \\ 2 & 0 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 5 & 2 \\ 5 & -10 & 0 \\ -1 & 5 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{1}{5}\begin{bmatrix} -3 & 5 & 2 \\ 5 & -10 & 0 \\ -1 & 5 & -1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -3 & 5 & 2 \\ 5 & -10 & 0 \\ -1 & 5 & -1 \end{bmatrix}\begin{bmatrix} 29 \\ 13 \\ 16 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 10 \\ 15 \\ 20 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$x = 2, y = 3, z = 4$

(iii) No. of chairs = 2

No of tables = 3

No, of Cots = 4

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12.(i) Given, $y e^{y}\, dx = (y^3 + 2x e^{y})\, dy$

$$\Rightarrow \quad y e^{y} \frac{dx}{dy} = y^3 + 2x e^{y}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{2}{y}.x + y^2 e^{-y}$$

$$\Rightarrow \quad \frac{dx}{dy} - \frac{2}{y}.x = y^2 e^{-y} \qquad \ldots(i)$$

It is a linear differential equation in the form of $\frac{dx}{dy} + Px = Q$.

$$P = -\frac{2}{y} \text{ and } Q = y^2 e^{-y}$$

$$I.F. = e^{\int -\frac{2}{y}.dy} \qquad (\because I.F. = e^{\int P\,dy})$$

$$= e^{-2\log|y|}$$

$$= e^{\log|y|^{-2}}$$

$$= |y^{-2}| = \frac{1}{y^2}$$

Therefore, general solution of (i) is

$$x.I.F. = \int Q \times I.F.\, dy + c$$

$$\Rightarrow \quad x.\frac{1}{y^2} = \int y^2 e^{-y}.\frac{1}{y^2}\, dy + c$$

$$\Rightarrow \quad x\cdot\frac{1}{y^2} = \int e^{-y}dy + c$$

$$\Rightarrow \quad x\cdot\frac{1}{y^2} = \frac{e^{-y}}{-1} + c \qquad ...(ii)$$

Given that $x = 0$ when $y = 1$

$$\Rightarrow \quad 0 = -e^{-1} + c$$

$$\Rightarrow \quad c = e^{-1}$$

Substituting the value of c in (ii), the required particular solution of the given equation is

$$\frac{x}{y^2} = -e^{-y} + e^{-1}$$

i.e., $x = y^2(-e^{-y} + e^{-1})$

OR

(ii) Let $I = \int_0^{\frac{3}{2}} |x\cos\pi x|\,dx$

In $0 \le x \le \frac{1}{2}, x\cos\pi x \ge 0,\ x \ge 0$

In $\frac{1}{2} \le x \le \frac{3}{2}, x\cos\pi x \le 0,\ x \ge 0$

$$\Rightarrow \quad I = \int_0^{\frac{1}{2}} x\cos\pi x\,dx - \int_{\frac{1}{2}}^{\frac{3}{2}} x\cos\pi x\,dx$$

$$= \left[x\frac{(\sin\pi x)}{\pi} - \int(1)\cdot\frac{(\sin\pi x)}{\pi}dx\right]_0^{\frac{1}{2}} - \left[x\frac{(\sin\pi x)}{\pi} - \int(1)\cdot\frac{(\sin\pi x)}{\pi}dx\right]_{\frac{1}{2}}^{\frac{3}{2}}$$

$$\Rightarrow \quad I = \left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{(\pi)^2}\right]_0^{\frac{1}{2}} - \left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{(\pi)^2}\right]_{\frac{1}{2}}^{\frac{3}{2}}$$

$$= \left[\frac{\frac{1}{2}\sin\frac{\pi}{2}}{\pi} + \frac{\cos\frac{\pi}{2}}{\pi^2} - 0 - \frac{\cos 0}{\pi^2}\right] - \left[\frac{\frac{3}{2}\sin\frac{3\pi}{2}}{\pi} + \frac{\cos\frac{3\pi}{2}}{\pi^2} - \frac{\frac{1}{2}\sin\frac{\pi}{2}}{\pi} - \frac{\cos\frac{\pi}{2}}{\pi^2}\right]$$

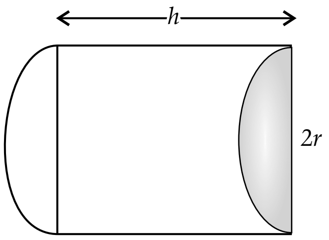
$$\Rightarrow \quad I = \left[\frac{1}{2\pi} - \frac{1}{\pi^2}\right] - \left[\frac{3}{2\pi}(-1) + 0 - \frac{1}{2\pi} - 0\right]$$

$$= \left[\frac{1}{2\pi} - \frac{1}{\pi^2} + \frac{3}{2\pi} + \frac{1}{2\pi}\right]$$

$$\therefore \quad I = \frac{1}{\pi}\left[\frac{5}{2} - \frac{1}{\pi}\right]$$

13. **(i)** Let r be the radius and h be the height of half cylinder

$$\text{Volume} = \frac{1}{2}\pi r^2 h = \text{V (constant)} \qquad ....(1)$$



Total surface area of half cylinder is

$$\text{S} = 2\left(\frac{1}{2}\pi r^2\right) + \pi rh + 2rh \qquad ....(2)$$

From (1) put the value of h in (2)

$$\text{S} = (\pi r^2) + \pi r\left(\frac{2V}{\pi r^2}\right) + 2r\left(\frac{2V}{\pi r^2}\right)$$

$$\text{S} = (\pi r^2) + \left(\frac{1}{r}\right)\left[\frac{4V}{\pi} + 2V\right]$$

$$\frac{dS}{dr} = (2\pi r) + \left(\frac{-1}{r^2}\right)\left[\frac{4V}{\pi} + 2V\right] \qquad ....(3)$$

From maxima/minima $\frac{dS}{dr} = 0$

$$\Rightarrow (2\pi r) + \left(\frac{-1}{r^2}\right)\left[\frac{4V}{\pi} + 2V\right] = 0$$

$$\Rightarrow \quad (2\pi r) = \left(\frac{1}{r^2}\right)\left[\frac{4V + 2V\pi}{\pi}\right]$$

$$\Rightarrow \quad \pi r^3 = V\left[\frac{2+\pi}{\pi}\right]$$

$$\Rightarrow \quad V = \frac{\pi^2 r^3}{\pi + 2} \qquad ....(4)$$

From (1) and (4),

$$\Rightarrow \quad \frac{1}{2}\pi r^2 h = \frac{\pi^2 r^3}{\pi + 2}$$

$$\Rightarrow \quad \frac{h}{2r} = \frac{\pi}{\pi + 2}$$

$\Rightarrow$ height : diameter $= \pi : (\pi + 2)$

Differentiating (3) with respect to r

$$\frac{d^2S}{dr^2} = (2\pi) + \left(\frac{2}{r^3}\right)\left[\frac{4V}{\pi} + 2V\right] = \text{positive}$$

(as all quantities are +ve)

So, S is minimum when

height : diameter $= \pi : (\pi + 2)$ Hence Proved

OR

(ii) $OQ = r\cos\theta$

$QR = r\sin\theta$

$A(\theta)$ = Area of the rectangle

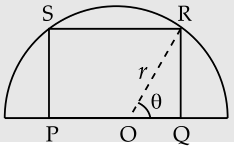
$= 2OQ \times QR$

$= 2\,r\cos\theta . r\sin\theta$

$= r^2 \sin 2\theta$

For maximum and minimum area

$$\frac{dA}{d\theta} = 2r^2\cos 2\theta = 0$$



$$\cos 2\theta = 0$$

$$\Rightarrow \quad 2\theta = \frac{\pi}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{4} \text{ is the critical point}$$

$$\frac{d^2A}{d\theta^2} = -4r^2 \sin 2\theta$$

$$\left(\frac{d^2A}{d\theta^2}\right)_{\theta=\frac{\pi}{4}} = -4r^2 < 0$$

$\Rightarrow$ Area is maximum at $\theta = \frac{\pi}{4}$

So, sides of rectangle are

$$QR = r\sin\theta = r.\sin\frac{\pi}{4} = \frac{r}{\sqrt{2}} = \frac{\sqrt{2}r}{2}$$

and $PQ = 2OQ = 2r\cos\theta = 2r\cos\left(\frac{\pi}{4}\right)$

$$= 2r\frac{1}{\sqrt{2}} = \sqrt{2}r$$

Maximum area $= PQ \times RQ$

$$= \sqrt{2}r \times \frac{\sqrt{2}r}{2}$$

$$= r^2$$

[ISC Marking Scheme, 2016]

14. (i)

	2	3	4	5
2	(2, 2)	(2, 3)	(2, 4)	(2, 5)
3	(3, 2)	(3, 3)	(3, 4)	(3, 5)
4	(4, 2)	(4, 3)	(4, 4)	(4, 5)
5	(5, 2)	(5, 3)	(5, 4)	(5, 5)

(ii) Total no. of order of pairs having one larger number are

$$16 - 4 = 12$$

(iii)

X	3	4	5
P ($X = x$)	2/12	4/12	6/12

(iv) $$P(x < 5) = \frac{2}{12} + \frac{4}{12} = \frac{6}{12} = \frac{1}{2}$$

(v) Expected value $(\bar{X}) = 3\times\frac{1}{6} + 4\times\frac{1}{3} + 5\times\frac{1}{2}$

$$= \frac{1}{2} + \frac{4}{3} + \frac{5}{2}$$

$$= \frac{3+8+15}{6}$$

$$= \frac{26}{6} = \frac{13}{3}$$

$\therefore$ Required answer $= \frac{13}{3}$

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Section-B

15. (i) Option (b) is correct.

Explanation:

Vector $\lambda\vec{a}$ is a unit vector if

$$|\lambda\vec{a}| = 1$$

$$\Rightarrow \quad |\lambda||\vec{a}| = 1$$

$$\Rightarrow \quad |\vec{a}| = \frac{1}{|\lambda|} \qquad [\lambda \neq 0]$$

$$\Rightarrow \quad a = \frac{1}{|\lambda|} \qquad [|\vec{a}| = a]$$

Therefore, vector $\lambda\vec{a}$ is a unit vector if $a = \frac{1}{|\lambda|}$.

(ii) Option (a) is correct.

Explanation:

Given equation $\quad 2x - 3y + z = 0$

Which is parallel to $2x - 3y + z = k$ ...(i)

It passes through (1, –1, 2)

Then, $\quad 2\times1 - 3(-1) + 2 = k$

$$\Rightarrow \quad k = 7$$

Putting the value of k in eq. (i)

$$2x - 3y + z = 7$$

$$\Rightarrow \quad 2x - 3y + z - 7 = 0$$

(iii) Option (a) is correct.

Explanation: We know that,

$$\cos2\alpha + \cos2\beta + \cos2\gamma = 1$$

$$\Rightarrow \left(\frac{1+\cos2\alpha}{2}\right) + \left(\frac{1+\cos2\beta}{2}\right) + \left(\frac{1+\cos2\gamma}{2}\right) = 1$$

$$\Rightarrow \quad 3 + \cos2\alpha + \cos2\beta + \cos2\gamma = 2$$

$$\Rightarrow \quad \cos2\alpha + \cos2\beta + \cos2\gamma = -1$$

(iv) $$\sin\theta = \frac{|-\hat{i} - \hat{j} + \hat{k}|}{\sqrt{2}\cdot\sqrt{2}}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

or $\quad \theta = 60°$

or $\quad \theta = \frac{\pi}{3}$

(v) Equation of diagonal of cube from O (0, 0, 0) to F(2, 2, 2)

$$\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(2\hat{i} + 2\hat{j} + 2\hat{k} - 0\hat{i} - 0\hat{j} - 0\hat{k})$$

$$\Rightarrow \vec{r} = \lambda(2\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\Rightarrow \vec{r} = 2\lambda(\hat{i} + \hat{j} + \hat{k})$$

$\Rightarrow \vec{r} = \mu(\hat{i} + \hat{j} + \hat{k})$, where $\mu = 2\lambda$

[ISC Marking Scheme, 2024]

16. (i) Let $\quad \vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$

$$\Rightarrow \quad |\vec{a}| = |\sqrt{1^2 + 2^2 + 2^2}|$$

$$= |\sqrt{1+4+4}| = \sqrt{9} = 3$$

So, its Unit Vector,

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i}+2\hat{j}+2\hat{k}}{3}$$

$$27\hat{a} = 27\times\frac{\hat{i}+2\hat{j}+2\hat{k}}{3}$$

$$= 9(\hat{i}+2\hat{j}+2\hat{k})$$

$$\text{Required vector} = 9\hat{i}+18\hat{j}+18\hat{k}$$

OR

(ii)
$$(\vec{r}\times\hat{i}) = (x\hat{i}+y\hat{j}+z\hat{k})\times\hat{i} = -y\hat{k}+z\hat{j}$$

$$(\vec{r}\times\hat{j}) = (x\hat{i}+y\hat{j}+z\hat{k})\times\hat{j} = x\hat{k}-z\hat{i}$$

$$\therefore\ (\vec{r}\times\hat{i}).(\vec{r}\times\hat{j}) = (0\hat{i}+z\hat{j}-y\hat{k}).(-z\hat{i}+0\hat{j}+x\hat{k})$$

$$= -xy$$

$$\therefore\ (\vec{r}\times\hat{i}).(\vec{r}\times\hat{j})+xy = 0$$

17. (i) Equations of lines are:

$$\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5} \text{ and } \frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$$

Here, $x_1 = 5, y_1 = 7, z_1 = -3;$
$x_2 = 8, y_2 = 4, z_2 = 5,$
$a_1 = 4, b_1 = 4, c_1 = -5;$
$a_2 = 7, b_2 = 1, c_2 = 3$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 3 & -3 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$= 3(17) + 3(47) + 8(-24) = 0$

$\therefore$ Lines are coplanar.

OR

(ii) The equation of a line passing through two points P(–1, 3, 2) and Q(–4, 2, –2) is given by :

$$\frac{x+1}{-4+1}=\frac{y-3}{2-3}=\frac{z-2}{-2-2}$$

OR
$$\frac{x+1}{-3}=\frac{y-3}{-1}=\frac{z-2}{-4}$$

OR
$$\frac{x+1}{3}=\frac{y-3}{1}=\frac{z-2}{4} \quad \text{...(i)}$$

Now, if the three points $P(-1, 3, 2)$, $Q(-4, 2, -2)$ and $R(5, 5, \lambda)$ are collinear, then the coordinates of point R must satisfy the equation (i).

$$\frac{5+1}{3}=\frac{5-3}{1}=\frac{\lambda-2}{4}$$

$$\Rightarrow \quad \frac{6}{3}=\frac{2}{1}=\frac{\lambda-2}{4}$$

$$\Rightarrow \quad \lambda-2 = 8 \quad \text{i.e.,} \quad \lambda = 10$$

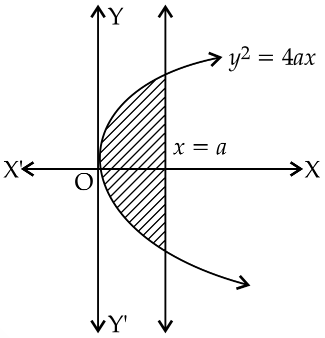
Examiner's Comment

- After expressing the equation of the line in Cartesian symmetric form, a few candidates failed to find λ by substituting coordinates of (5, 5, λ) in the equation of the line. Candidates who used vector form of the line also made a similar mistake.

Answering Tip

- Equation of a line in all forms needs to be understood well. Direction ratios of parallel lines and perpendicular lines need clarity of understanding.

18. (i)



(ii)
$$\text{Required area} = \int_0^a y\,dx$$

$$= \int_0^a 2\sqrt{a}\sqrt{x}\,dx$$

$$= 2.\sqrt{a}\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^a$$

$$= \frac{4\sqrt{a}}{3}(a^{3/2}-0)$$

$$= \frac{4a^2}{3}\text{ units}^2$$

(iii)
$$\text{Required area} = 2\int_0^a y\,dx$$

$$= 2\times\frac{4a^2}{3}$$

$$= \frac{8a^2}{3}\text{ unit}^2$$

Section-C

19. (i) Option **(a)** is correct.

Explanation: For $MC < AC \Rightarrow AC$ decreases with x and AC curve is falling.

(ii) Option (a) is correct.

Explanation: The purpose is to explain the variation of the dependent variable.

(iii)
$$x + 2y - 5 = 0 \quad \text{...(i)}$$
$$2x+3y = 8 \quad \text{...(ii)}$$

Let eq. (i), y on x and eq. (ii) be x on y

$$\Rightarrow \quad b_{yx} = -\frac{1}{2} \text{ and } b_{xy} = -\frac{3}{2}$$

Since both b_{yx} and b_{xy} are of same sign and

$$b_{yx}\times b_{xy} = -\frac{1}{2}\times-\frac{3}{2}=\frac{3}{4}<1$$

$\therefore$ Our assumption is true.

Hence eq. (i) i.e., $x + 2y - 5 = 0$ is a line of regression of y on x. From eq (ii), the regression line of x on y is $2x + 3y = 8$.

[ISC Marking Scheme 2023]

(iv) Given, $C(x) = 3x^2 - 6x + 5$

$$\text{Average cost} = \frac{C(x)}{x}$$

$$= 3x - 6 + \frac{5}{x}$$

$$(\text{Average cost})_{\text{at } x = 2} = 3 \times 2 - 6 + \frac{5}{2}$$

$$= \frac{5}{2} = 2.5$$

[ISC Marking Scheme 2023]

(v) Given, cost of overhaul of an engine is ₹10,000.

Operating cost per hour = $2x - 240$.

Total cost for the engine to run for 300 hours after overhaul

$$= 10,000 + \int_0^{300} (2x - 240)dx$$

$$= 10,000 + \left[\frac{2x^2}{2} - 240x\right]_0^{300}$$

$$= 10,000 + \left[x^2 - 240x\right]_0^{300}$$

$$= 10,000 + \left[(300)^2 - (240)(300)\right] - 0$$

$$= 10,000 + 90,000 - 72,000$$

$$= 1,00,000 - 72,000$$

$$= ₹\ 28,000$$

20. **(i)** Let the number of articles be x.

Total selling price = ₹$60x$

Total cost price = $35x + 250$

(a) Profit function $P(x)$ = S.P . – C.P.

$$= 60x - 35x - 250$$

$$= 25x - 250$$

Hence, profit function is $25x - 250$.

(b) Break-even point i.e., $P(x) = 0$

$$25x - 250 = 0$$

$$\Rightarrow \quad x = 10 \text{ units}$$

OR

(ii) Given, average revenue = $AR = 25 - \frac{x}{4}$

Since, $AR = \frac{R}{x}$

$$= \frac{p.x}{x} = p$$

$$\therefore \quad p = 25 - \frac{x}{4}$$

Total revenue $R(x) = p.x = 25x - \frac{x^2}{4}$

Marginal revenue, $MR = \frac{d}{dx}R(x)$

$$= \frac{d}{dx}\left(25x - \frac{x^2}{4}\right)$$

$$= 25 - \frac{x}{2}$$

[ISC Marking Scheme 2023]

21. **(i)** Given, $n = 102$, $\Sigma x = 510$, $\Sigma y = 7140$, $\Sigma x^2 = 4150$, $\Sigma y^2 = 740200$, $\Sigma xy = 54900$

We know that, regression equation of y on x is

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

So, $\bar{x} = \frac{\Sigma x}{n} = \frac{510}{102} = 5$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{7140}{102} = 70$$

$$b_{xy} = \frac{\Sigma xy - n\bar{x}\,\bar{y}}{\Sigma x^2 - n(\bar{x})^2}$$

$$= \frac{54900 - (102)(5)(70)}{4150 - 102(5)^2}$$

$$= \frac{54900 - 35700}{4150 - 2550}$$

$$= \frac{19200}{1600} = 12$$

Regression line y on x is

$$y - 70 = 12(x - 5)$$

$$\Rightarrow \quad y = 12x - 60 + 70$$

$$\Rightarrow \quad y = 12x + 10$$

$$\Rightarrow \quad y = 2(6x + 5)$$

[ISC Marking Scheme 2023]

OR

(ii) Given : Correlation coefficient

$r = 0.6$

Variance of x = 225

Variance of y = 400

Mean of x = 10

Mean of y = 20

$$\sigma_x = \sqrt{\text{Variance of } x}$$

$$= \sqrt{225} = \pm 15$$

$$\sigma_y = \sqrt{\text{Variance of } y}$$

$$= \sqrt{400} = \pm 20$$

$$b_{yx} = r\frac{\sigma_x}{\sigma_y}$$

$$= 0.6 \times \frac{15}{20} = 0.45$$

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Regression line y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$
$$y - 20 = 0.8(x - 10)$$
$$y = 0.8x - 8 + 20$$
$$y = 0.8x + 12 \quad \text{...(i)}$$

Regression line x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$
$$x - 10 = 0.45(y - 20)$$
$$x = 0.45y - 9 + 10$$

Regression line x on $y = 0.45y + 1$ ...(ii)

(b) When $x = 2$

$y = 0.8 \times 2 + 12 = 13.6$

[ISC Marking Scheme 2020]

Examiner's Comment

- Many candidates applied incorrect formulae in the process of calculating b_{yx} and b_{xy}. There by made errors in calculating other two values also.

Answering Tip

- Learn formulae for calculating regression coefficients b_{yx} and b_{xy} correctly.

22. Let x packets of food P and y packets of food Q be mixed, then to maximize/minimize units of vitamin A

Objective function is $Z = 6x + 3y$

Subject of constraints $12x + 3y \geq 240$ (for calcium units)

$4x + 20y \geq 460$ (for Iron units)

$3x + 2y \leq 150$ (for cholesterol units)

$x \geq y \geq 0$ (non-negativity constraints)

$12x + 3y = 240$

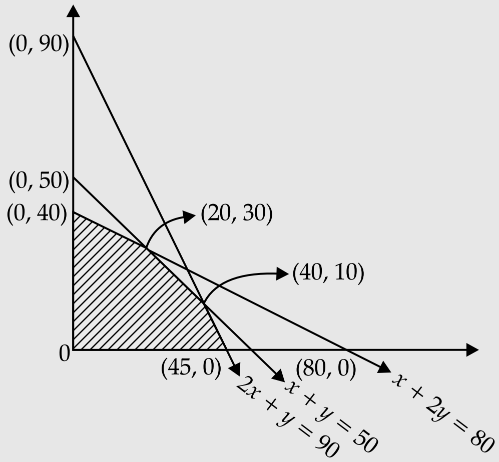
x	0	20	10
y	80	0	40

$4x + 2y = 460$

x	0	115	50
y	23	0	13

$3x + 2y = 150$

x	0	50	70
y	75	0	30



The value of Z (Units of vitamin A)$=6x + 3y$

At A (15,20), $Z = 6 \times 15 + 3 \times 20 = 150$ units

At B (40,15), $Z = 6 \times 40 + 3 \times 15 = 285$ units

At C (2,72), $Z = 6 \times 2 + 3 \times 72 = 228$ units

(i) Minimum units vitamin A = 150 at 15 packets of P 20 packet of Q are mixed.

(ii) (Maximum units of vitamin A = 285 When 40 packets of food P and 15 packets of Q are mixed.

[ISC Marking Scheme 2020]

❑❑

Sample Question Paper-3

MATHEMATICS

Class-12

SOLVED

Time Allowed : 3 hours **Maximum Marks : 80**

You are allowed an additional 15 minutes for only reading the paper.
You must NOT start writing during reading time.
The question paper has 12 printed pages.
The Question Paper is divided into three sections and has 22 questions in all.

Section A is compulsory and has fourteen questions.
You are required to attempt all questions either from Section B or Section C.
Section B and Section C have four questions each.
Internal choices have been provided in two questions of 2 marks, two questions of 4 marks and two questions of 6 marks in Section A.
Internal choices have been provided in one question of 2 marks and one question of 4 marks each in Section B and Section C.
While attempting Multiple Choice Questions in Section A, B and C, you are required to write only ONE option as the answer.
The intended marks for questions or parts of questions are given in the brackets [].
All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
Mathematical tables and graph papers are provided.
Instruction to Supervising Examiner:
Kindly read aloud the instructions given above to all the candidates present in the examination hall.

Section-A [65 marks]

1. In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed. **[15]**

(i) Matrices A and B will be inverse of each other only if

(a) $AB = BA$ **(b)** $AB = BA = 0$ **(c)** $AB = 0, BA = I$ **(d)** $AB = BA = I$

[Understanding]

(ii) The value of $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ is

(a) $\frac{\pi}{2}$ **(b)** $\frac{2\pi}{3}$ **(c)** $\frac{\pi}{6}$ **(d)** $\frac{\pi}{12}$ **[Recall]**

(iii) The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is

(a) $\frac{\pi}{2}$ **(b)** $\frac{3\pi}{2}$ **(c)** $\frac{5\pi}{2}$ **(d)** $\frac{7\pi}{2}$ **[Application]**

(iv) Assertion (A): The order and the degree of differential equation $2+\left(\frac{dy}{dx}\right)^3 = \frac{d^3y}{dx^3}$ are 3 and 1 respectively.

Reason (R): The order of a differential equation is the order of the highest derivative appearing in the differential equation. The degree of the differential equation is the power of the highest order derivative.
Which of the following is correct?

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true. **[Analysis]**

(v) If $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then the value of $P(B/A)$ is

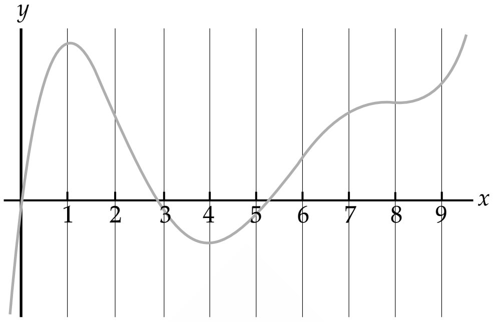
(a) $\frac{1}{8}$ **(b)** $\frac{4}{7}$ **(c)** $\frac{5}{7}$ **(d)** $\frac{7}{8}$ A I **[Application]**

(vi) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = x$, then what is the value of $\begin{vmatrix} a & 2b & c \\ 4d & 8e & 4f \\ g & 2h & i \end{vmatrix}$?

(a) $8x$ **(b)** $12x$ **(c)** $4x$ **(d)** $2x$

[Understanding]

(vii) Shown below is the graph of a function, $f(x)$.



Statement 1: $(-\infty, 1)$, $(4, 8)$ & $(8, \infty)$ are the intervals on which the $f(x)$ increases.

Statement 2: $(-\infty, 1)$, $(4, 8)$ & $(-8, -\infty)$ are the intervals on which the $f(x)$ increases.

Which of the following is correct?

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false. **[Application]**

(viii) If $x = a\cos\theta$, $y = a\sin\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$ will be

(a) –1 **(b)** 1 **(c)** 0 **(d)** 2

(ix) **Statement 1:** Let A be the set of all 50 students of Class – IX in a school. Let $f : A \to N$ be a function defined by $f(x)$ = roll number of student 'x'. Then the function 'f' is one-one.

Statement 2: Let A be the set of all 50 students of Class – IX in a school. Let $f : A \to N$ be a function defined by $f(x)$ = roll number of student 'x'. Then the function 'f' is onto.

Which one of the following is correct?

(a) Statement 1 implies Statement 2.

(b) Statement 2 implies Statement 1.

(c) Statement 1 is true only if Statement 2 is true.

(d) Statements 1 and 2 are independent of each other. **[Understanding]**

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(x) **Assertion (A):** Let the matrices $A = \begin{bmatrix} -3 & 2 \\ -5 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 \\ 5 & -3 \end{bmatrix}$ be such that $A^{100}B = BA^{100}$

Reason (R): $AB = BA$ implies $A^nB = BA^n$ for all positive integers n.

Which of the following is correct?

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true. **[Application]**

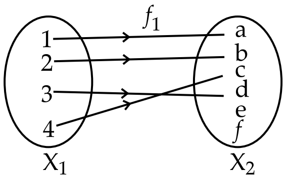
(xi) If two balls are drawn from a bag containing 3 white, 4 black and 5 red balls. Then, the probability that the drawn balls are of different colours is

(a) $\frac{1}{66}$ **(b)** $\frac{3}{66}$ **(c)** $\frac{19}{66}$ **(d)** $\frac{47}{66}$ **[Understanding]**

(xii) If $A = \begin{vmatrix} -8 & -2 & 1 \\ -3 & -5 & 0 \\ 6 & -9 & 4 \end{vmatrix}$, write the minor of the element a_{31}. **[Analysis]**

(xiii) Let L be a set of all straight lines in a plane. What is the relation R on L defined as 'perpendicular to' ?

[Understanding]



(xiv) The function $f : X \to Y$ is one-one. Give reason. **[Understanding]**

(xv) 50 tickets in a box are numbered 00, 01, 02, ..., 49. One ticket is drawn randomly from the box. Find the probability of the ticket having the product of its digits 7, given that the sum of the digits is 8. **[Application]**

2. **(i)** Find $\frac{dy}{dx}$, if $x^3 + y^3 = 3axy$. **AI** **[Understanding] [2]**

OR

(ii) The edge of a variable cube is increasing at the rate of 10 cm/s. How fast is the volume of the cube increasing when the edge is 5 cm long ? **[Understanding]**

3. Evaluate: $\int xe^x\,dx$ **[Evaluate] [2]**

4. Prove that the curves $y^2 = 4x$ and $x^2 + y^2 - 6x + 1 = 0$ touch each other at the point (1, 2). **[Application] [2]**

5. **(i)** Evaluate: $\int_{-1}^{2}\frac{|x|}{x}dx$. **AI** **[Evaluate] [2]**

OR

(ii) Evaluate: $\int_4^5 |x-5|\,dx$. **[Evaluate]**

6. Solve for : $5\tan^{-1} x + 3\cot^{-1} x = 2\pi$ **[Analysis] [2]**

7. Prove that : $\tan^{-1}\frac{1}{2} = \frac{\pi}{4} - \tan^{-1}\frac{1}{3}$. **AI** **[Application] [4]**

8. Evaluate: $\int_0^{\pi}\frac{x\tan x}{\sec x + \tan x}dx$ **AI** **[Application] [4]**

9. **(i)** Water is dripping out from a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at the uniform rate of 2 cm^2/s in the surface, through a tiny hole at the vertex of bottom. When the slant height of water level is 4 cm, find the rate of decrease of the slant height of the water. **[Application] [4]**

OR

(ii) Solve the following differential equation:

$$\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$$ **[Analysis]**

10. **(i)** A group of Class XI students designed a game for Class XII students to be played at the farewell programme. They kept two sets of cards on the stage. Each set contained four cards numbered 1, 2, 3 and 4. Each student of Class XII had to draw one card from each set simultaneously.

The score a student received was defined as the difference between the numbers shown on the cards. The possible outcome of the score is given in the table below:

Numbers on Card 1

Numbers on Card 2		**1**	**2**	**3**	**4**
	1	0	p	2	3
	2	1	0	p	2
	3	q	1	0	p
	4	3	q	1	0

(a) Calculate the values of p and q in the above table.

(b) Reshma plays the game once. Let the random variable 'S' represents Reshma's score.
Construct the probability distribution for 'S'

s			
$P(S = s)$			

(c) Find the probability that Reshma's score is at most 2. **AI** **[Analysis] [4]**

OR

(ii) For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

- 960 of the total applications were the folk genre.
- 192 of the folk applications were for the below 18 category.
- 104 of the classical applications were for the 18 and above category.

(a) What is the probability that an application selected at random is for the 18 and above category provided it is under the classical genre? Show your work.

(b) An application selected at random is found to be under the below 18 category. Find the probability that it is under the folk genre. Show your work.

(c) Write the rule of Total Probability.

11. Garima owns a cafe that serves burgers, samosas and juice bottles. On a particular day, 100 items in total were sold. The number of burgers sold was thrice the number of samosas sold. Also, the number of samosas sold was 10 more than the number of juice bottles sold.

(i) Form a set of simultaneous equations for the above information.

(ii) Solve the set of equations formed in (i) by matrix method.

(iii) Hence, find the number of items sold in each category. **[Application] [6]**

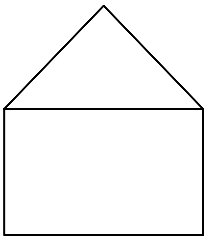
12. (i) Solve the following differential equation:

$$2ye^{x/y}\,dx + (y - 2xe^{x/y})dy = 0, \text{ given } x = 0 \text{ and } y = 1$$

AI **[Application]**

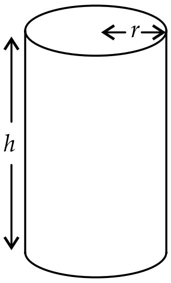
OR

(ii) Evaluate: $\int_0^{\pi} e^{2x}\sin\left(\frac{\pi}{4}+x\right)dx$ AI **[Evaluate] [6]**



13. (i) Mrs. Roy designs a window in her son's study room so that the room gets maximum sunlight. She designs the window in the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the window that will admit maximum sunlight into the room. AI **[Application] [6]**

OR



(ii) Sumit has bought a closed cylindrical dustbin. The radius of the dustbin is '*r*' cm and height is '*h*' cm. It has a volume of $20\pi\text{cm}^3$.

(a) Express '*h*' in terms of '*r*', using the given volume.

(b) Prove that the total surface area of the dustbin is $2\pi r^2 + \frac{40\pi}{r}$

(c) Sumit wants to paint the dustbin. The cost of painting the base and top of the dustbin is 2 per cm^2 and the cost of painting the curved side is ₹ 25 per cm^2. Find the total cost in terms of '*r*', for painting the outer surface of the dustbin including the base and top.

(d) Calculate the minimum cost for painting the dustbin. **[Application]**

14. A biased four-sided die with faces labelled 1, 2, 3 and 4 is rolled and recorded. Let X be the result obtained when the die is rolled. The probability distribution for X is given in the following table where p and q are constants.

x	1	2	3	4
$P(X = x)$	p	0.3	q	0.1

For the probability distribution, it is known that $E(X) = 2$.

(i) Find p and q.

(ii) Also, find $P(X > 2)$.

(iii) Ajay play a game with this four-sided die. In this game he is allowed a maximum of five rolls. His score is calculated by adding the results of each roll. He wins the game if his score is at least 10. After 3 rolls, Ajay has score of four points. Assuming the rolls of the die are independent, find the probability that Ajay wins the game. **[Analysis] [6]**

Section-B [15 marks]

15. In subparts (i) to (iii) choose the correct options and in subparts (iv) and (v), answer the questions as instructed.**[5]**

(i) Consider the following statements and choose the correct option :

Statement 1: The position vector of the point which divides the join of points $2\vec{a}-3\vec{b}$ and $\vec{a}+\vec{b}$ in the ratio 3 : 1. is $\frac{5\vec{a}}{4}$.

Statement 2: The position vector of a point R dividing the line segments joining the points P and Q, whose position vectors are $\vec{p}$ and $\vec{q}$ in the ratio $m : n$ internally, is given by $\frac{m\vec{p}-n\vec{q}}{m-n}$.

Which of the following is correct?

(a) Only Statement 1
(b) Only Statement 2
(c) Both Statements 1 and 2
(d) Neither Statement 1 nor Statement 2 AI **[Recall]**

(ii) If a line makes an angle α, β and γ with positive direction of the coordinate axes, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ will be:

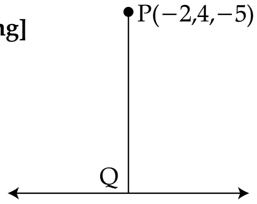
(a) 1 **(b)** 3 **(c)** –2 **(d)** 2 **[Application]**

(iii) The direction cosines of x-axis are-

(a) <1, 1, 1> **(b)** <0, 0, 0> **(c)** <1, 0, 0> **(d)** <0, 1, 1>

A I **[Understanding]**

(iv) If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 5\hat{i} - 3\hat{j} + \hat{k}$ Find the projection of $\vec{b}$ on $\vec{a}$ **A I** **[Understanding]**



(v) Find the distance of the point (–2, 4, –5) from the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$. **[Analysis]**

16. **(i)** Write a vector of magnitude of 52 units in the direction of the vector $4\hat{i} - 3\hat{j} + 12\hat{k}$. **A I** **[Application] [2]**

OR

(ii) The position vectors of the points A and B are (2, −3 , 2)and (2, 3, 1) respectively, then find the area of triangle OAB.

A I **[Application]**

17. **(i)** Find the equation of the lines passing through the point (2, l, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$. **A I** **[Application] [4]**

OR

(ii) Find the image of the point (2, –1, 5) in the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$. Also, find the length of the perpendicular from the point (2, –1, 5) to the line. **A I** **[Application]**

18. **(i)** Sketch the region enclosed bounded by the curve, $y^2 = 4x$ and $x^2 = 4y$

(ii) Evaluate: $\int (y_2 - y_1)dx$

(iii) Hence find the area bounded by the curve, $y^2 = 4x$ and $x^2 = 4y$. **A I** **[4]**

Section-C [15 marks]

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.**[5]**

(i) Which condition is true if AC is increases with x and AC curve is rising.

(a) $MC < AC$ **(b)** $MC = AC$ **(c)** $MC > AC$ **(d)** None of these **[Recall]**

(ii) Read the following statements and choose the correct option:

(I) The slope of the regression line of Y on X is also called the correlation coefficient of X on Y.

(II) The slope of the regression line of Y on X is also called the correlation coefficient of Y on X.

(III) The slope of the regression line of Y on X is also called the regression coefficient of X on Y.

(IV) The slope of the regression line of Y on X is also called the regression coefficient of Y on X.

Which of the following is correct?

(a) Only IV is correct **(b)** Only III and II are correct

(c) Only II and IV are correct **(d)** Only III and IV are correct **[Recall]**

(iii) By using the data $\bar{x} = 25$, $\bar{y} = 30$; $b_{yx} = 1.6$ and $b_{xy} = 0.4$, What is the regression equation of y on x? What is the most likely value of y when $x = 60$? **[Understanding]**

(iv) The marginal revenue of a manufacturing facility is given by the function $\text{MR}(x) = 408 - 17x$. The manufacturer should limit the production to 24 units. Why? **A I** **[Understanding]**

(v) Find the marginal cost function (MC), if the cost function is: $C(x) = \frac{x^3}{3} + 5x^2 - 16x + 2$. **[Understanding]**

20. **(i)** A company produces a commodity with ₹ 24,000 as fixed cost. The variable cost estimated to be 25% of the total revenue received on selling the product, is at the rate of ₹ 8 per unit. Find the break-even point.

A I **[Application] [2]**

OR

(ii) The average cost function associated with producing and marketing x units of an item is given by $AC = 2x - 11 + \frac{50}{x}$. Find the range of values of the output x, for which AC is increasing. **A I** **[Application]**

21. **(i)** Find the regression coefficients b_{yx}, b_{xy} and correlation coefficient 'r' for the following data: (2, 8), (6, 8), (4, 5), (7, 6), (5, 2). **[Application] [4]**

OR

(ii) The following table shows the mean, the standard deviation and the coefficient of correlation of two variables x and y.

Series	x	y
Mean	8	6
Standard deviation	12	4
Coefficient of correlation	0.6	

Calculate:

(a) the regression coefficient b_{xy} and b_{yx}.

(b) the probable value of y when $x = 20$. **[Application]**

22. The standard weight of a special purpose brick is 5 kg and it must contain two basic ingredients B_1 and B_2 cost ₹ 5 per kg and B_2 cost ₹ 8 per kg. Strength considerations dictate that the brick should not contain more than 4 of B_1 and a minimum of 2 kg of B_2. Since the demand for the product is likely to be related to the price of the bricks, find the minimum cost of the brick satisfying the above conditions.

(a) Formulate this situation as an L.P.P.

(b) Solve it graphically. **[Analysis] [4]**

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SOLUTIONS

Sample Question Paper - 3

Section-A

1. **(i) Option (d) is correct.**

Explanation: We know that if A is a square matrix of order m and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is said to be the inverse of A.

In this case, it is clear that A is the inverse of B.

Thus, matrices A and B will be the inverse of each other only if AB = BA = I.

(ii) Option (d) is correct.

Explanation:
$$\int_1^{\sqrt{3}}\frac{dx}{1+x^2} = \left[\tan^{-1}x\right]_1^{\sqrt{3}}$$
$$= \tan^{-1}\sqrt{3}-\tan^{-1}1$$
$$= \frac{\pi}{3}-\frac{\pi}{4} = \frac{\pi}{12}$$

(iii) Option (a) is correct.

Explanation : We have, $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$

$$= \cos^{-1}\left[\cos\left(2\pi-\frac{\pi}{2}\right)\right] \qquad \left[\because \cos\left(2\pi-\frac{\theta}{2}\right) = \cos\frac{\theta}{2}\right]$$
$$= \cos^{-1}\cos\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \qquad \left[\because \cos^{-1}(\cos x) = x,\ if\ x\in[0,\pi]\right]$$
$$\Rightarrow \cos^{-1}\left(\cos\frac{3\pi}{2}\right) = \frac{\pi}{2}$$

(iv) Option (a) is correct.

Explanation: $2+\left(\frac{dy}{dx}\right)^3 = \frac{d^3y}{dx^3}$ is the differential equation of order 3 because highest order derivative of y w.r.t. x is $\frac{d^3y}{dx^3}$ and degree is 1 because the power of highest order derivative $\frac{d^3y}{dx^3}$ is 1.

(v) Option (d) is correct.

Explanation :

Since,
$$P(A) = \frac{4}{5} \ \&\ P(A\cap B) = \frac{7}{10}$$
$$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{\frac{7}{10}}{\frac{4}{5}} = \frac{7}{8}$$

(vi) Option (a) is correct.

***Explanation:* Given,**
$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = x \qquad \ldots(\text{i})$$

Now,
$$\begin{vmatrix} a & 2b & c \\ 4d & 8e & 4f \\ g & 2h & i \end{vmatrix}$$
$$= 4\begin{vmatrix} a & 2b & c \\ d & 2e & f \\ g & 2h & i \end{vmatrix} \quad \text{(By taking 4 common from 2nd row)}$$
$$= 4\times 2\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \quad \text{(By taking 2 common from 2nd column)}$$
$$= 4\times2\times x \quad \text{[Using (i)]}$$
$$=8x$$

(vii) Option (a) is correct.

Explanation: It is evident from the graph, where the function is increasing.

⇒ The function increases at the following intervals: $(-\infty,1)$, $(4, 8)$ & $(8, \infty)$

Hence, Statement 1 is true and Statement 2 is false.

(viii) Option (c) is correct.

Explanation : $x = a\cos\theta$ and $y = a\sin\theta$

$$\therefore \quad \frac{dx}{d\theta} = \frac{d}{d\theta}(a\cos\theta) = -a\sin\theta$$
and
$$\frac{dy}{d\theta} = \frac{d}{d\theta}(a\sin\theta) = a\cos\theta$$
Now,
$$\frac{dy}{dx} = \frac{dy}{d\theta}\times\frac{d\theta}{dx} = \frac{a\cos\theta}{-a\sin\theta} = -\cot\theta$$
At $\theta = \frac{\pi}{2}$,
$$\frac{dy}{dx} = \cot\frac{\pi}{2} = 0$$

(ix) Option (d) is correct.

Explanation: A is a set of all 50 students of class IX in a school.

We know that, none of the two students will have same roll number so f must be one-one.

However, we can assume that roll numbers are from 1 to 50 and thus 51 in natural numbers will not have any image of any element of x under f. Hence, f is not onto.

Hence, Statements 1 and 2 are independent of each other.

(x) Option (a) is correct.

Explanation:
$$AB = \begin{bmatrix} -3 & 2 \\ -5 & 4 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$
$$BA = \begin{bmatrix} 4 & -2 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

Reason, $AB = BA$

$A.A.B = A.B.\,A$

{∴ Both sides multiply by A}

$A^2B = B.\,A.\,A$ $\{AB = BA\}$

$A^2B = BA^2$

So, $A^nB = BA^n$

Hence, Reason is true and correct explanation of Assertion

(xi) Option (a) is correct.

Explanation: Required probability that the drawn balls are of different colour

$$= \frac{{}^3C_1 \times {}^4C_1}{{}^{12}C_2} + \frac{{}^3C_1 \times {}^5C_1}{{}^{12}C_2} + \frac{{}^4C_1 \times {}^5C_1}{{}^{12}C_2}$$

$$= \frac{3\times 4}{\frac{12\times 11}{2}} + \frac{3\times 5}{\frac{12\times 11}{2}} + \frac{4\times 5}{\frac{12\times 11}{2}}$$

$$= \frac{12}{66} + \frac{15}{66} + \frac{20}{66} = \frac{47}{66}$$

(xii) Given $A = \begin{vmatrix} -8 & -2 & 1 \\ -3 & -5 & 0 \\ 6 & -9 & 4 \end{vmatrix}$

$\therefore$ Minor of $a_{31} = \begin{vmatrix} -2 & 1 \\ -5 & 0 \end{vmatrix}$

$= 0 + 5 = 5$

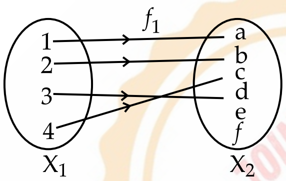
(xiii) $L_1 \perp L_2$, $L_2 \perp L_3$ then $L_1 \parallel L_3$

$\therefore$ Relation is not transitive

$L_1 \perp L_2$ then $L_2 \perp L_1$

$\therefore$ Relation is symmetric.

(xiv) A function $f : X \to Y$ is defined to be one-one, if the images of distinct elements of X under f are distinct. Here, the function has distinct elements having distinct images. Thus, it is one-one.



(xv) Let, A = Event that sum of digits on selected tickets is 8

$\therefore$ $A = \{08, 17, 26, 35, 44\}$

$n(A) = 5$

Let, B = Event that the product of digits is 7.

$\therefore$ $B = \{17\}$

Implies, $n(B) = 1$

Also, $n(A \cap B) = 1$ as only 17 is the number with sum 8 and product 7.

$$\text{Probability} = \frac{n(A\cap B)}{n(A)} = \frac{1}{5}$$

2. (i) $x^3 + y^3 = 3axy$

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) = \frac{d}{dx}(3axy)$$

$$3x^2 + 3y^2\frac{dy}{dx} = 3a\frac{d}{dx}(xy)$$

$$x^2 + y^2\frac{dy}{dx} = a\left\{x\frac{d}{dx}y + y\frac{d}{dx}x\right\}$$

$$x^2 + y^2\frac{dy}{dx} = a\left\{x\frac{dy}{dx} + y(1)\right\}$$

$$y^2\frac{dy}{dx} - ax\frac{dy}{dx} = ay - x^2$$

$$(y^2 - ax)\frac{dy}{dx} = ay - x^2$$

$$\therefore \quad \frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$$

[ISC Marking Scheme, 2020]

Examiner's Comment

- Some candidates apply wrong formula for product. Some leave the question in the middle as they do not combined the terms having *dy/dx*.

Answering Tip

- Learn all the formulae and combined the terms of *dy/dx* together before giving the final answer.

OR

(ii) Let the edge of cube be x cm.

$\therefore$ $\frac{dx}{dt} = 10$ cm/s (given)

Volume of cube $(V) = x^3$

$$\Rightarrow \quad \frac{dV}{dt} = \frac{d}{dt}x^3 = 3x^2\frac{dx}{dt}$$

$$\Rightarrow \quad \left(\frac{dV}{dt}\right)_{x=5} = 3(5)^2(10) = 750 \text{ cm}^3/\text{s}.$$

$\therefore$ Volume of cube increasing by 750 cm^3/s when edge of cube is 5 cm.

[ISC Marking Scheme, 2020]

Examiner's Comment

- Few students are not able to form this mathematically.

Answering Tip

- Practice adequate problems from this topic as it is important from exam point of view.

3. Let $I = \int xe^x dx$

On integrating by parts

$$\int u.v dx = u\int v dx - \int\left(\frac{d}{dx}u\int v dx\right)dx$$

Therefore,

$$I = xe^x - \int 1.e^x dx$$

$$I = xe^x - \int e^x dx$$

$$I = xe^x - e^x + C$$

[ISC Marking Scheme, 2023-24]

Examiner's Comment

- Students find it difficult to select the first function when solving problems using integration by parts.

Answering Tip

- Students should remember to apply ILATE rule for selecting first function.

4. We have, $y^2 = 4x$ and $x^2 + y^2 - 6x + 1 = 0$

Since, both the curves touch each other at (1, 2), i.e., curves are passing through (1, 2).

$$\Rightarrow \quad 2y\cdot\frac{dy}{dx} = 4$$

and $2x + 2y\dfrac{dy}{dx} = 6$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{4}{2y} = m_1$$

and $\dfrac{dy}{dx} = \dfrac{6-2x}{2y} = m_2$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1,2)} = \frac{4}{4} = 1$$

and $\left(\dfrac{dy}{dx}\right)_{(1,2)} = \dfrac{6-2\cdot1}{2\cdot2} = \dfrac{4}{4} = 1$

$$\Rightarrow \quad m_1 = 1 \text{ and } m_2 = 1$$

Thus, we see that slope of both the curves are equal to each other, i.e., $m_1 = m_2 = 1$ at the point (1, 2). Hence, both the curves touch each other.

5. **(i)** Let $\quad I = \displaystyle\int_{-1}^{2}\frac{|x|}{x}dx$

Since, $\quad \dfrac{|x|}{x} = \begin{cases} \dfrac{-x}{x}, & x<0 \\ \dfrac{x}{x}, & x>0 \end{cases}$

$$= \begin{cases} -1, & x<0 \\ 1, & x>0 \end{cases}$$

$$\therefore \quad I = \int_{-1}^{0}(-1)dx + \int_{0}^{2}(1)dx$$

$$= [-x]_{-1}^{0} + [x]_0^2$$

$$= -[0-(-1)] + (2-0)$$

$$= -1 + 2 = 1$$

OR

(ii) $\displaystyle\int_4^5|x-5|\,dx = \int_4^5 -(x-5)dx$

$$= \left[\frac{x^2}{2} - 5x\right]_4^5$$

$$= -\left[\frac{25}{2} - 25 - \frac{16}{2} + 20\right]$$

$$= \frac{1}{2}$$

[ISC 2020 Marking Scheme]

Examiner's Comment

- Few students have no idea how to solve (take positive or negative sign the modulus within the given limits.

Answering Tip

- Understand the concept of sign taken for modulus within the given limits by doing various similar questions.

6. Given, $5\tan^{-1}x + 3\cot^{-1}x = 2\pi$ **...(i)**

We know that,

$$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow \quad \cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$$

$$\therefore \; 5\tan^{-1}x + 3\left(\frac{\pi}{2} - \tan^{-1}x\right) = 2\pi \qquad \text{[From eq (i)]}$$

$$\Rightarrow \; 5\tan^{-1}x + \frac{3\pi}{2} - 3\tan^{-1}x = 2\pi$$

$$\Rightarrow \quad 2\tan^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow \quad \tan^{-1}x = \frac{\pi}{4}$$

$$\Rightarrow \quad x = \tan\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \quad x = 1$$

7. $\quad \text{RHS} = \dfrac{\pi}{4} - \tan^{-1}\dfrac{1}{3}$

$$= \tan^{-1}1 - \tan^{-1}\frac{1}{3}$$

$$= \tan^{-1}\frac{1-\frac{1}{3}}{1+\frac{1}{3}}$$

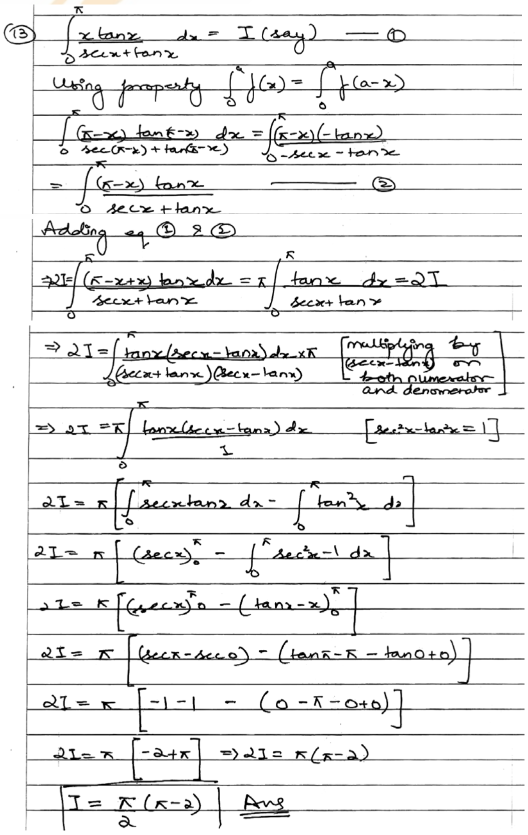
$$= \tan^{-1}\frac{2}{4}$$

$$= \tan^{-1}\frac{1}{2}$$

$= \text{LHS}$ **Hence proved.**

8.

Topper Answer's, 2019



⑬ $\displaystyle\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx = I\text{(say)}$ —— ①

Using property $\displaystyle\int_0^a f(x) = \int_0^a f(a-x)$

$$\int_0^{\pi}\frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}dx = \int_0^{\pi}\frac{(\pi-x)(-\tan x)}{-\sec x-\tan x}$$

$$= \int_0^{\pi}\frac{(\pi-x)\tan x}{\sec x+\tan x} \quad \text{——} \quad ②$$

Adding eq ① & ②

$$\Rightarrow 2I = \int_0^{\pi}\frac{(\pi-x+x)\tan x\,dx}{\sec x+\tan x} = \pi\int_0^{\pi}\frac{\tan x}{\sec x+\tan x}dx = 2I$$

$$\Rightarrow 2I = \int_0^{\pi}\frac{\tan x(\sec x-\tan x)dx\times\pi}{(\sec x+\tan x)(\sec x-\tan x)} \quad \left[\begin{array}{l}\text{multiplying by}\\ (\sec x-\tan x) \text{ on}\\ \text{both numerator}\\ \text{and denomerator}\end{array}\right]$$

$$\Rightarrow 2I = \pi\int_0^{\pi}\frac{\tan x(\sec x-\tan x)dx}{1} \quad [\sec^2x-\tan^2x=1]$$

$$2I = \pi\left[\int_0^{\pi}\sec x\tan x\,dx - \int_0^{\pi}\tan^2x\,dx\right]$$

$$2I = \pi\left[(\sec x)_0^{\pi} - \int_0^{\pi}\sec^2x-1\,dx\right]$$

$$2I = \pi\left[(\sec x)_0^{\pi} - (\tan x-x)_0^{\pi}\right]$$

$$2I = \pi\left[(\sec\pi-\sec 0) - (\tan\pi-\pi-\tan 0+0)\right]$$

$$2I = \pi\left[-1-1 - (0-\pi-0+0)\right]$$

$$2I = \pi\left[-2+\pi\right] \Rightarrow 2I = \pi(\pi-2)$$

$$\boxed{I = \frac{\pi(\pi-2)}{2}} \quad \underline{\underline{\text{Ans}}}$$

9. (i) Let r be the radius and l be the slant height of water surface

$$\sin\frac{\pi}{4} = \frac{r}{l}$$

or, $$r = l.\frac{1}{\sqrt{2}}$$

Curved surface, $$S = \pi rl$$

$$\Rightarrow \quad S = \pi.l.\frac{1}{\sqrt{2}}.l$$

$$\Rightarrow \quad = \frac{1}{\sqrt{2}}\pi l^2$$

Now, $$\frac{dS}{dt} = \frac{1}{\sqrt{2}}\pi.2l\frac{dl}{dt}$$

Given that, $$\frac{dS}{dt} = -2\text{ cm}^2/\text{s}$$

$$\therefore \quad -2 = \frac{\pi}{\sqrt{2}}.2l\frac{dl}{dt}$$

$$\Rightarrow \quad \frac{dl}{dt} = \frac{-\sqrt{2}}{\pi l}$$

At $l = 4$ cm

$$\left(\frac{dl}{dt}\right)_{l=4} = \frac{-\sqrt{2}}{\pi.4} = \frac{-\sqrt{2}}{4.\pi}\text{ cm/s}$$

[ISC Marking Scheme 2018]

Examiner's Comment

- Many candidates considered volume as function instead of surface area. Majority of candidates did not apply proper sign through it was specified in the question that find the rate of decrease of slant height of the water.

Answering Tip

- The difference between rate of increase and rate of decrease need to be understood carefully. Practice a number of problems based on applications of derivatives.

10. (i) (a) $p = 1, q = 2$

(b)

s	0	1	2	3
$P(S = s)$	$\frac{4}{16} = \frac{1}{4}$	$\frac{6}{16} = \frac{3}{8}$	$\frac{4}{16} = \frac{1}{4}$	$\frac{2}{16} = \frac{1}{8}$

(c)
$$P(x \le 2) = 1 - P(x > 2)$$
$$= 1 - P(x = 3)$$
$$= 1 - \frac{2}{16}$$
$$= \frac{14}{16} = \frac{7}{8}$$

[ISC Specimen Paper Marking Scheme 2024]

OR

(ii) (a) According to the given information, we construct the following table.

Given, total applications = 2000

	Folk Genre	Classical Genre
	960 (given)	2000 – 960 = 1040
Below 18	192 (given)	1040 – 104 = 936
18 or Above 18	960 – 192 = 768	104 (given)

Let E_1 = Event that application for folk genre

E_2 = Event that application for classical genre

A = Event that application for below 18

B = Event that application for 18 or above 18

$$\therefore \quad P(E_2) = \frac{1040}{2000}$$

and $$P(B \cap E_2) = \frac{104}{2000}$$

Required Probability $$= \frac{P(B \cap E_2)}{P(E_2)} = \frac{\frac{104}{2000}}{\frac{1040}{2000}} = \frac{1}{10}$$

(b) Required probability $$= P\left(\frac{\text{folk}}{\text{below 18}}\right)$$

$$= P\left(\frac{E_1}{A}\right) = \frac{P(E_1 \cap A)}{P(A)}$$

Now, $$P(E_1 \cap A) = \frac{192}{2000}$$

and $$P(A) = \frac{192 + 936}{2000} = \frac{1128}{2000}$$

$\therefore$ Required probability $$= \frac{\frac{192}{2000}}{\frac{1128}{2000}} = \frac{192}{1128} = \frac{8}{47}$$

(c) Consider an event E which occurs via two different events A and B. The probability of E is given by the value of total probability as:

$$P(E) = P(A \cap E) + P(B \cap E)$$

$$P(E) = P(A)P\left(\frac{E}{A}\right) + P(B)P\left(\frac{E}{B}\right)$$

11. Let no. of burgers = x

Let no. of samosas = y

Let no. of juice bottles = z

A.T.Q.

Total items sold were 100

$$\Rightarrow \quad x + y + z = 100 \quad \ldots\text{(i)}$$

The number of burgers sold was thrice the no. of samosas sold

$$\Rightarrow \quad x = 3y$$

$$\Rightarrow \quad x - 3y = 0 \quad \ldots\text{(ii)}$$

The no. of samosas sold was 10 more than the number of juice bottles

$$y = z + 10$$

$$\Rightarrow \quad y - z = 10 \quad \ldots\text{(iii)}$$

(i)
$$x + y + z = 100$$
$$x - 3y = 0$$
$$y - z = 10$$

(ii) $$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -3 & 0 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 100 \\ 0 \\ 10 \end{pmatrix}$$

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B \qquad \ldots\text{(i)}$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -3 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

$$|A| = (1)(3-0) - 1(-1-0) + 1(1+0)$$
$$= 3 + 1 + 1 = 5 \neq 0$$

$\Rightarrow A^{-1}$ exists

$$\text{adj. } A = \begin{pmatrix} 3 & 2 & 3 \\ 1 & -1 & 1 \\ 1 & -1 & -4 \end{pmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{5}\begin{pmatrix} 3 & 2 & 3 \\ 1 & -1 & 1 \\ 1 & -1 & -4 \end{pmatrix}$$

$$X = A^{-1}B = \frac{1}{5}\begin{pmatrix} 3 & 2 & 3 \\ 1 & -1 & 1 \\ 1 & -1 & -4 \end{pmatrix}\begin{pmatrix} 100 \\ 0 \\ 10 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} 300+30 \\ 100+10 \\ 100-40 \end{pmatrix} = \begin{pmatrix} 66 \\ 22 \\ 12 \end{pmatrix}$$

(iii) $x = 66, y = 22$ and $z = 12$

12. **(i)** $2ye^{x/y}\,dx + (y - 2xe^{x/y})\,dy = 0$

$$\frac{dx}{dy} = -\left(\frac{y - 2xe^{x/y}}{2ye^{x/y}}\right)$$

This is a homogeneous differential equation.

Let, $x = Vy$

$$\frac{dx}{dy} = V + y\frac{dV}{dy}$$

$$V + y\frac{dV}{dy} = -\left(\frac{y - 2Vye^{V}}{2ye^{V}}\right)$$

$$y\frac{dV}{dy} = -\left(\frac{1 - 2Ve^{V}}{2e^{V}}\right) - V$$

$$y\frac{dV}{dy} = \frac{-1 + 2Ve^{V} - 2Ve^{V}}{2e^{V}}$$

$$y\frac{dV}{dy} = -\frac{1}{2e^{V}}$$

$$e^{V}dV = -\frac{1}{2}\frac{dy}{y}$$

$$\int e^{V}dV = -\frac{1}{2}\int\frac{dy}{y}$$

$$e^{V} = -\frac{1}{2}\log y + C$$

$$e^{x/y} = -\frac{1}{2}\log y + C$$

$$x = 0, y = 1$$

$$e^{\circ} = -\frac{1}{2}\log 1 + C$$

$$2e^{x/y} + \log y = 2$$
$$\log y = 2\,(1 - e^{x/y})$$

OR

(ii) $\int_0^{\pi} e^{2x}\sin\left(\frac{\pi}{4} + x\right)dx$

Let $I = \int_0^{\pi} e^{2x}\cdot\sin\left(\frac{\pi}{4} + x\right)dx$

Integrating by parts, without limits

$$I = \sin\left(\frac{\pi}{4} + x\right)\cdot\frac{e^{2x}}{2} - \int\cos\left(\frac{\pi}{4} + x\right)\cdot\frac{e^{2x}}{2}dx$$

$$= \sin\left(\frac{\pi}{4} + x\right)\cdot\frac{e^{2x}}{2} - \frac{1}{2}\int e^{2x}\cdot\cos\left(\frac{\pi}{4} + x\right)dx$$

$$= \sin\left(\frac{\pi}{4} + x\right)\cdot\frac{e^{2x}}{2} - \frac{1}{2}\left[\cos\left(\frac{\pi}{4} + x\right)\cdot\frac{e^{2x}}{2} - \int -\sin\left(\frac{\pi}{4} + x\right)\cdot\frac{e^{2x}}{2}dx\right]$$

$$= \sin\left(\frac{\pi}{4} + x\right)\cdot\frac{e^{2x}}{2} - \frac{1}{4}e^{2x}\cos\left(\frac{\pi}{4} + x\right) - \frac{1}{4}\int e^{2x}\cdot\sin\left(\frac{\pi}{4} + x\right)dx$$

$$\Rightarrow I\left(1 + \frac{1}{4}\right) = \frac{1}{4}e^{2x}\left\{2\sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right)\right\}$$

$$\Rightarrow \qquad I = \frac{1}{5}e^{2x}\left\{2\sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right)\right\}$$

Now, $\int_0^{\pi} e^{2x}\sin\left(\frac{\pi}{4} + x\right)dx$

$$= \frac{1}{5}\left[e^{2x}\left\{2\sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right)\right\}\right]_0^{\pi}$$

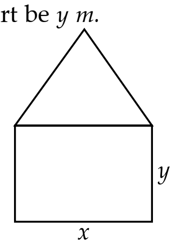
$$= \frac{1}{5}\left[e^{2\pi}\left\{2\sin\left(\pi + \frac{\pi}{4}\right) - \cos\left(\pi + \frac{\pi}{4}\right)\right\} - 2\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right]$$

$$= \frac{1}{5}\left[e^{2\pi}\left(-2\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{5}\left[-\frac{e^{2\pi}}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right]$$

$$\therefore \int_0^{\pi} e^{2x}\sin\left(\frac{\pi}{4} + x\right)dx = -\frac{1}{5\sqrt{2}}[e^{2\pi} + 1]$$

13. **(i)** Let the length of window be x m and breadth of rectangle part be y m.



$$\text{Perimeter} = 12\text{ m}$$

$$3x + 2y = 12$$

$$y = 6-\frac{3x}{2} \qquad \text{...(i)}$$

Area of window $A = xy+\frac{\sqrt{3}}{4}x^2$

$$A = x\left(6-\frac{3x}{2}\right)+\frac{\sqrt{3}}{4}x^2$$

$$A = 6x-\frac{3x^2}{2}+\frac{\sqrt{3}}{4}x^2$$

$$\frac{dA}{dx} = 6+\left(\frac{-3}{2}+\frac{\sqrt{3}}{4}\right)2x$$

For max./min. put $\frac{dA}{dx} = 0$

$$6+\left(\frac{\sqrt{3}}{4}-\frac{3}{2}\right)2x = 0$$

$$\left(\frac{\sqrt{3}-6}{4}\right)x = -3$$

$$x = \frac{12}{6-\sqrt{3}} = \frac{12(6+\sqrt{3})}{36-3}$$

$$= \frac{4}{11}(6+\sqrt{3})$$

$$\frac{d^2A}{dx^2} = 0+2\left(\frac{\sqrt{3}}{4}-\frac{3}{2}\right) < 0$$

Hence, area is maximum at $x = \frac{4}{11}(6+\sqrt{3})$

$$y = 6-\frac{3x}{2}$$

$$y = 6-\frac{12}{22}(6+\sqrt{3})$$

$$y = \frac{66-36-6\sqrt{3}}{11} = \frac{30-6\sqrt{3}}{11}$$

$$y = \frac{6(5-\sqrt{3})}{11}$$

Hence, dimension of window be $\frac{4}{11}(6+\sqrt{3})$ m and $\frac{6}{11}(5-\sqrt{3})$ m

OR

(ii) **(a)** Volume of cylinder $= 20\pi$

$$\pi r^2h = 20\pi$$

$$h = \frac{20}{r^2}$$

(b) $\text{T.S.A} = 2\pi r^2 + 2\pi rh$

$$= 2\pi r^2+\frac{40\pi}{r}$$

(c) Cost of painting $= 2 \times 2\pi r^2 + 25 \times \frac{40\pi}{r}$

$$= 4\pi r^2+\frac{1000\pi}{r}$$

$$C(r) = 4\pi r^2+\frac{1000\pi}{r}$$

$$\frac{dC}{dr} = 8\pi r-\frac{1000\pi}{r^2}$$

For max/min $\frac{dC}{dr} = 0$

$$8\pi r-\frac{1000\pi}{r} = 0$$

$$r^3 = \frac{1000}{8}$$

$$r = \frac{10}{2} = 5\text{ cm}$$

$$\frac{d^2C}{dr^2} = 8\pi+\frac{2000\pi}{r^3}$$

$$\left(\frac{d^2C}{dr^2}\right)_{r=5} = 8\pi+\frac{2000\pi}{125} > 0$$

Hence, cost is minimum at r = 5cm

(d) Minimum cost $C(r) = 4\pi(5)^2+\frac{1000\pi}{5}$

$$= ₹\ 300\,\pi$$

$$= ₹\ \frac{300\times 22}{7}$$

$$= ₹\ 942.85 \qquad \text{(Approx)}$$

14. **(i)** We know that

$$\Sigma P_i = 1$$

$\therefore$ $p + 0.3 + q + 0.1 = 1$

$\Rightarrow$ $p + q = 0.6$ ...(i)

Also, $E(x) = \Sigma x\, P_i(x)$

$\Rightarrow$ $2 = p + 0.6 + 3q + 0.4$

[Given E(x) = 2]

$\Rightarrow$ $p + 3q = 1$ ...(ii)

On solving Eqs. (i) and (ii), we get

$p = 0.4$ and $q = 0.2$...(iii)

(ii) Now, $p\,(X > 2) = 1 - p(X \le 2)$

$= 1 - [p\,(1) + p(2)]$

$= 1 - [p + 0.3]$

$= 1 - [0.4 + 0.3]$ [From eq. (iii)]

$= 1 - 0.7$

$= 0.3$

(iii) Ajay already scored 4 points in 3 rolls. So, if in remaining 2 rolls, if he gets 6 or more points, then only he will win the game.

Total possible cases of next two rolls are {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)}

Favourable cases are {(2, 4), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)}

Required probability $= \frac{6}{16} = \frac{3}{8}$

Section-B

15. **(i) Option (a) is correct.**

Explanation: Let the position vector of the R divides the join of points $2\vec{a}-3\vec{b}$ and $\vec{a}+\vec{b}$.

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$\therefore$ Position vector, $R = \dfrac{3(\vec{a}+\vec{b})+1(2\vec{a}-3\vec{b})}{3+1} = \dfrac{5\vec{a}}{4}$

Hence, Statement 1 is correct.
Statement 2 is incorrect because the given formula is for external division.

(ii) Option (d) is correct

Explanation:

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$
$= 1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$
$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$
$= 3 - 1 = 2$

(iii) Option (c) is correct.

Explanation :

D.C′ of x axis $= <1, 0, 0>$

(iv) Projection $\vec{b}$ on $\vec{a}$ $= \dfrac{\vec{a}.\vec{b}}{|\vec{a}|}$

$$= \frac{(2\hat{i}+\hat{j}+2\hat{k}).(5\hat{i}-3\hat{j}+\hat{k})}{\sqrt{2^2+1^2+2^2}}$$

$$= \frac{10-3+2}{3}$$

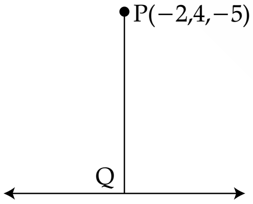
$= 3$ units

(v) Here, P $(-2, 4, -5)$ is the given point.
Any point Q on the line is given by $(3\lambda-3, 5\lambda+4, (6\lambda-8)$,

$$\overrightarrow{(PQ)} = \overrightarrow{(OQ)} - \overrightarrow{(OP)}$$

$$\overline{PQ} = (3\lambda-1)\hat{i} + 5\lambda\hat{j} + (6\lambda-3)\hat{k}.$$

Since, $\overline{PQ} \perp (3\hat{i}+5\hat{j}+6\hat{k})$,we have
$3(3\lambda-1) + 5(5\lambda) + 6(6\lambda-3) = 0$



$9\lambda + 25\lambda + 36\lambda = 21$, i.e., $\lambda = \dfrac{3}{10}$

Thus $\overline{PQ} = -\dfrac{1}{10}\hat{i} + \dfrac{15}{10}\hat{j} - \dfrac{12}{10}\hat{k}$

Hence, $\left|\overline{PQ}\right| = \dfrac{1}{10}\sqrt{1+225+144} = \sqrt{\dfrac{37}{10}}$ units

16. (i) Let, $\vec{a} = 4\hat{i} - 3\hat{j} + 12\hat{k}$

$\Rightarrow \quad |\vec{a}| = |\sqrt{4^2+(-3)^2+12^2}\,|$

$|\sqrt{16+9+144}\,| = \sqrt{169} = 13$

So, its unit vector is:

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{4\hat{i}-3\hat{j}+12\hat{k}}{13}$$

$$52\hat{a} = 52 \times \frac{4\hat{i}-3\hat{j}+12\hat{k}}{13}$$

$$= 4(4\hat{i}-3\hat{j}+12\hat{k})$$

Required vector $= 16\hat{i} - 12\hat{j} + 48\hat{k}$

OR

(ii) $A = (2, -3, 2),\ B = (2, 3, 1)$

$\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$

Area of triangle OAB $= \dfrac{1}{2}\left|\overline{OA} \times \overline{OB}\right|$

$$= \frac{1}{2}\left|(2\hat{i}-3\hat{j}+2\hat{k}) \times (2\hat{i}+3\hat{j}+\hat{k})\right|$$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left|\{\hat{i}(-3-6) - \hat{j}(2-4) + \hat{k}(6+6)\}\right|$$

$$= \frac{1}{2}\left|\{\hat{i}(-9) - \hat{j}(-2) + \hat{k}(12)\}\right|$$

$= \dfrac{1}{2}\left|-9\hat{i} + 2\hat{j} + 12\hat{k}\right|$ square units

Or area of ΔOAB

$= \dfrac{1}{2}\sqrt{81+4+144} = \dfrac{1}{2}\sqrt{299}$ square units

[ISC Marking Scheme 2024]

Examiner's Comment

- Some students commit error in writing the formula for area of triangle OAB. They did not put $\dfrac{1}{2}$ with $|\overrightarrow{OA} \times \overrightarrow{OB}|$.

Answering Tip

- Always remember the correct formula for area of triangle OAB is:

  Area of triangle $OAB = \dfrac{1}{2}|\overrightarrow{OA} \times \overrightarrow{OB}|$.

17. (i) Let the direction ratios of the line be a, b, c and the direction ratios of two given lines are $<1, 2, 3>$ and $<-3, 2, 5>$

Since, the line is perpendicular to the other two lines therefore

$$a + 2b + 3c = 0$$
$$-3a + 2b + 5c = 0$$

$$\frac{a}{10-6} = \frac{b}{-9-5} = \frac{c}{2+6} \Rightarrow \frac{a}{4} = \frac{b}{-14} = \frac{c}{8}$$

$$\Rightarrow \quad \frac{a}{2} = \frac{b}{-7} = \frac{c}{4}$$

Direction ratios of required line are $<2, -7, 4>$
And the line passes through the point $(2, 1, 3)$
Hence, the equation will be :

$\Rightarrow \dfrac{x-2}{2} = \dfrac{y-1}{-7} = \dfrac{z-3}{3}$ **[ISC Marking Scheme 2019]**

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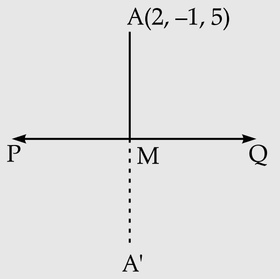
OR

(ii) Let A(2, –1, 5) be the point and PQ be the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$.



Let M be any point on the line PQ is

$$M(10r+11, -4r-2, -11r-8)$$

d.r.´s of AM $10r+9, -4r-1, -11r-13$

AM is perpendicular to PQ

$$\therefore \quad a_1a_2+b_1b_2+c_1c_2=0$$

$$10(10r+9)-4(-4r-1)-11(-11r-13)=0$$

$$100r+90+16r+4+121r+143=0$$

$$237r=-237$$

$$\therefore \quad r=-1$$

$\therefore$ M is (1, 2, 3)

M is the mid point of AA′,

$$\frac{2+x}{2}=1, \frac{-1+y}{2}=2, \frac{5+z}{2}=3$$

Coordinates of A′, are (0, 5, 1)

Length of the perpendicular $=|\sqrt{(-1)^2+3^2+(-2)^2}\,|$

$=|\sqrt{14}\,|$ units

[ISC Marking Scheme 2016]

Examiner's Comment

- Candidate made mistakes in calculating a point M on the line PQ and then applying the condition to prove that AM ⊥ PQ, i.e., $a_1a_2+b_1b_2+c_1c_2=0$.
  Some made errors while calculating the length of perpendicular.

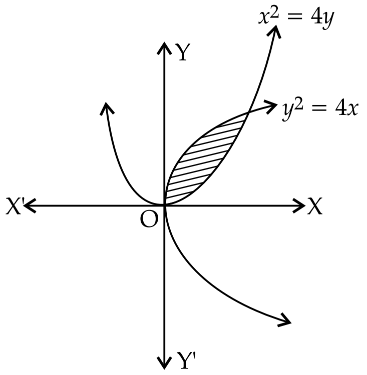
Answering Tip

- Need to emphasise on complete understanding of perpendicularity condition at different stages. Do practice in solving problems based on the concept of image of a given point with reference to a line.

18. (i)

$$y^2 = 4x$$
$$x^2 = 4y$$

Points of intersection of the curves are (0, 0) and (4, 4)



(ii)

$$\int(y_2-y_1)dx = \int\left(2\sqrt{x}-\frac{x^2}{4}\right)dx$$
$$= \frac{2x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^3}{12}$$

(iii)

$$\text{Required area} = \int_0^4 (y_2-y_1)dx$$
$$= \left(\frac{2x^{3/2}}{\frac{3}{2}}-\frac{x^3}{12}\right)_0^4$$
$$= \left[\frac{4}{3}x^{3/2}-\frac{x^3}{12}\right]_0^4$$
$$= \left[\frac{4}{3}\times 8-\frac{64}{12}-0\right]$$
$$= \frac{32}{3}-\frac{16}{3}$$
$$= \frac{16}{3}\text{ units}^2$$

Section-C

19. (i) Option (c) is correct.

Explanation:

For $MC > AC$

⇒ AC increases with x and AC curve is rising.

(ii) Option (a) is correct.

Explanation: The purpose is to explain the variation of the dependent variable.

(iii) Here, the given values are:

$\bar{x}=25, \bar{y}=30, b_{yx}=1.6$ and $b_{xy}=0.4$

(a) Regression equation y on x is given as:

$$y-\bar{y} = b_{yx}(x-\bar{x})$$
$$y-30 = 1.6(x-25)$$
$$y-30 = \frac{16}{10}(x-25)$$
$$5y-150 = 8x-200$$
$$8x-5y-50 = 0 \qquad \text{...(i)}$$

(b) Put $x = 60$ in eq. (i), we obtain

$$8(60)-5y-50 = 0$$
$$5y = 480-50$$
$$5y = 430$$
$$y = 86$$

(iv)

$$MR(x) = 408 - 17x$$

$$\int MR(x) = \int (408 - 17x)dx = 408x - 17\frac{x^2}{2} + C$$

$$R(x) = 408x - \frac{17}{2}x^2 + C$$

When $x = 0, R(x) = 0$

$\therefore$ $C = 0$

$$R(x) = 408x - \frac{17}{2}x^2$$

$$R'(x) = 408 - 17x$$

For Maximum/Minimum

$$R'(x) = 0$$

$$408 - 17x = 0$$

$$\Rightarrow \quad x = \frac{408}{17} = 24$$

$$R''(x) = -17 < 0$$

Hence, Maximum revenue obtained at $x = 24$

$\therefore$ Manufacturer should limit the production to 24 units.

[ISC Specimen Paper Marking Scheme 2024]

(v)

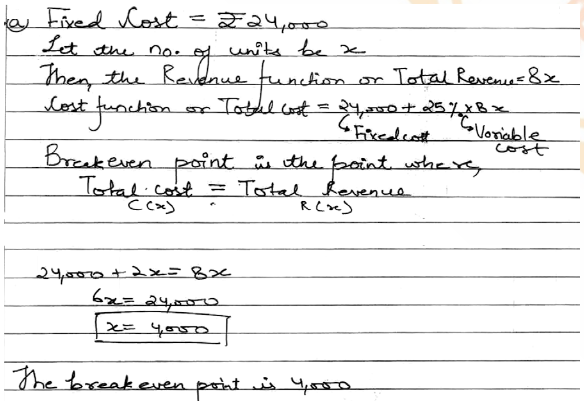
$$C(x) = \frac{x^3}{3} + 5x^2 - 16x + 2$$

$$M.C. = \frac{3x^2}{3} + 10x - 16$$

$$= x^2 + 10x - 16$$

20. **(i)**

Topper's Answer, 2019



ⓐ Fixed Cost = ₹24,000

Let the no. of units be x

Then the Revenue function or Total Revenue = $8x$

Cost function or Total cost = 24,000 + 25% × 8x

↳ Fixed cost ↳ Variable cost

Breakeven point is the point where,

Total cost = Total Revenue

C(x) R(x)

24,000 + 2x = 8x

6x = 24,000

$\boxed{x = 4,000}$

The breakeven point is 4,000

OR

(ii) Given, average cost function, $AC = 2x - 11 + \frac{50}{x}$

$$\text{Now,} \quad \frac{d}{dx}(AC) = \frac{d}{dx}\left(2x - 11 + \frac{50}{x}\right)$$

$$= 2 - \frac{50}{x^2}$$

AC increases, $\frac{d}{dx}(AC) > 0$

$$\Rightarrow \quad 2 - \frac{50}{x^2} > 0$$

$$\Rightarrow \quad x^2 - 25 > 0$$

$$\Rightarrow \quad (x-5)(x+5) > 0$$

$$\Rightarrow \quad x < -5 \text{ or } x > 5$$

$\because$ x is positive, AC increases, when $x > 5$.

[ISC Marking Scheme 2018]

Examiner's Comment

- Most of the candidates attempted the first part correctly but while calculating the value of x when AC was increasing, some candidates included the negative value of x.

Answering Tip

- Analyse in detail about conditions of increasing and decreasing functions and simplification of problems based on those concepts.

21. (i)

x_i	y_i	x_i^2	y_i^2	x_iy_i
2	8	04	64	16
6	8	36	64	48
4	5	16	25	20
7	6	49	36	42
5	2	25	04	10
$\Sigma x_i = 24$	$\Sigma y_i = 29$	$\Sigma x_i^2 = 130$	$\Sigma y_i^2 = 193$	$\Sigma x_i y_i = 136$

$n = 5$

$$b_{yx} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$$

$$= \frac{136 - \frac{24 \times 29}{5}}{130 - \frac{(24)^2}{5}}$$

$$= \frac{680 - 696}{5(130 - 115.2)}$$

$$\therefore \quad b_{yx} = \frac{-16}{5(14.8)} = \frac{-16}{74}$$

$$= \frac{-8}{37} = -0.2162 \text{ (approx.)}$$

$$b_{xy} = \frac{\Sigma xy - \frac{1}{n}\Sigma x \Sigma y}{\Sigma y^2 - \frac{1}{n}(\Sigma y)^2}$$

$$= \frac{136 - \frac{1}{5}24 \times 29}{193 - \frac{1}{5}(29)^2}$$

$$= \frac{680 - 696}{5(193 - 168.2)} = \frac{-16}{5 \times 24.8}$$

$$= \frac{-16}{124} = \frac{-4}{31}$$

$$\therefore \quad b_{xy} = -0.129 \text{ (approx.)}$$

$$r = \pm\sqrt{b_{xy} \times b_{yx}}$$

$$= -\sqrt{\frac{8}{37} \times \frac{4}{31}} = -\sqrt{\frac{32}{1147}}$$

$$= -\sqrt{0.02790}$$

$$r = -0.1670 \text{ (approx.)}$$

[ISC Marking Scheme 2020]

OR

(ii) Given: Correlation coefficient $r = 0.6$

$$\text{Mean of } x = \bar{x} = 8$$
$$\text{Mean of } y = \bar{y} = 6$$
$$\text{S.D. of } x = \sigma_x = 12$$
$$\text{S.D. of } y = \sigma_y = 4$$

(a)
$$b_{xy} = \frac{r\sigma_x}{\sigma_y} = \frac{0.6 \times 12}{4}$$
$$= 1.8$$
$$b_{yx} = \frac{r\sigma_y}{\sigma_x} = \frac{0.6 \times 4}{12}$$
$$= 0.2$$

(b) Regression line y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$
$$y - 6 = 0.2\,(x - 8)$$
$$y = 0.2x - 1.6 + 6$$
$$= 0.2x + 4.4$$

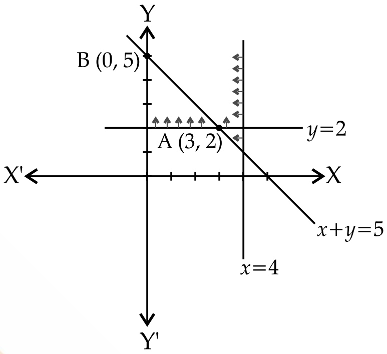
At $x = 20$
$$= 0.2 \times 20 + 4.4$$
$$= 4 + 4.4$$
$$y = 8.4$$

[ISC Marking Scheme 2023]

22. Let x kg of B_1 and y kg of B_2 are taken for making brick.

Here, $Z = 5x + 8y$ is the cost which is objective function and is to be maximised subjected to the following constraints.

$$x + y = 5 \quad \text{...(i)}$$
$$x \le 4 \quad \text{...(ii)}$$
$$y \ge 2 \quad \text{...(iii)}$$
$$x \ge 0, y \ge 0 \quad \text{...(iv)}$$



Therefore, maximum or minimum value of objective function Z exist on end points of line (constraint) (i) in feasible region i.e., at A or B.

At A (3, 2), $Z = 5 \times 3 + 8 \times 2 = 15 + 16 = 31$

At B (0, 5), $Z = 5 \times 0 + 8 \times 5 = 0 + 40 = 40$

Hence, cost of brick is minimum when 3 kg of B_1 and 2 kg of B_2 are taken.

❑❑

Sample Question Paper-4

MATHEMATICS

Class-12

SOLVED

Time Allowed : 2½ hours **Maximum Marks : 80**

You are allowed an additional 15 minutes for only reading the paper.
You must NOT start writing during reading time.
The question paper has 12 printed pages.
The Question Paper is divided into three sections and has 22 questions in all.

Section A is compulsory and has fourteen questions.
You are required to attempt all questions either from Section B or Section C.
Section B and Section C have four questions each.
Internal choices have been provided in two questions of 2 marks, two questions of 4 marks and two questions of 6 marks in Section A.
Internal choices have been provided in one question of 2 marks and one question of 4 marks each in Section B and Section C.
While attempting Multiple Choice Questions in Section A, B and C, you are required to write only ONE option as the answer.
The intended marks for questions or parts of questions are given in the brackets [].
All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
Mathematical tables and graph papers are provided.
Instruction to Supervising Examiner:
Kindly read aloud the instructions given above to all the candidates present in the examination hall.

Section-A [65 marks]

1. In subparts **(i)** to **(xi)** choose the correct options and in subparts **(xii)** to **(xv)**, answer the questions as instructed.

(i) If $A^2 - A + I = 0$, then the inverse of A is:

(a) A **(b)** $A + I$ **(c)** $I - A$ **(d)** $A - I$ **[Understanding] [1]**

(ii) Given that $\frac{1}{y}+\frac{1}{x}=\frac{1}{12}$ and y decreases at a rate of 1 cms^{-1}, find the rate of change of x when $x = 5$ cm and $y = 1$ cm

(a) 25 cm/s **(b)** 24 cm/s **(c)** 12 cm/s **(d)** 20 cm/s **[Recall][1]**

(iii) The domain of function $\cos^{-1}(2x - 1)$ is:

(a) $[0, 1]$ **(b)** $[-1, 1]$ **(c)** $(-1, 1)$ **(d)** $[0, \pi]$

AI **[Application][1]**

(iv) Assertion: The order of differential equation is $5=\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^4$ is:

Reason: The order of a differential equation is the order of the highest derivative appearing in the differential equation.

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true. **[Analysis] [1]**

(v) If ***A*** and ***B*** are two events such that $P(A) \neq 0$ and $P(B/A) = 1$, then

(a) $A \subset B$ **(b)** $B \subset A$ **(c)** $B = \phi$ **(d)** $A = \phi$

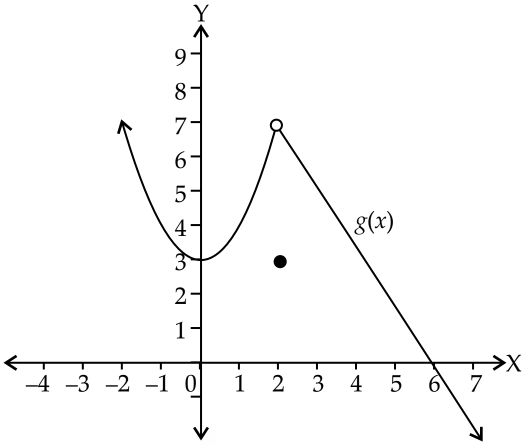
AI **[Application] [1]**

(vi) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = p$, then what is the value of $\begin{vmatrix} 9a & 9b & 72c \\ d & e & 8f \\ g & h & 8i \end{vmatrix}$?

(a) $8p$ **(b)** $72p$ **(c)** $9p$ **(d)** None of the above

[Understanding] [1]

(vii) Shown below is the graph of a function, $g(x)$. **[1]**



Statement 1: $g(x)$ is neither continuous nor differentiable at $x = 2$
Statement 2: $g(x)$ is not continuous but differentiable at $x = 2$

Which of the following is true?

(a) Statement 1 is true and Statement 2 is false. (b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true. (d) Both the statements are false. **[Application] [1]**

(viii) Find the derivative of $y = \log x + \dfrac{1}{x}$ with respect to x.

(a) $\dfrac{x^2}{x-1}$ **(b)** $\dfrac{x+1}{x^2}$ **(c)** $\dfrac{x-1}{x^2}$ **(d)** None of the above

[Application] [1]

(ix) **Statement 1:** The function $f : N \to N$ given by $f(x) = 5x$ is one to one and onto.
Statement 2: The function $f : R \to R$ given by $f(x) = 5x$ is one to one and onto.
Which one of the following options is correct?
(a) Statement 1 implies Statement 2.
(b) Statement 2 implies Statement 1.
(c) Statement 1 is true only if Statement 2 is true.
(d) Statements 1 and 2 are independent of each other **[Understanding] [1]**

(x) Assertion **(A)** : If the order of A is 3×4, then order of B is 3×4 and the order of C is 5×4, then the order of (A^TB) C^T is 4×5.

Reason **(R)**: The multiply an $m\times n$ matrix by $n\times p$ matrix then n must be the same and result is an $m\times p$ matrix. Also, A be a matrix of order $m\times n$ then the order of transpose matrix is $n\times m$.

Which of the following is correct?

(a) Both A and R are true, and R is the correct explanation of A.

(b) Both A and R are true, but R is not the correct explanation of A.

(c) A is true but R is false.

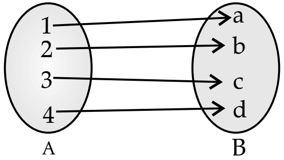
(d) A is false and R is true. **[Application] [1]**

(xi) If Z is a random variable defined as the number of heads minus number of tails in a simultaneous toss of 6 coins, the all possible values of Z are:
(a) {–6, –4, –2, 0, 2, 4, 6} **(b)** {0, 2, 4, 6} **(c)** {–6, –4, –2, 0, 2} **(d)** {–4, –2, 0, 2, 4}
[Understanding] [1]

(xii) If $A = \begin{vmatrix} 6 & 8 & -3 \\ -3 & -13 & 0 \\ 6 & -8 & 0 \end{vmatrix}$ write the minor of the element a_{13}. **[Analysis] [1]**

(xiii) If a relation R on the set A={1, 2, 3} be defined by R={(1, 2), (2, 1)} then classify the function R.
[Understanding] [1]

(xiv) Check whether the below represented figure shows an onto function or not. Give reason. **[Understanding] [1]**



(xv) Teena is practicing for an upcoming Rifle Shooting tournament. The probability of her shooting the target in the 1st, 2nd, 3rd and 4th shots are 0.4, 0.3, 0.2 and 0.1 respectively. Find the probability of at least one shot of Teena hitting the target.

[Application] [1]

2. **(i)** $y = e^{\sin x^2}$, find $\dfrac{dy}{dx}$. **[Understanding] [2]**

OR

(ii) The length x of a rectangle is decreasing at the rate of 5 cm/min and the width y increasing at the rate of 4 cm/min, find the rate of change of its area when $x = 5$ cm and $y = 8$ cm. AI **[Understanding]**

3. Evaluate: $\int \log_{10} x\, dx$ **[Evaluate] [2]**

4. Find the point on the curve $y = x^2 + 2x - 1$, where the tangent is parallel to x-axis. **[Analysis] [2]**

5. **(i)** Evaluate : $\int_0^2 \sqrt{4-x^2}\, dx$ AI **[Evaluate] [2]**

OR

(ii) Evaluate: $\int_0^{\pi/4} \log(1+\tan\theta)\, d\theta$ **[Evaluate] [2]**

6. Solve:

$\sin(2\tan^{-1}x) = 1.$ **[Application] [2]**

7. If $\tan^{-1}\left(\dfrac{x-1}{x+1}\right)+\tan^{-1}\left(\dfrac{2x-1}{2x+1}\right)=\tan^{-1}\left(\dfrac{23}{36}\right)$ then prove that $24x^2 - 23x - 12 = 0$ AI **[Application] [4]**

8. Evaluate: $\int_{-6}^{3} |x+3|\, dx$ AI **[Application] [4]**

9. **(i)** A 13 m long ladder is leaning against a wall, touching the wall at a certain height from the ground level. The bottom of the ladder is pulled away from the wall, along the ground, at the rate of 2 m/s. How fast is the height of the wall decreasing when the foot of the ladder is 5 m away from the wall? **[Application] [4]**

OR

(ii) If the area enclosed between the curves $y = ax^2$ and $x = ay^2$, $a > 0$ is 27 sq. unit, find the value of 'a'. **[Analysis]**

10. **(i)** There are two boxes, Box 1 and Box 2. Each box has 8 red balls, 5 green balls, and 7 blue balls. Sahil takes one ball from Box 1 and places it in Box 2. Then he draws two balls from Box 2 simultaneously.

(a) What is the probability of Sahil selecting a green ball from Box 1?

(b) After Sahil transfers a ball from Box 1 to Box 2, what is the probability of drawing one red ball and one green ball simultaneously from Box 2?

(c) Given that the balls drawn from Box 2 are red and green, what is the probability that the ball transferred from Box 1 to Box 2 was green? AI **[Analysis] [4]**

OR

(ii) There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an aeroplane are 0.3 and 0.2 respectively. Both of them fired one shell at an aeroplane at the same time.

(a) How is Bayes' theorem different from conditional probability?

(b) Write the rule of Total Probability.

(c) What is the probability that the shell fired from exactly one of them hit the plane? **[Application]**

11. A shopkeeper has 3 varieties of pen '*A*', '*B*' and '*C*'. Meenu purchased 1 pen of each variety for a total of ₹ 21. Jeevan purchased 4 pens of 'A' variety, 3 pen say 'B' variety and 2 pens of 'C' variety for ₹ 60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for ₹ 70.

(i) Form a set of simultaneous equations for the above information.

(ii) Solve the set of equations formed in **(i)** by matrix method.

(iii) Hence, find the cost of each variety of pen. **[Application] [6]**

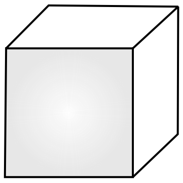
12. **(i)** Evaluate: $\int \dfrac{x+7}{x^2+4x+7}\, dx$ AI **[Application][6]**

OR

(ii) Evaluate : $\int_0^{\pi/2} \dfrac{\cos^2 x}{1+\sin x\cos x}\, dx$ AI **[Evaluate]**

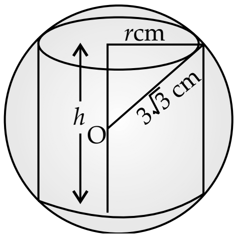
13. **(i)** A village chief wants to build a cuboidal water tank with a square base and an open top to store 50,000 Liters of water. The cost of the stainless-steel sheet required for construction is ₹ 60,000 per square metre.

Find the dimensions of the tank that will minimise the cost of the stainless steel sheet and determine the minimum cost. AI **[Application] [6]**



OR

(ii) A paper weight manufacturer is designing a paper weight in the form of a cylinder inscribed in a sphere. He wants to keep the volume of the cylinder to be maximum. The radius of the sphere is $3\sqrt{3}$ cm. Find the maximum volume of the cylinder.



[Application]

14. An urn contains 25 balls of which 10 balls are red and the remaining green. A ball is drawn at random from the urn, the colour is noted and the ball is replaced. If 6 balls are drawn in this way, find the probability that :

(i) All the balls are red.

(ii) Not more than 2 balls are green.

(iii) Number of red balls and green balls are equal. **AI [Analysis] [6]**

Section-B [15 marks]

15. In subparts **(i)** to **(iii)** choose the correct options and in subparts **(iv)** and **(v)**, answer the questions as instructed.**[5]**

(i) Consider the following statements and choose the correct option:

Statement 1: The cross product of two vectors $\vec{a}$ and $\vec{b}$ is defined by $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\ \tilde{n}$ where θ is the angle between $\vec{a}$ and $\vec{b}, 0 \le \theta \le \pi$ and $\tilde{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

Statement 2 : The dot product of two vectors $\vec{a}$ and $\vec{b}$ is defined by $\vec{a}\,.\,\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

where θ is the angle between $\vec{a}$ and $\vec{b}, 0 \le \theta \le \pi$

Which of the following is correct?

(a) Only Statement 1 **(b)** Only Statement 2

(c) Both Statements 1 and 2 **(d)** Neither Statement 1 nor Statement 2 **AI [Recall]**

(ii) If $P(1, 0, -3)$ is the foot of perpendicular from the origin to the plane, then the Cartesian equation of the plane is:

(a) $x + y - 3z = 12$ **(b)** $x - 3z = 5$ **(c)** $x - 3z = 10$ **(d)** $x + y - 3z = 10$

[Analysis]

(iii) The direction cosines of y-axis are:

(a) $<1, 1, 1>$ **(b)** $<0, 1, 0>$ **(c)** $<1, 0, 0>$ **(d)** $<0, 1, 1>$

AI [Understanding]

(iv) If $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} - \gamma\hat{k}$ such that they are perpendicular to each other, then the value of γ.

AI [Understanding]

(v) Find the equation of the plane passing through $(-2, 1, 3)$ and perpendicular to the line having direction ratios $<3, 1, 5>$. **[Analysis]**

16. **(i)** If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are non-zero vectors, then prove that either $\vec{b} = \vec{c}$ or $\vec{a} = (\vec{b} - \vec{c})$ are parallel.

AI [Application] [2]

OR

(ii) If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $|\vec{a} \times \vec{b}| = \vec{a}\,.\,\vec{b}$, find the angle between $\vec{a}$ and $\vec{b}$.

AI [Application]

17. **(i)** A line passes through the point $(2, -1, 3)$ and is perpendicular to the lines

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k}) \text{ and}$$

$$\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k}).$$

Obtain its equation. **AI [Application] [4]**

OR

(ii) Find the equation of the plane passing through the point $(1, 1, -1)$ and perpendicular to the planes $x + 2y + 3z = 7$ and $2x - 3y + 4z = 0$. **[Application]**

18. **(i)** Sketch the region enclosed bounded by the curve, $y^2 = 16x$ and $y = 4mx$

(ii) If the area is bounded by the parabola $y^2 = 16x$ and the line $y = 4mx$ is $\frac{1}{12}$ sq units, then using integration, find the value of m $(m > 0)$. **AI [4]**

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Section-C [15 marks]

19. In subparts **(i)** and **(ii)** choose the correct options and in subparts **(iii)** to **(v)**, answer the questions as instructed. **[5]**

(i) If the demand function is given by $p = 1500 - 2x - x^2$ then find the marginal revenue when $x = 10$

(a) 1160 **(b)** 1600 **(c)** 1100 **(d)** 1200 **[Recall]**

(ii) Read the following statements and choose the correct option:

(I) The straight line graph of the linear equation $Y = a + bX$, the slope will be upward if $b = 0$
(II) The straight line graph of the linear equation $Y = a + bX$, the slope will be upward if $b < 0$
(III) The straight line graph of the linear equation $Y = a + bX$, the slope will be upward if $b > 0$
(IV) The straight line graph of the linear equation $Y = a + bX$, the slope will be upward if $b \neq 0$

Which of the following is correct?

(a) Only III is correct **(b)** Only III and II are correct
(c) Only II and IV are correct **(d)** Only III and I are correct **[Recall]**

(iii) The equations of two lines of regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find the mean value of x and y. **[Understanding]**

(iv) If the total cost function is given by $C = x + 2x^3 - \frac{7}{2}x^2$, find the Marginal Average Cost function **(MAC)**.

A I **[Understanding]**

(v) The manufacturer of a pen fixes its selling price at ₹ 45, and the cost function is $C(x) = 30x + 240$. The manufacturer will begin to earn profit if he sells more than 16 pens. Why? Give one reason. **[Understanding]**

20. (i) A monopolist's demand function is $x = 60 - \frac{p}{5}$. At what level of output will marginal revenue be zero?

A I **[Application] [2]**

OR

(ii) The Average Cost function associated with producing and marketing x units of an item is given by $AC = x + 5 + \frac{36}{x}$.

(a) Find the Total Cost function.

(b) Find the range of values of x for which Average Cost is increasing. A I **[Application]**

21. (i) Find the equation of the regression line of y on x, if the observations (x, y) are as follows :
(1, 4), (2, 8), (3, 2), (4, 12), (5, 10), (6, 14), (7, 16), (8, 6), (9, 18).
Also, find the estimated value of y when $x = 14$. A I **[Application] [4]**

OR

(ii) The line of regression of marks in Statistics **(X)** and marks in Accountancy **(Y)** for a class of 50 students is $3y - 5x + 180 = 0$. The average score in Accountancy is 44 and the variance of marks in Statistics is $\left(\frac{9}{16}\right)^{th}$ of variance of marks in Accountancy.

(a) Find the average score in Statistics.

(b) Find the coefficient of correlation between marks in Statistics and marks in Accountancy. **[Application]**

22. A soap company manufactures two varieties of soap using two machines. The time taken on each machine to manufacture one batch of each variety of soap is given below.

	Machine 1	**Machine 2**
Soap A	1 hour	1 hour
Soap B	2 hours	1 hour

Machine 1 and Machine 2 cannot be operational for more than 12 and 10 hours in a day, respectively. The profit earned from one batch of Soap A and one batch of Soap B is ₹ 3000 and ₹ 4000, respectively.

Find the number of batches of each kind of soap that should be made daily to maximise the profit.

(i) Formulate this situation as an L.P.P.

(ii) Solve it graphically **[Analysis] [4]**

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SOLUTIONS

Sample Question Paper - 4

Section-A

1. (i) Option (c) is correct.

Explanation:

$$A^2 - A + I = 0$$

Multiplying with A^{-1}

$\Rightarrow \quad A - I + A^{-1} = 0$

$\Rightarrow \quad A^{-1} = I - A$

(ii) Option (a) is correct.

Explanation: $\dfrac{1}{y} + \dfrac{1}{x} = \dfrac{1}{12}$

Derivative on both sides w.r.t. t :

$$\frac{d}{dt}\frac{1}{y} + \frac{d}{dt}\frac{1}{x} = \frac{d}{dt}\frac{1}{12}$$

$$-\frac{1}{y^2}\frac{dy}{dt} - \frac{1}{x^2}\frac{dx}{dt} = 0$$

$$\frac{1}{x^2}\frac{dx}{dt} = -\frac{1}{y^2}\frac{dy}{dt}$$

$$\frac{dx}{dt} = -\frac{x^2}{y^2}\frac{dy}{dt}$$

Substitute the given values and get :

$$\frac{dx}{dt} = -\frac{25}{1}(-1) = 25 \text{ cm/s}$$

(iii) Option (a) is correct.

Explanation:

We have $\cos^{-1}(2x - 1)$

$\Rightarrow -1 \le 2x - 1 \le 1 \qquad \therefore \{ x [-1, 1]]$

$\Rightarrow 0 \le 2x \le 2$

$\Rightarrow 0 \le x \le 1$

$\Rightarrow x \in [0, 1]$

(iv) Option (a) is correct.

Explanation: $5 = \dfrac{d^2y}{dx^2} - \left(\dfrac{dy}{dx}\right)^4$ is the differential equation of order 2 because highest order derivative of y w.r.t. x is $\dfrac{d^2y}{dx^2}$.

(v) Option (a) is correct.

Explanation: $P(A) \neq 0$ and $P\left(\dfrac{B}{A}\right) = 1$

$\therefore \quad P\left(\dfrac{B}{A}\right) = \dfrac{P(A \cap B)}{P(A)}$

$$1 = \frac{P(A \cap B)}{P(A)}$$

$\Rightarrow \quad P(A) = P(A \cap B)$

$\therefore \quad A \subset B$

(vi) Option (b) is correct.

Explanation: Given, $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = p \qquad \ldots(i)$

Now, $\begin{vmatrix} 9a & 9b & 72c \\ d & e & 8f \\ g & h & 8i \end{vmatrix} = 9\begin{vmatrix} a & b & 8c \\ d & e & 8f \\ g & h & 8i \end{vmatrix}$

(By taking 9 common from 1st row)

$$= 9 \times 8 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

(By taking 8 common from 1st column)

$= 9 \times 8 \times p \qquad$ [Using (i)]

$= 72p$

(vii) Option (a) is correct.

Explanation: It is evident from the graph, where the function is discontinuous.

$\Rightarrow$ The function discontinuous at $x=2$

Also, if a function is not continuous then it is not differentiable.

Thus, $g(x)$ is neither continuous nor differentiable at $x = 2$

Hence, Statement 1 is true, and Statement 2 is false.

(viii) Option (c) is correct.

Explanation: $y = \log x + \dfrac{1}{x}$

$$\frac{dy}{dx} = \frac{d}{dx}\log x + \frac{d}{dx}\frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x^2} = \frac{x-1}{x^2}$$

(ix) Option (d) is correct.

Explanation:

Statement 1: To check one–one:

$$5x_1 = 5x_2$$

i.e., $\quad x_1 = x_2$

Therefore, the function is one–one.

To check onto:

Let $\quad y = 5x$

$\Rightarrow \quad x = \dfrac{y}{5}$

If $y = 1$, then $x = \dfrac{1}{5} = 0.2$, which is not a natural number.

Thus, the function is not one-one.

Statement 2 : To check one-one:

$$5x_1 = 5x_2$$

i.e. $\quad x_1 = x_2$

Therefore, the function is one-one.

To check onto:

Let, $\quad y = 5x$

$\Rightarrow \quad x = \dfrac{y}{5}$

If $y = 1$, then $x = \dfrac{1}{5} = 0.2$, which is a real number

Thus, the function is onto.

Hence, Statements 1 and 2 are independent of each other.

(x) Option (a) is correct.

Explanation:

$$
\begin{aligned}
&\text{Order of } A = 3 \times 4\\
&\text{Order of } B = 3 \times 4\\
&\text{Order of } C = 5 \times 4\\
\text{The,}\quad &\text{Order of } A^T = 4 \times 3\\
\text{and}\quad &\text{Order of } C^T = 4 \times 5\\
\text{Now,}\quad &\text{Order of } (A^TB) = 4 \times 4\\
\text{Thus,}\quad &\text{Order of } (A^TB)C^T = 4 \times 5
\end{aligned}
$$

(xi) Option (a) is correct.

Explanation:

Here, Z = No. of heads – No. of tails

No. of heads	No. of tails	Values of Z
0	6	$0 - 6 = -6$
1	5	$1 - 5 = -4$
2	4	$2 - 4 = -2$
3	3	$3 - 3 = 0$
4	2	$4 - 2 = 2$
5	1	$5 - 1 = 4$
6	0	$6 - 0 = 6$

(xii) Given,
$$A = \begin{vmatrix} 6 & 8 & -3 \\ -3 & -13 & 0 \\ 6 & -8 & 0 \end{vmatrix}$$

$$
\begin{aligned}
\therefore\quad \text{Minor of } a_{13} &= \begin{vmatrix} -3 & -13 \\ 6 & -8 \end{vmatrix}\\
&= 24 - (-78)\\
&= 24 + 78\\
&= 102
\end{aligned}
$$

(xiii) $A=\{1,2,3\}$ and $R=\{(1,2),(2,1)\}$ it is reflexive since $(1,2) \in R$ and $(2,1) \in R$.

(xiv) In the given function, element 'b' has no pre-image. So, it is not onto.

(xv) Probability of at least one hitting the target
$$
\begin{aligned}
&= 0.4 \times 0.3 \times 0.2 \times 0.1\\
&= 0.0024
\end{aligned}
$$

Probability of not hitting the target is
$$
\begin{aligned}
&= (1 - 0.4)\,(1 - 0.3)\,(1 - 0.2)\,(1 - 0.1)\\
&= 0.6 \times 0.7 \times 0.8 \times 0.9\\
&= 0.336 \times 0.9\\
&= 0.3024
\end{aligned}
$$

Probability of at least one shot hitting the target is
$$1 - 0.3024 = 0.6976$$

2. (i) Given, $y = e^{\sin x^2}$

Let $y = e^z$, where $z = \sin t$ and $t = x^2$

On differentiating both sides of t w.r.t. x, we get
$$\frac{dt}{dx} = 2x \qquad \ldots\text{(i)}$$

On differentiating both sides of z w.r.t. x, we get
$$\frac{dz}{dt} = \cos t \qquad \ldots\text{(ii)}$$

On differentiating both sides of y w.r.t. z, we get
$$\frac{dy}{dz} = e^z \qquad \ldots\text{(iii)}$$

Using chain rule of derivative,
$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dt} \times \frac{dt}{dx} = e^z \times \cos t \times 2x$$

[from eqs. (i), (ii) and (iii)]

$$
\begin{aligned}
&= 2x\, e^{\sin t} \cos t\\
&= 2xe^{\sin x^2} \cos x^2
\end{aligned}
$$

OR

(ii) Let A denote the area of rectangle at instant t.

$$
\begin{aligned}
\therefore\quad A &= xy \qquad \text{(area of rectangle)}\\
\text{Given,}\quad \frac{dx}{dt} &= -5 \text{ cm/min}\\
\frac{dy}{dt} &= 4 \text{ cm/min}\\
\therefore\quad \frac{dA}{dt} &= x\frac{dy}{dt} + y\frac{dx}{dt}\\
\Rightarrow\quad \left(\frac{dA}{dt}\right)_{x=5,\,y=8} &= 5 \times 4 + 8 \times (-5) \text{ cm}^2\text{/min}\\
\Rightarrow\quad \frac{dA}{dt} &= (20 - 40) \text{ cm}^2\text{/min}\\
\frac{dA}{dt} &= -20 \text{ cm}^2\text{/min}
\end{aligned}
$$

Here, (–) ve sign shows that area is decreasing at the rate of 20 cm^2/min.

3.
$$
\begin{aligned}
\int (\log_{10} x)dx &= \int (\log_{10} x).1dx\\
&= (\log_{10} x)x - \int \left(\frac{1}{x}\log_{10} e\right) x\, dx\\
&\qquad\qquad \text{[Using integration by parts]}\\
&= x\log_{10} x - \log_{10} e \int 1dx\\
&= x\log_{10} x - x\log_{10} e + C
\end{aligned}
$$

4. Given curve, $y = x^2 + 2x - 1$

Differentiating w.r.t. x

$$
\begin{aligned}
\Rightarrow\quad \frac{dy}{dx} &= 2x + 2\\
&= \text{slope of tangent}
\end{aligned}
$$

Since, tangent is parallel to x-axis

$$
\begin{aligned}
\text{Slope of tangent} &= 0\\
\Rightarrow\quad 2x + 2 &= 0\\
\Rightarrow\quad 2x &= -2\\
\Rightarrow\quad x &= -1\\
\therefore\quad y &= (-1)^2 + 2(-1) - 1\\
&= 1 - 2 - 1 = -2
\end{aligned}
$$

$\therefore$ (–1,– 2) is the required point.

[ISC Specimen Paper Marking Scheme 2024]

Examiner's Comment

- This question was solved correctly by most of the candidates. A few candidates made mistakes while finding the point of contact at which the equation of tangent to the curve where tangent is parallel to the x-axis.

Answering Tip

- Understand the geometrical interpretation of differentiation with ample examples.

5. (i) $\int_0^2 \sqrt{4-x^2}\,dx = \int_0^2 \sqrt{2^2-x^2}\,dx$

$$= \left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= [0+2\sin^{-1}(1)]-[0+2\sin^{-1}0]$$

$$= 2\times\frac{\pi}{2}-(0+2\times0)=\pi$$

OR

(ii) Let $I = \int_0^{\pi/4}\log(1+\tan\theta)\,d\theta$ ...(i)

$$= \int_0^{\frac{\pi}{4}}\log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\frac{\pi}{4}} \log\left[1+\left\{\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right\}\right]d\theta$$

$$\left[\because \tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}\right]$$

$\therefore$ $I = \int_0^{\frac{\pi}{4}}[\log 2-\log(1+\tan\theta)]d\theta$ ...(ii)

Adding eqs. (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{4}}\log 2\,d\theta$$

$$= \log 2[\theta]_0^{\frac{\pi}{4}} = \log 2.\frac{\pi}{4}$$

$\therefore$ $I = \frac{\pi}{8}.\log 2$

[ISC Marking Scheme, 2017]

Examiner's Comment

- Many candidates made errors in applying the properties of definite integrals correctly and some candidates attempted it by integration by parts.

Answering Tip

- Learn the properties of definite integrals well and do adequate practice to apply them appropriately.

6. $\sin(2\tan^{-1}x) = 1$

$$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$\therefore$ $\left(\frac{2x}{1+x^2}\right) = 1$

$\Rightarrow$ $x^2-2x+1 = 0$

$\Rightarrow$ $(x-1)^2 = 0$

$\Rightarrow$ $x-1 = 0 \Rightarrow x = 1$

Alternative method:

$\sin(2\tan^{-1}x) = 1,$

$\sin(2\tan^{-1}x) = \sin\frac{\pi}{2}$

$\Rightarrow$ $2\tan^{-1}x = \frac{\pi}{2}$

$\Rightarrow$ $\tan^{-1}x = \frac{\pi}{4}$

$\Rightarrow$ $\tan^{-1}x = \frac{\pi}{4}$

$\Rightarrow$ $x = 1$ **[ISC Marking Scheme, 2019]**

7. $\tan^{-1}\left(\frac{x-1}{x+1}\right)+\tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\left(\frac{23}{36}\right)$

$$\left[\tan^{-1}A+\tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-1}{x+1}+\frac{2x-1}{2x+1}}{1-\left(\frac{x-1}{x+1}\right)\left(\frac{2x-1}{2x+1}\right)}\right) = \tan^{-1}\left(\frac{23}{36}\right)$$

$$\Rightarrow \tan^{-1}\left[\frac{(x-1)(2x+1)+(2x-1)(x+1)}{(x+1)(2x+1)-(x-1)(2x-1)}\right] = \tan^{-1}\left(\frac{23}{36}\right)$$

$$\Rightarrow \tan^{-1}\left[\frac{(2x^2-x-1)+(2x^2+x-1)}{(2x^2+3x+1)-(2x^2-3x+1)}\right] = \tan^{-1}\left(\frac{23}{36}\right)$$

$$\Rightarrow \tan^{-1}\left[\frac{4x^2-2}{6x}\right] = \tan^{-1}\left(\frac{23}{36}\right)$$

$$\Rightarrow \frac{4x^2-2}{6x} = \frac{23}{36}$$

$$\Rightarrow \frac{2x^2-1}{3x} = \frac{23}{36}$$

$\Rightarrow$ $36(2x^2-1) = 23(3x)$

$\Rightarrow$ $12(2x^2-1) = 23x$

$\Rightarrow$ $24x^2-23x-12 = 0$ **Hence Proved**

8. $\int_{-6}^{3}|x+3|\,dx = \int_{-6}^{-3}-(x+3)dx+\int_{-3}^{3}(x+3)dx$

$$\left[\because \int_a^b f(x)dx = \int_a^c f(x)dx+\int_c^b f(x)dx\right]$$

$x+3>0 \Rightarrow x>-3$

$x+3<0 \Rightarrow x<-3$

$\Rightarrow -(x+3)>0$

$$= \int_{-3}^{-6}(x+3)dx+\int_{-3}^{3}(x+3)dx$$

$$\left[\because \int_a^b f(x)dx = \int_b^a -f(x)dx\right]$$

$$= \left[\frac{x^2}{2}+3x\right]_{-3}^{-6}+\left[\frac{x^2}{2}+3x\right]_{-3}^{3}$$

$= 22.5$ **[ISC Marking Scheme, 2019]**

Examiner's Comment

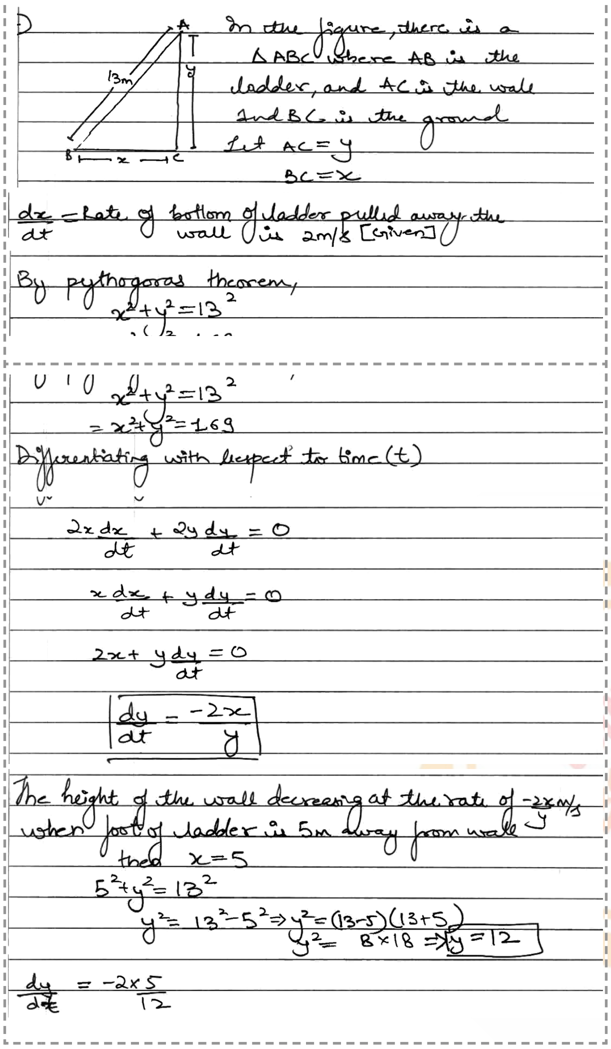
- Many candidates made errors while splitting the given interval into two parts. Some candidates made calculation errors even after applying correct limits.

Answering Tip

- Understand the concept of splitting the given interval into two or more intervals in case of absolute functions and properties of definite integrals.

9. (i)

Topper Answer, 2019



In the figure, there is a ΔABC where AB is the ladder, and AC is the wall And BC is the ground

Let $AC = y$

$BC = x$

$\frac{dx}{dt}$ = Rate of bottom of ladder pulled away the wall is 2m/s [Given]

By pythogoras theorem,

$x^2+y^2=13^2$

$x^2+y^2=13^2$

$= x^2+y^2=169$

Differentiating with respect to time (t)

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

$2x + y\frac{dy}{dt} = 0$

$\boxed{\frac{dy}{dt} = \frac{-2x}{y}}$

The height of the wall decreasing at the rate of $\frac{-2x}{y}$ m/s when foot of ladder is 5m away from wall then $x=5$

$5^2+y^2=13^2$

$y^2 = 13^2-5^2 \Rightarrow y^2=(13-5)(13+5)$

$y^2 = 8\times 18 \Rightarrow \boxed{y=12}$

$\frac{dy}{d\cancel{t}} = \frac{-2\times 5}{12}$

Examiner's Comment

- Several candidates could not form the function. Many candidates made errors while differentiating the function with reference to time and ended up differentiating with respect to 'x' instead of time 't'.

Answering Tip

- Understand the concept of application of derivatives under different topics.

OR

(ii) Given, $y = ax^2$, $x = ay^2$

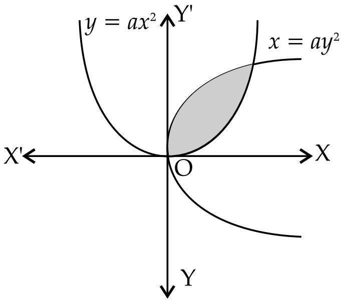
Substitute value of y in x :

$$x = a(ax^2)^2$$

$$x = 0, \frac{1}{a} \text{ and } y = 0, \frac{1}{a}$$

Point of intersection is (0, 0) and $\left(\frac{1}{a}, \frac{1}{a}\right)$

$$\Rightarrow \int_0^{\frac{1}{a}} \left[\sqrt{\frac{x}{a}} - ax^2\right] dx = 27$$



$$\Rightarrow \left[\frac{1}{\sqrt{a}}\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{ax^3}{3}\right]_0^{\frac{1}{a}} dx = 27$$

$$\Rightarrow \frac{2}{3a^2} - \frac{1}{3a^2} = 27$$

$$\Rightarrow a^2 = \frac{1}{81} = \pm\frac{1}{9}$$

10. (i) (a) P(green ball from box 1) = $\frac{5}{20} = \frac{1}{4}$

(b) There are three cases:

Case 1:

Let ball transfer from box 1 to box 2 was red

P(drawing one red and one green ball)

$$= \frac{{}^9C_1 \times {}^5C_1}{{}^{21}C_1} = \frac{9\times 5\times 2}{21\times 20}$$

Case 2:

Let ball transfer from box 1 to box 2 was green

P(drawing one red and one green ball)

$$= \frac{{}^8C_1 \times {}^6C_1}{{}^{21}C_2} = \frac{8\times 6\times 2}{21\times 20}$$

Case 3:

Let ball transfer from box 1 to box 2 was blue.

P(drawing one red ball and one green ball)

$$= \frac{{}^8C_1 \times {}^5C_1}{{}^{21}C_2} = \frac{8\times 5\times 2}{21\times 20}$$

P(after transfer one ball from box 1 to box 2, then drawing one red ball and one green ball from box 2)

$$= \frac{9\times 5\times 2}{21\times 20} + \frac{8\times 6\times 2}{21\times 20} + \frac{8\times 5\times 2}{21\times 20}$$

$$= \frac{45}{210} + \frac{48}{210} + \frac{40}{210} = \frac{133}{210}$$

(c) Let E_1: Red ball transfer from box 1 to box 2

Let E_2: Green ball transfer from box 1 to box 2

Let E_3: Blue ball transfer from box 1 to box 2

A: After transfer one ball from box 1 to box 2, then one red ball and one green ball

$$P(E_1) = \frac{8}{20}$$

$$P(E_2) = \frac{5}{20}$$

$$P(E_3) = \frac{7}{20}$$

$$P\left(\frac{A}{E_1}\right) = \frac{{}^9C_1 \times {}^5C_1}{{}^{21}C_1} = \frac{9\times5\times2}{21\times20} = \frac{45}{210}$$

$$P\left(\frac{A}{E_2}\right) = \frac{{}^8C_1 \times {}^6C_1}{{}^{21}C_2} = \frac{8\times6\times2}{21\times20} = \frac{48}{210}$$

$$P\left(\frac{A}{E_3}\right) = \frac{{}^8C_1 \times {}^5C_1}{{}^{21}C_2} = \frac{8\times5\times2}{21\times20} = \frac{40}{210}$$

Now, $P\left(\frac{E_2}{A}\right)$

$$= \frac{P(E_2).P\left(\frac{A}{E_2}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right) + P(E_3).P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{5}{20}\times\frac{48}{210}}{\frac{8}{20}\times\frac{45}{210} + \frac{5}{20}\times\frac{48}{210} + \frac{7}{20}\times\frac{40}{210}} = \frac{3}{11}$$

Examiner's Comment

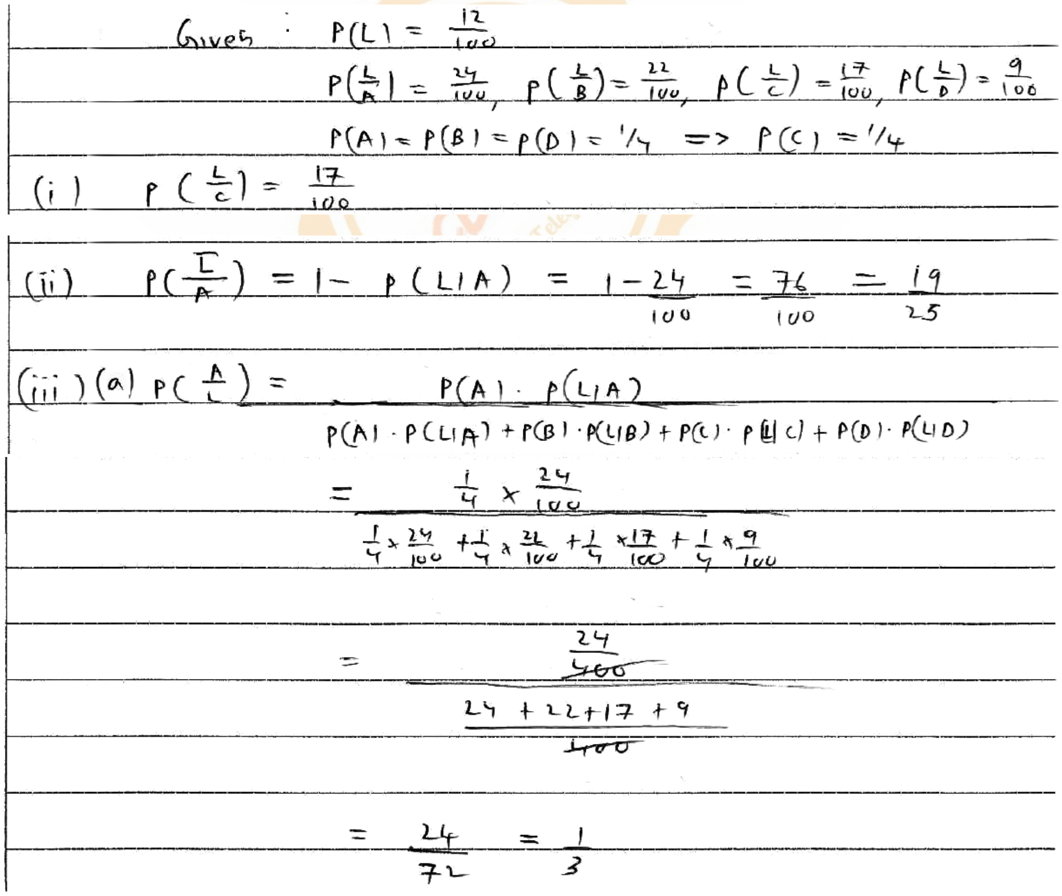
- Most of the candidates attempted this question correctly but a few got confused while presenting the solution and result in proper way.

Answering Tip

- Understand the concepts of probability by doing plenty of examples.

OR

(ii) **Topper Answer, 2023**



Given : $P(L) = \frac{12}{100}$

$P\left(\frac{L}{A}\right) = \frac{24}{100}$, $P\left(\frac{L}{B}\right) = \frac{22}{100}$, $P\left(\frac{L}{C}\right) = \frac{17}{100}$, $P\left(\frac{L}{D}\right) = \frac{9}{100}$

$P(A) = P(B) = P(D) = 1/4 \Rightarrow P(C) = 1/4$

(i) $P\left(\frac{L}{C}\right) = \frac{17}{100}$

(ii) $P\left(\frac{\bar{L}}{A}\right) = 1 - P(L|A) = 1 - \frac{24}{100} = \frac{76}{100} = \frac{19}{25}$

(iii) (a) $P\left(\frac{A}{L}\right) = \dfrac{P(A)\cdot P(L|A)}{P(A)\cdot P(L|A) + P(B)\cdot P(L|B) + P(C)\cdot P(L|C) + P(D)\cdot P(L|D)}$

$$= \frac{\frac{1}{4}\times\frac{24}{100}}{\frac{1}{4}\times\frac{24}{100} + \frac{1}{4}\times\frac{22}{100} + \frac{1}{4}\times\frac{17}{100} + \frac{1}{4}\times\frac{9}{100}}$$

$$= \frac{\frac{24}{\cancel{400}}}{\frac{24+22+17+9}{\cancel{400}}}$$

$$= \frac{24}{72} = \frac{1}{3}$$

11. **(i)** Let the cost of 'A' variety pen be x, the cost of 'B' variety pen be y, the cost of 'C' variety pen be z.
According to the question,

$$x + y + z = 21$$
$$4x + 3y + 2z = 60$$
$$6x + 2y + 3z = 70$$

(ii) It can be written as,

$$\begin{bmatrix}1 & 1 & 1\\4 & 3 & 2\\6 & 2 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}21\\60\\70\end{bmatrix}$$

i.e., $AX = B$

where, $A = \begin{bmatrix}1 & 1 & 1\\ 4 & 3 & 2\\ 6 & 2 & 3\end{bmatrix}, X = \begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B = \begin{bmatrix}21\\ 60\\ 70\end{bmatrix}$

Now, $|A| = \begin{bmatrix}1 & 1 & 1\\ 4 & 3 & 2\\ 6 & 2 & 3\end{bmatrix}$

$= 1\,(9-4) - 1(12-12) + 1(8-18)$

$= 5 - 10 = -5 \neq 0$

$\Rightarrow A^{-1}$ exists.

$$\begin{aligned}
A_{11} &= 9-4 = 5\\
A_{12} &= -(12-12) = 0\\
A_{13} &= (8-18) = -10\\
A_{21} &= -(3-2) = -1\\
A_{22} &= 3-6 = -3\\
A_{23} &= -(2-6) = 4\\
A_{31} &= 2-3 = -1\\
A_{32} &= -(2-4) = 2\\
A_{33} &= 3-4 = -1
\end{aligned}$$

$$adj\,A = \begin{bmatrix}5 & 0 & -10\\ -1 & -3 & 4\\ -1 & 2 & -1\end{bmatrix}^T = \begin{bmatrix}5 & -1 & -1\\ 0 & -3 & 2\\ -10 & 4 & -1\end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(adj\,A)$$

Also, $AX = B$

$\Rightarrow \quad X = A^{-1}B$

$\Rightarrow \quad X = \frac{1}{|A|}(adj\,A)\,B$

$$= \frac{1}{|A|}\begin{bmatrix}5 & -1 & -1\\ 0 & -3 & 2\\ -10 & 4 & -1\end{bmatrix}\begin{bmatrix}21\\ 60\\ 70\end{bmatrix}$$

$$= \frac{1}{-5}\begin{bmatrix}105-60-70\\ 0-180+140\\ -210+240-70\end{bmatrix}$$

$$= \frac{1}{-5}\begin{bmatrix}-25\\ -40\\ -40\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}5\\ 8\\ 8\end{bmatrix}$$

$\therefore x = 5, y = 8, z = 8$

(iii) Cost of 'A' variety pen = ₹ 5

'B' variety pen = ₹ 8

'C' variety pen = ₹ 8

12. Given $\int \frac{x+7}{x^2+4x+7}dx$

$$\int \frac{x+7}{x^2+4x+7}dx = \int \frac{\frac{1}{2}(2x+4)+5}{x^2+4x+7}dx$$

$$\int \frac{x+7}{x^2+4x+7}dx = \frac{1}{2}\int \frac{2x+4}{x^2+4x+7}dx + \int \frac{5}{x^2+4x+7}dx$$

$$\int \frac{x+7}{x^2+4x+7}dx = \frac{1}{2}\log|x^2+4x+7| + \int \frac{5}{(x+2)^2+3}dx$$

$$\int \frac{x+7}{x^2+4x+7}dx = \frac{1}{2}\log|x^2+4x+7|$$

$$+\frac{5}{\sqrt{3}}\tan^{-1}\left|\frac{x+2}{\sqrt{3}}\right| + C$$

OR

(ii) Let $I = \int_0^{\pi/2} \frac{\cos^2 x}{1+\sin x\cos x}dx$ ...(i)

Apply property

$$\int_0^a f(a)\,dx = \int_0^a f(a-x)\,dx$$

$$I = \int_0^{\pi/2} \frac{\cos^2\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}dx$$

$\therefore \quad I = \int_0^{\pi/2} \frac{\sin^2 x}{1+\cos x\sin x}dx$ ...(ii)

Adding eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{1}{1+\sin x\cos x}dx$$

$[\because \sin^2\theta + \cos^2\theta = 1]$

Dividing N and D by $\cos^2 x$, we get

$\therefore \quad 2I = \int_0^{\pi/2} \frac{\sec^2 x}{\sec^2 x + \tan x}dx$

Let $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

Also, when $x = 0, t = 0$ and $x = \frac{\pi}{2}, t = \infty$

$\therefore$

$$2I = \int_0^{\infty} \frac{dt}{t^2+t+1}$$

$$= \int_0^{\infty} \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$2I = \frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)\right]_0^{\infty}$$

$$2I = \frac{2}{\sqrt{3}}\left[\frac{\pi}{2} - \frac{\pi}{6}\right]$$

$\therefore \quad 2I = \frac{2\pi}{3\sqrt{3}}$

or $\quad I = \frac{\pi}{3\sqrt{3}}$

[ISC Marking Scheme, 2018]

Examiner's Comment

- A number of candidates could not attempt the question correctly due to incorrect use of properties of definite integrals and using incorrect methods to evaluate integrals.

Answering Tip

- Practice as many problems as you can for successful application of properties of definite integrals and its further simplification.

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13. **(i)** **For least cost:**

Let $l = x$, $b = x$ and $h = y$

$\Rightarrow V = lbh = x \times x \times y = x^2y$

And TSA $= x^2 + 4xy$

V is constant

Cost least when TSA least

$$S = x^2 + 4xy$$

$$\Rightarrow \quad S = x^2 + \frac{4V}{x}$$

$$\Rightarrow \quad \frac{dS}{dx} = 2x - \frac{4V}{x^2}$$

$$\Rightarrow \quad \frac{d^2S}{dx^2} = 2 + \frac{8V}{x^3}$$

For maximum and minimum value of x

$$\frac{dS}{dx} = 0$$

$$\Rightarrow \quad 2x - \frac{4V}{x^2} = 0$$

$$\Rightarrow \quad 2x^3 = 4V$$

$$\Rightarrow \quad 2x^3 = 4x^2y$$

$$\Rightarrow \quad x = 2y$$

$$\frac{d^2S}{dx^2} = 2 + \frac{8V}{x^3} > 0 \ \forall \ x$$

Here S is minimum, when $x = 2y$

$$l = x = 2y$$
$$b = x = 2y$$
$$h = y = y$$

$$\Rightarrow \quad 50000 = 2y \times 2y \times y$$

$$\Rightarrow \quad 50000 = 4y^3$$

$$\Rightarrow \quad 12500 = y^3$$

$$\Rightarrow \quad y = 23.20$$

$$\Rightarrow \quad x = 2(23.20)$$

$$= 46.40$$

Now, $l = 46.4$, $b = 46.4$ and $h = 23.2$

TSA of cuboid – area of square

$\Rightarrow 2(lb + bh + hl) - (\text{side})^2$

$\Rightarrow 2(2152.96 + 1076.48 + 1076.48) - 2152.96$

$\Rightarrow 2(4305.92) - 2152.96$

$\Rightarrow 8611.84 - 2152.96$

$\Rightarrow 6458.88 \text{ m}^2$

Cost of 1m^2 steel = ₹60000

Cost of building the tank

= 60000×6458.88

= ₹38,75,32,800

OR

(ii) Maximum volume of a cylinder which can fit inside a sphere

$$V_c = \pi R^2 h$$

$$\Rightarrow \quad \left(\frac{h}{2}\right)^2 + R^2 = r^2 \qquad \text{(from figure)}$$

$$\Rightarrow \quad \frac{h^2}{4} + R^2 = r^2$$

$$\Rightarrow \quad R^2 = r^2 - \frac{h^2}{4}$$

Now,
$$V_c = \pi\left(r^2 - \frac{h^2}{4}\right)h$$

$$V_c = \pi r^2 h - \pi\frac{h^3}{4}$$

$$\frac{dV}{dh} = \pi r^2 - \frac{3\pi}{4}h^2 = 0$$

$$\pi r^2 = \frac{3\pi}{4}h^2$$

$$\frac{4r^2}{3} = h^2$$

$$h = \frac{2r}{\sqrt{3}}$$

$$\frac{dv}{dh} = \pi r^2 - \frac{3\pi}{4}h^2$$

$$\frac{d^2v}{dh^2} = \frac{-6\pi}{4}h < 0$$

$\therefore$ Volume is maximum

$$V_c = \pi r^2\left(\frac{2r}{\sqrt{3}}\right) - \frac{\pi}{4}\left(\frac{2r}{\sqrt{3}}\right)^3$$

$$V_c = \pi r^2\left(\frac{2r}{\sqrt{3}}\right) - \pi\left(\frac{2r^3}{3\sqrt{3}}\right)$$

$$V_c = \frac{6\pi r^3 - 2\pi r^3}{3\sqrt{3}} = \frac{4\pi r^3}{3\sqrt{3}}$$

$$r = 3\sqrt{3} \quad \text{(given)}$$

$$\therefore \quad V_c = \frac{4\pi(3\sqrt{3})^3}{3\sqrt{3}} = 4\pi(3\sqrt{3})^2$$

$$= 4\pi(27) = 108\pi$$

14. **(i)** Number of red balls : 10, number of green balls: 15

$$P(R) = \frac{10}{25} = \frac{2}{5};\ P(G) = \frac{15}{25} = \frac{3}{5}$$

$$p = \frac{2}{5},\ q = \frac{3}{5}$$

Number of ways of drawing a red ball :

$$X = 0, 1, 2, 3, 4, 5, 6$$

$$P(X = r) = {}^nC_r . p^r . q^{n-r}$$

(i) All are red

$$P(X = 6) = {}^6C_6\left(\frac{2}{5}\right)^6\left(\frac{3}{5}\right)^0 = \left(\frac{2}{5}\right)^6$$

(ii) Not more than 2 are green :

$$P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6)$$

$$P(X \geq 4) = {}^6C_4\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^2 + {}^6C_5\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right)^1 + {}^6C_6\left(\frac{2}{5}\right)^6\left(\frac{3}{5}\right)^0$$

(iii) Equal number of balls *i.e.*, no. of red balls and green balls are equal

$$P(X = 3) = {}^6C_3\left(\frac{2}{5}\right)^3\cdot\left(\frac{3}{5}\right)^3 = 20 \times \frac{8}{125} \times \frac{27}{125} = \frac{864}{3125}$$

[ISC Marking Scheme, 2017]

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Section-B

15. (i) Option (c) is correct.

Explanation: Both the statement are correct because both are definitions of cross product and dot product of vectors.

(ii) Option (c) is correct.

Explanation: $P = (1, 0, -3)$

$$\vec{OP} = \hat{i} - 3\hat{k}$$

Equation of the required plane is

$$1(x-1) + 0(y-0) - 3(z+3) = 0$$

$$\Rightarrow \quad x - 3z - 10 = 0$$

$$\Rightarrow \quad x - 3z = 10$$

(iii) Option (b) is correct.

Explanation:

D.C′ of y-*axis* $= <0, 1, 0>$

(iv) When two vectors $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}.\vec{b} = 0$.

Therefore, $(2\hat{i} + 4\hat{j} - \hat{k}).(3\hat{i} - 2\hat{j} - \gamma\hat{k}) = 0$

$$\Rightarrow \quad 6 - 8 + \gamma = 0$$

$$\Rightarrow \quad \gamma = 2$$

(v) Given that, the equation of the plane is perpendicular to the line having direction ratios <3, 1, 5>

∴ Normal to the line is parallel to the line

∴ Required equation of plane passing through (–2, 1, 3) is $3(x + 2) + 1(y - 1) + 5(z - 3) = 0$

$$\therefore \quad 3x + y + 5z = 10$$

16. (i)

$$\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$$

$$\vec{a} \times \vec{b} - \vec{a} \times \vec{c} = 0$$

$$\vec{a} \times (\vec{b} - \vec{c}) = 0$$

Either $\vec{a}$ and $\vec{b} - \vec{c}$ is parallel vectors

or $\quad \vec{b} - \vec{c} = 0$

$$\vec{b} = \vec{c}$$

OR

(ii)

$$|\vec{a} \times \vec{b}| = \vec{a}.\vec{b}$$

$$|\vec{a}||\vec{b}|\sin\theta = |\vec{a}||\vec{b}|\cos\theta$$

$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4} \text{ or } 45°$$

17. (i) Given line are $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

A line perpendicular to the given lines will be in the direction of

$$(2\hat{i} - 2\hat{j} + \hat{k}) \times (\hat{i} + 2\hat{j} + 2\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix}$$

$$= -6\hat{i} - 3\hat{j} + 6\hat{k} \text{ or, } \vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$$

Position vector of given point (2, –1, 3) is.

$$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$$

Using $\vec{r} = \vec{a} + \lambda\vec{b}$, the required vector equation of line is :

$$\vec{r} = 2\hat{i} - \hat{j} + 3\hat{k} + \lambda(2\hat{i} + \hat{j} - 2\hat{k})$$

And, cartesian form of the line is :

$$\frac{x-2}{2} = \frac{y+1}{1} = \frac{z-3}{-2}$$

Examiner's Comment

- Most candidates did not apply the concept of perpendicular lines and condition for the same correctly. Some candidates made calculation errors while applying cross multiplication rule to find direction ratios.

Answering Tip

- Understand the different methods of finding the equation of straight line by applying different types of conditions.

OR

(ii) Let the equation of plane passing through point (1, 1, – 1) be

$$a(x-1) + b(x-1) + c(x+1) = 0 \quad \text{...(i)}$$

Eq (i) is perpendicular to the plane $x + 2y + 3z - 7 = 0$

$$\therefore \quad 1.a + 2.b + 3.c = 0$$

$$\Rightarrow \quad a + 2b + 3c = 0 \quad \text{...(ii)}$$

Again eq (i) is perpendicular to plane $2x - 3y + 4z = 0$

$$\therefore \quad 2.a - 3.b + 4.c = 0$$

$$\Rightarrow \quad 2a - 3b + 4c = 0 \quad \text{...(iii)}$$

On solving eqs (ii) and (iii), we get

$$\frac{a}{8+9} = \frac{b}{6-4} = \frac{c}{-3-4}$$

$$\Rightarrow \quad \frac{a}{17} = \frac{b}{2} = \frac{c}{-7} = k$$

∴ $a = 17k$, $b = 2k$ and $c = -7k$

Putting the values of a, b and c in eq (i), we get

$$17k(x-1) + 2k(y-1) - 7k(z+1) = 0$$

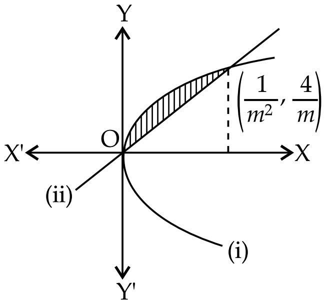
$$\Rightarrow \quad 17(x-1) + 2(y-1) - 7(z+1) = 0$$

$$\Rightarrow \quad 17x + 2y - 7z - 17 - 2 - 7 = 0$$

$$\Rightarrow \quad 17x + 2y - 7z - 26 = 0$$

[ISC Marking Scheme 2023]

18. (i)



Let, $\quad y^2 = 16x \quad$ ...(i)

$\quad y = 4mx \quad$ ...(ii)

By eq. (i) & (ii)

$$(4mx)^2 = 16x$$

or, $\quad 16x(m^2x - 1) = 0$

or, $\quad x = 0$, or, $x = \dfrac{1}{m^2}$

(ii) According to a question., area bounded by the curves

$$\int_0^{\frac{1}{m^2}} [y_{(i)} - y_{(ii)}]dx = \frac{1}{12}$$

$$\Rightarrow \int_0^{\frac{1}{m^2}} (4\sqrt{x} - 4mx)\, dx = \frac{1}{12}$$

$$\Rightarrow 4\int_0^{\frac{1}{m^2}} \sqrt{x}\, dx - 4m \int_0^{\frac{1}{m^2}} x\, dx = \frac{1}{12}$$

$$\Rightarrow \frac{8}{3}\left[x^{3/2}\right]_0^{\frac{1}{m^2}} - 2m\left[x^2\right]_0^{\frac{1}{m^2}} = \frac{1}{12}$$

$$\Rightarrow \frac{8}{3}\left(\frac{1}{m^2}\right)^{3/2} - 2m\left(\frac{1}{m^2}\right)^2 = \frac{1}{12}$$

$$\Rightarrow \frac{8}{3}\frac{1}{m^3} - \frac{2}{m^3} = \frac{1}{12}$$

$$\Rightarrow \left(\frac{8}{3} - 2\right)\frac{1}{m^3} = \frac{1}{12}$$

$$\Rightarrow \frac{2}{3}\frac{1}{m^3} = \frac{1}{12}$$

$$\Rightarrow m = 2$$

Section-C

19. (i) Option (a) is correct.

Explanation: Given, $p = 1500 - 2x - x^2$

Revenue function,

$$R = px$$
$$R = 1500x - 2x^2 - x^3$$

$$\text{Marginal revenue} = \frac{dR}{dx}$$
$$= \frac{d}{dx}(1500x - 2x^2 - x^3)$$
$$= 1500 - 4x - 3x^2$$

Marginal revenue at $x = 10$ is

$$= 1500 - 4(10) - 3(10)^2$$
$$= 1500 - 40 - 300 = 1160$$

[ISC Marking Scheme 2023]

(ii) Option (a) is correct.

Explanation: The straight line graph of the linear equation $Y = a + bX$, slope will be upward if $b > 0$.

(iii) $4x + 3y + 7 = 0$ ....(i)

$3x + 4y + 8 = 0$ ....(ii)

From (i) & (ii) $7x + 7y + 15 = 0$

By adding $x + y + \frac{15}{7} = 0$ ....(iii)

By subtracting $x - y - 1 = 0$ ....(iv)

From (iii) & (iv) $2x + \frac{8}{7} = 0$

$$x = \frac{-4}{7}$$

From (iii) $y = \frac{-11}{7}$

Hence, m mean value of x and y is $\frac{-4}{7}$ and $\frac{-11}{7}$.

(iv)

$$C = x + 2x^3 - \frac{7}{2}x^2$$

$$\text{Average cost} = \frac{C}{x} = 1 + 2x^2 - \frac{7}{2}x$$

$$\text{M.A.C} = \frac{d}{dx}\left(1 + 2x^2 - \frac{7}{2}x\right) = 4x - \frac{7}{2}$$

(v)

$$R(x) = p \times x = 45x$$
$$C(x) = 30x + 240$$
$$P(x) = R(x) - C(x)$$
$$= 45x - 30x - 240$$
$$= 15x - 240$$

To get profit $P(x) > 0$

$$15x - 240 > 0$$
$$x > \frac{240}{15}$$
$$x > 16$$

20. (i)

$$x = 60 - \frac{p}{5}$$

$\Rightarrow$ $300 - 5x = p$

$$R(x) = p \times x = x(300 - 5x)$$

$$\text{M.R.} = \frac{d}{dx}(300x - 5x^2) = 300 - 10x$$

Put, $\text{M.R} = 0$

$$300 - 10x = 0$$

$\therefore$ $x = 30$

(ii) **(a)** Total cost function $= x \times \text{A.C.} = x^2 + 5x + 36$

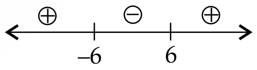
(a)

$$AC = x + 5 + \frac{36}{x}$$

$$\frac{d(AC)}{dx} = 1 + 0 - \frac{36}{x^2}$$

For increasing $\frac{d(AC)}{dx} > 0$

$$1 - \frac{36}{x^2} > 0$$
$$x^2 - 36 > 0$$
$$(x - 6)(x + 6) > 0$$



Range of increasing average cost is $x > 6$.

21. (i)

x_i	y_i	$x_i y_i$	x_i^2
1	4	4	1
2	8	16	4
3	2	6	9
4	12	48	16
5	10	50	25
6	14	84	36
7	16	112	49
8	6	48	64
9	18	162	81
$\sum x_i = 45$	$\sum y_i = 90$	$\sum x_i y_i = 530$	$\sum x^2_i = 285$

$$\therefore \quad b_{yx} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$$

$$= \frac{9 \times 530 - 45 \times 90}{9 \times 285 - (45)^2} = \frac{4}{3}$$

∴ Regression equation of y on x

$$y - 10 = \frac{4}{3}(x-5)$$

$$y = \frac{4}{3}x + \frac{10}{3}$$

Now, when $x = 14$

$$y = \frac{4}{3}\times 14 + \frac{10}{3} = 22$$

Examiner's Comment

- Many candidates wrote incorrect formula for regression coefficients which caused error in finding the regression equation y on x. Several candidate were unable to interpret the result of y for $x = 14$.

Answering Tip

- Practice a variety of problems based on regression.

OR

(ii) $3y - 5x + 180 = 0$

$$\bar{y} = 44, n = 50$$

$$\text{Variance of X} = \frac{9}{16}\ \text{Variance of Y}$$

Regression equation X on Y is

$$5x = 3y + 180$$

$$x = \frac{3}{5}y + \frac{180}{5} = \frac{3}{5}y + 36$$

$$\therefore \quad b_{xy} = \frac{3}{5} = 0.6$$

(a)

$$a = \bar{x} - b_{xy}\,\bar{y}$$

$$\bar{x} = a + b_{xy}\,\bar{y} = 36 + 0.6 \times 44$$

$$\bar{x} = 36 + 26.4 = 62.4$$

(b)

$$\sigma_x^2 = \frac{9}{16}\sigma_y^2$$

$$\therefore \quad \frac{\sigma_x}{\sigma_y} = \frac{3}{4}$$

$$b_{xy} = r \times \frac{\sigma_x}{\sigma_y}$$

$$\frac{3}{5} = r \times \frac{3}{4}$$

$$\therefore \quad r = \frac{4}{5} = 0.8$$

22. Let the number of batches of Soap A be x and Soap B be y.

Objective Function: To maximise the profit

$$Z = 3000x + 4000y$$

Subject to Constraints :

$$x + 2y \le 12$$

$$x + y \le 10$$

$$x \ge 0,\ y \ge 0$$

$x + 2y = 12$

x	0	12	6
y	6	0	3

$x + y = 10$

x	0	10	5
y	10	0	5

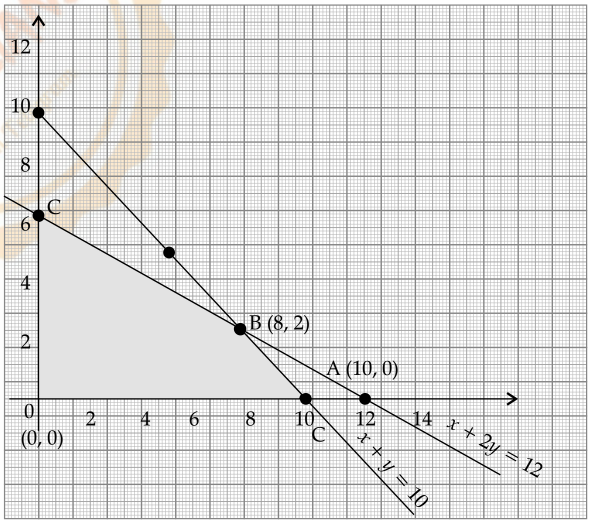
Feasible region

OABCO $\quad Z = 3000x + 4000y$

At O (0, 0) Z	3000 × 0 + 4000 × 0	0
At A (10, 0) Z	3000 × 10 + 4000 × 0	30000
At B (8, 2) Z	3000 × 8 + 4000 × 2	32000 max
At C (0, 6) Z	3000 ×0 + 4000 × 6	24000

For maximum profit, number of batches of soap A is 8 and soap B is 2

∴ Max Profit = ₹ 32000.



❑❑

Sample Question Paper-5

MATHEMATICS
Class-12

SOLVED

Time Allowed : 3 hours **Maximum Marks : 80**

You are allowed an additional 15 minutes for only reading the paper.
You must NOT start writing during reading time.
The question paper has 12 printed pages.
The Question Paper is divided into three sections and has 22 questions in all.

Section A is compulsory and has fourteen questions.
You are required to attempt all questions either from Section B or Section C.
Section B and Section C have four questions each.
Internal choices have been provided in two questions of 2 marks, two questions of 4 marks and two questions of 6 marks in Section A.
Internal choices have been provided in one question of 2 marks and one question of 4 marks each in Section B and Section C.
While attempting Multiple Choice Questions in Section A, B and C, you are required to write only ONE option as the answer.
The intended marks for questions or parts of questions are given in the brackets [].
All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
Mathematical tables and graph papers are provided.
Instruction to Supervising Examiner:
Kindly read aloud the instructions given above to all the candidates present in the examination hall.

Section-A [65 marks]

1. In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed. **[15]**

(i) If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, then the matrix A + A^T is

(a) Symmetric matrix **(b)** Skew-symmetric matrix
(c) Diagonal matrix **(d)** Identity matrix **[Understanding]**

(ii) If $\int\left(\cot x - \operatorname{cosec}^2 x\right)e^x dx = e^x f(x) + c$ then $f(x)$ then $f(x)$ will be

(a) $\cot x + \operatorname{cosec} x$ **(b)** $\cot^2 x$ **(c)** $\cot x$ **(d)** $\operatorname{cosec} x$ **[Recall]**

(iii) The value of $\tan^{-1}1 + \cos^{-1}\frac{1}{2}$ is

(a) $\frac{5\pi}{12}$ **(b)** $\frac{\pi}{4}$ **(c)** $\frac{\pi}{3}$ **(d)** $\frac{7\pi}{12}$ AI **[Application]**

(iv) Assertion (A): The degree of differential equation $\left(\frac{d^4y}{dx^4}\right)^2 - \left(\frac{dy}{dx}\right)^3 - 6 = 0$ is 2.

Reason (R): The degree of the differential equation is the power of the highest order derivative.
Which of the following is correct?
(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true. **[Analysis]**

(v) For the following probability distribution

X	−4	−3	−2	−1	0
P (X)	0.1	0.2	0.3	0.2	0.2

E(X) is equal to

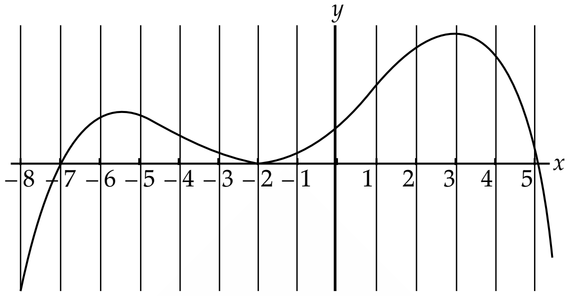
(a) 0 **(b)** −1 **(c)** −2 **(d)** −1.8 **[Application]**

(vi) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = k$, then what is the value of $\begin{vmatrix} 6a & b & c \\ 6d & e & f \\ 48g & 8h & 8i \end{vmatrix}$?

(a) $48k$ **(b)** $6k$ **(c)** $8k$ **(d)** None of these

[Understanding]

(vii) Shown below is the graph of a function, $g(x)$.



Statement 1: $(-\infty, -6)$ & $(-2, 3)$ are the intervals on which the $g(x)$ increases.
Statement 2: $(-\infty, -2)$ & $(-2, 5)$ are the intervals on which the $g(x)$ increases.
Which of the following is correct?

(a) Statement 1 is true and Statement 2 is false. **(b)** Statement 2 is true and Statement 1 is false.
(c) Both the statements are true. **(d)** Both the statements are false. **[Application]**

(viii) Differentiation of $\log(1 + x^2)$ with respect to $\tan^{-1} x$ is

(a) $\frac{1}{1+x^2}$ **(b)** $2x$ **(c)** $\frac{-1}{1+x^2}$ **(d)** $-x$ **[Application]**

(ix) **Statement 1:** Relation $R = \{(1, 1), (2, 2), (3, 3)\}$ on the set $\{1, 2, 3\}$ is symmetric, reflexive and transitive.
Statement 2: Relation $R = \{(1, 1), (2, 2), (3, 3)\}$ on the set $\{1, 2, 3\}$ is an equivalence relation.
Which one of the following is correct? **[Understanding]**

(a) Statement 1 implies Statement 2.
(b) Statement 2 implies Statement 1.
(c) Statement 1 is true only if Statement 2 is true.
(d) Statements 1 and 2 are independent of each other.

(x) Matrix P is a skew-symmetric matrix of odd order.
Assertion (A): The inverse of matrix P does not exist.
Reason (R): The determinant of matrix P is zero.
Which of the following is correct? **[Application]**

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true.

(xi) For the following probability distribution :

X	1	2	3	4
P(X)	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

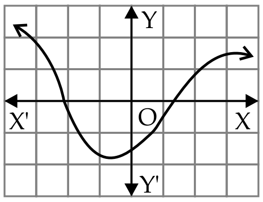
E(X^2) is equal to

(a) 3 **(b)** 5 **(c)** 7 **(d)** 10 **[Understanding]**

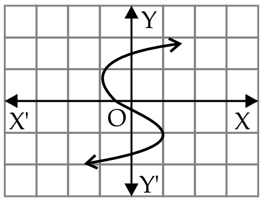
(xii) If $A = \begin{vmatrix} 2 & -6 & 0 \\ 0 & 9 & -4 \\ 5 & -8 & 0 \end{vmatrix}$, write the minor of the element a_{32}. **[Analysis]**

(xiii) If a relation $\mathbb{R}$ in $\mathbb{R}$ as aRb and $a \geq b$.
Then check whether R is an equivalence relation or not. **[Understanding]**

(xiv) Which one of the following graphs is a function of x ? **[Understanding]**



Graph A



Graph B

(xv) Dobleron, a chocolate is known for its distinctive shape and its taste. The quality assurance team of the brand undertakes quality test in two steps:

Step 1: 30 out of every 990 chocolate bars are randomly picked for physical examination, where the shape of the bar is examined.

Step 2: For every 30 chocolate bars whose shape is examined, 1 chocolate bar is randomly picked for chemical examination, where the composition and taste of the chocolate is examined.

If one chocolate bar is randomly picked out of 990 bars, what is the probability that it was not chemically examined, provided it was physically examined? **[Application]**

2. **(i)** If $x = a\left(\frac{1-t^2}{1+t^2}\right)$ and $y = \frac{2at}{1+t^2}$, prove that $\frac{dy}{dx} = -\frac{x}{y}$. Hence, prove that $\frac{d^2y}{dx^2} = -\left(\frac{x^2+y^2}{y^3}\right)$. **[Understanding] [2]**

OR

(ii) Find the point on the curve $y^2 = 8x + 3$ for which the y-coordinate changes 8 times as coordinate of x. AI **[Understanding]**

3. Evaluate: $\int_0^{2\pi} \frac{1}{\left(1+e^{\sin x}\right)} dx$ **[Evaluate] [2]**

4. Find a point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the line joining the chord through the points (2, 0) and (4, 4). **[Application] [2]**

5. **(i)** Evaluate: $\int \frac{5-3x^2}{6-x^3+5x} dx$ AI **[Evaluate] [2]**

OR

(ii) Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x| dx$ **[Evaluate]**

6. Prove that : $\cos[\tan^{-1}\{\cot(\sin^{-1} x)\}] = x$ **[Analysis] [2]**

7. Solve for x : $\sin^{-1}\left(\frac{x}{2}\right) + \cos^{-1} x = \frac{\pi}{6}$ AI **[Application] [4]**

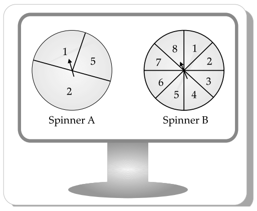
8. Evaluate: $\int x^2 \cos x \; dx$ AI **[Application] [4]**

9. **(i)** A circular disc of radius 3 cm is heated. Due to expansion, its radius increases at the rate of 0.05 cm/s. Find the rate of which its area increases when the radius is 3.2 cm. **[Application] [4]**

OR

(ii) Solve the differential equation $\frac{dy}{dx} = \frac{x+y+2}{2(x+y)-1}$ **[Analysis]**

10. **(i)** Rubiya, Thaksh, Shanteri, and Lilly entered a spinning zone for a fun game, but there is a twist: they don't know which spinner will appear on their screens until it is their turn to play. They may encounter one of the following spinners, or perhaps even both: **[4]**



Different combinations of numbers will lead to exciting prizes. Below are some of the rewards they can win:

l Get the number '5', from Spinner A and '8' from Spinner B, and you'll win a music player! You win a photo frame if Spinner A lands on a value greater than that of Spinner B!

(a) Thaksh spun both the spinners, A and B in one of his turns.

What is the probability that Thaksh wins a music player in that turn? Show your steps.

(b) Lilly spun spinner B in one of her turns.
What is the probability that the number she got is even given that it is a multiple of 3? Show your steps.

(c) Rubiya spun both the spinners.
What is the probability that she wins a photo frame? Show your work. AI **[Analysis] [4]**

OR

(ii) Senior students tend to stay up all night and therefore are not able to wake up on time in the morning. Not only this but their dependence on tuitions further leads to absenteeism in school. Among the students in Class XII, it is known that 30% of the students have 100% attendance. Previous year results report that 80% of all students who have 100% attendance attain an A grade and 10% of irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the Class XII.

Using the above information answer the following:

(a) Find the conditional probability that a student attains an A grade given that he is not a 100 % regular student.

(b) Find the probability of attaining A grade by the students of Class XII.

(c) Find the probability that student is 100% regular given that he attains an A grade.

11. A trust invested some money in two types of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹ 2,800 as interest. However, if trust had interchanged money in bonds, they would have got ₹ 100 less as interest.

(i) Form a set of simultaneous equations for the above information.

(ii) Solve the set of equations formed in (i) by matrix method.

(iii) Hence, find the amount invested by the trust. **[Application] [6]**

12. **(i)** Solve the following differential equation:

$$e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)+\left(1+e^{\frac{x}{y}}\right)\frac{dx}{dy}=0, \text{ when } x = 0 \text{ and } y = 1$$

AI **[Application] [6]**

OR

(ii) Evaluate: $\int\left(\frac{x^2}{x^4+1}\right)dx$ AI **[Evaluate]**

13. **(i)** Mr Aithal, a mathematics teacher, announces the following activity in his classroom and assures grand prizes for the winners.

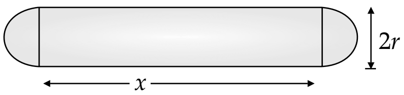
Instructions:

- Make a rectangular photo frame of total area using a chart paper.
- The frame should have a margin of 1.25 cm each at the top and the bottom.
- The frame should have a margin of 1 cm each on the left and the right sides.
- The area available at the centre to stick the photo should be maximum.

What must be the dimensions of such a photo frame? Show your work.

OR

(ii) A running track of 440 m is to be laid out enclosing a football field. The football field is in the shape of a rectangle with a semi-circle at each end. If the area of the rectangular portion is to be maximum, then find the length of its sides. Also calculate the area of the football field.



[Application]

14. In a class of 60 students, 30 opted for Mathematics, 32 opted for Biology and 24 opted for both Mathematics and Biology. If one of these students is selected at random, find the probability that

(i) The student opted for Mathematics or Biology.

(ii) The student has opted neither Mathematics nor Biology.

(iii) The student has opted Mathematics but not Biology. AI **[Analysis] [6]**

Section-B [15 marks]

15. In subparts (i) to (iii) choose the correct options and in subparts (iv) and (v), answer the questions as instructed.

(i) Consider the following statements and choose the correct option:

Statement 1: Two vectors are said to be like vectors if they have the same direction but different magnitude. **Statement 2:** Vector quantities do not have a specific direction.

Which of the following is correct?

(a) Only Statement 1 **(b)** Only Statement 2

(c) Both Statements 1 and 2 **(d)** Neither Statement 1 nor Statement 2 AI **[Recall]**

(ii) Find the intercepts on the coordinate axis cut off by the plane.

$\vec{r}.\left(2\hat{i}-4j+3k\right)=12$

(a) $a = 6, b = -3, c = 4$ **(b)** $a = 6, b = 3, c = 4$

(c) $a = -6, b = -3, c = 4$ **(d)** $a = 6, b = -3, c = -4$ **[Analysis]**

(iii) The direction cosines of the z-axis are-

(a) <0, 0, 1> **(b)** <0, 1, 0> **(c)** <1, 0, 0> **(d)** <1, 1, 1>

AI **[Understanding]**

(iv) Find the vector in the direction of the vector $\hat{i}-2\hat{j}+2\hat{k}$ that has magnitude 9. AI **[Understanding]**

(v) Find the vector equation of the plane with intercepts 3, – 4 and 2 on X, Y and Z-axis respectively. **[Application]**

16. **(i)** $A(1, 2, -3)$ and $B(-1, -2, 1)$ are the end points of a vector $\overrightarrow{AB}$, then find the unit vector in the direction of $\overrightarrow{AB}$.

AI **[Application] [2]**

OR

(ii) If $\hat{a}$ is a unit vector and $\left(2\vec{x}-3\hat{a}\right).\left(2\vec{x}+3\hat{a}\right)=91,$ find the value of $|\vec{x}|$. AI **[Application]**

17. **(i)** Find the shortest distance between the lines

$\vec{r}=3\hat{i}+2\hat{j}-4\hat{k}+\lambda\left(\hat{i}+2\hat{j}+2\hat{k}\right)$ and

$\vec{r}=5\hat{i}-2\hat{j}+\mu\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$

If the lines intersect find their point of intersection. AI **[Application] [4]**

OR

(ii) A line with positive direction cosine passes through the point P(−2, 3, 2) and makes equal angles with the coordinate axes. The line passes through the plane $3x - 4y + 2z = 11$ at point Q.

Find the length of the line segment PQ. **[Application]**

18. **(i)** Sketch the region enclosed bounded by the curve, $x^2 = 4y$ and the line $x = 4y - 2$.

(ii) Find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$. AI **[4]**

Section-C [15 marks]

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) At a toy factory, the production cost per day for x number of toys is given by the following function:

$C(x) = 0.0001x^3 - 0.01x^2 - 20x + 3500$

Which of these represents the approximate additional cost to produce the 201st toy?

(a) C(200) **(b)** AC(201) **(c)** MC(200) **(d)** MC(201) **[Recall]**

(ii) Read the following statements and choose the correct option:

(I) The slope of the regression line of X on Y is also called the regression coefficient of Y on X.

(II) The slope of the regression line of X on Y is also called the regression coefficient of X on Y.

(III) The slope of the regression line of X on Y is also called the correlation coefficient of X on Y.

(IV) The slope of the regression line of X on Y is also called the correlation coefficient of Y on X.

Which of the following is correct?

(a) Only II is correct **(b)** Only III is correct **(c)** Only I is correct **(d)** Only I and IV are correct

[Recall]

(iii) If the two regression coefficients are 0.8 and 0.2, then find the value of coefficient of correlation r. **[Understanding]**

(iv) Vishwanath runs a company that produces and sells a particular model of laptop. Its revenue function is given by $R(x) = 4x^3 + \frac{1}{5}x^2 + x$, where x is the number of laptops demanded and R(x) is in rupees.

What is the ideal unit price (in rupees) for Vishwanath to charge for each laptop if there is a demand for 100 laptops? AI **[Understanding]**

(v) The fixed cost of a product is ₹ 30,000 and its variable cost per unit is ₹ 800. If the demand function is $p(x) = 4500 - 100x$, find the break-even values. **[Understanding]**

20. (i) A bicycle spare parts manufacturing company decided to upgrade its production unit by installing advanced machinery. As a result of this investment, the cost of producing spare parts increased. The company used to sell 1000 spare parts daily at ₹ 50 per unit.

The increased daily cost of production $C(x)$ for x number of spare parts is given by $C(x) = \frac{99}{2}x + 525$

If the selling price remains the same, find how many more spare parts must the company sell daily to ensure no loss. A I **[Application] [2]**

OR

(ii) The government's Department of Culture is organising an art and craft exhibition cum sale to promote local artists and their work.

They want to charge a nominal amount as entry fee.

Profit, in rupees, generated by the entry tickets can be calculated using the function:

$P(x) = -8x^3 + 29400x - 60000$ where x is the price of each ticket.

What is the ideal price for each ticket that the department should set to maximise the profit? A I **[Application]**

21. (i) Shown below is a data set that includes the average temperature and the number of ice cream cones sold per day by an ice cream truck in a small town over the course of six days.

Temperature (°F) (X)	65	75	60	70	80	70
Cones sold (Y)	180	250	160	210	300	220

Find the line of regression using average temperature as the independent variable. A I **[Application] [4]**

OR

(ii) For observations of pairs (x, y) of the variables X and Y, the following results are obtained. $\Sigma x = 125$, $\Sigma y = 100$, $\Sigma x^2 = 1650$, $\Sigma y^2 = 1500$, $\Sigma xy = 50$, $n = 25$.

(a) Find the equation of the line of regression of x on y.

(b) Estimate the value of x, if $y = 5$. **[Application]**

22. A farm is engaged in breeding hens. The hens are fed products A and B grown in the farm which contains nutrients P, Q and R. One kilogram of product A contains 36 units, 3 units and 20 units of nutrients P, Q and R respectively, whereas one kilogram of product B contains 6 units, 12 units and 10 units of nutrients P, Q and R respectively. The minimum requirement of nutrients P, Q and R for a hen is 108, 36 and 100 units respectively. Product A costs ₹20 per kilogram and product B costs ₹40 per kilogram. Using linear programming, find the number of kilograms of products A and B to be produced to minimise the total cost.

(a) Formulate this situation as an L.P.P.

(b) Identify the feasible region from the rough sketch. **[Analysis] [4]**

❑❑

JOIN CASEIANS

SOLUTIONS

Sample Question Paper - 5

Section-A

1. **(i) Option (a) is correct.**

Explanation:

$$A = \begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$$

$$\Rightarrow \quad A^T = \begin{pmatrix}1 & 3\\ 2 & 4\end{pmatrix}$$

and $$A + A^T = \begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix} + \begin{pmatrix}1 & 3\\ 2 & 4\end{pmatrix} = \begin{pmatrix}2 & 5\\ 5 & 8\end{pmatrix} \quad \text{....(i)}$$

Now, $$(A + A^T)^T = \begin{pmatrix}2 & 5\\ 5 & 8\end{pmatrix} \quad \text{...(ii)}$$

From (i) and (ii),

$$A + A^T = (A + A^T)^T$$

Therefore, $A + A^T$ is a symmetric matrix.

(ii) Option (c) is correct.

Explanation: $\int (\cot x - \text{cosec}^2 x) e^x \, dx$

$f(x) = \cot x$ then $f'(x) = -\text{cosec}^2 x$

$$\int e^x f(x) + f'(x) dx = e^x f(x) + c$$

$\therefore \quad f(x) = \cot x$

(iii) Option (d) is correct.

Explanation:

$$\tan^{-1} 1 = \frac{\pi}{4} \text{ and } \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$$

$$\Rightarrow \tan^{-1} 1 + \cos^{-1}\frac{1}{2} = \frac{\pi}{4} + \frac{\pi}{3} = \frac{7\pi}{12}$$

(iv) Option (a) is correct.

Explanation: Degree is 2 because the power of highest order derivative $\frac{d^4y}{dx^4}$ is 2.

(v) Option (d) is correct.

Explanation:

$$\begin{aligned} E(X) &= \Sigma XP(X) \\ &= -4 \times (0.1) + (-3 \times 0.2) + (-2 \times 0.3) \\ &\quad + (-1 \times 0.2) + (0 \times 0.2) \\ &= 0.4 - 0.6 - 0.6 - 0.2 + 0 = -1.8 \end{aligned}$$

(vi) Option (a) is correct.

Explanation: Given, $$\begin{vmatrix} a & b & c\\ d & e & f\\ g & h & i \end{vmatrix} = k \quad \text{...(i)}$$

Now, $$\begin{vmatrix} 6a & b & c\\ 6d & e & f\\ 48g & 8h & 8i \end{vmatrix}$$

$$= 8 \begin{vmatrix} 6a & b & c\\ 6d & e & f\\ 6g & h & i \end{vmatrix}$$

(By taking 8 common from 3rd row)

$$= 6 \times 8 \begin{vmatrix} a & b & c\\ d & e & f\\ g & h & i \end{vmatrix}$$

(By taking 6 common from 1st column)

$$= 6 \times 8 \times k \quad \text{[Using (i)]}$$

$$= 48k$$

(vii) Option (a) is correct.

Explanation: It is evident from the graph, where the function is increasing.

⇒ The function increases at the following intervals: $(-\infty, -6)$ & $(-2, 3)$

Hence, Statement 1 is true and Statement 2 is false.

(viii) Option (b) is correct.

Explanation: Let $u = \log(1 + x^2)$ and $v = \tan^{-1} x$

$$\frac{du}{dx} = \frac{d}{dx}\log(1 + x^2) = \frac{1}{1 + x^2}(2x) = \frac{2x}{1 + x^2}$$

$$\frac{dv}{dx} = \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2}$$

$$\frac{du}{dv} = \frac{du}{dx} \times \frac{dx}{dv} = \frac{2x}{1 + x^2} \times \frac{1 + x^2}{1} = 2x$$

(ix) Option (c) is correct.

Explanation: The given relation

$$R = \{(1, 1), (2, 2), (3, 3)\}$$

is an identity relation.

We know that, identity relation is always an equivalence relation, therefore the given relation is an equivalence relation.

(x) Option (a) is correct.

Explanation: The determinant of a skew-symmetric matrix of odd order is always zero and if the determinant of a matrix is zero then the inverse of that matrix does not exist.

(xi) Option (d) is correct.

Explanation:

$$\begin{aligned} E(X^2) &= \Sigma X^2 P(X) \\ &= 1.\frac{1}{10} + 4.\frac{1}{5} + 9.\frac{3}{10} + 16.\frac{2}{5} \\ &= \frac{1}{10} + \frac{4}{5} + \frac{27}{10} + \frac{32}{5} \\ &= \frac{1 + 8 + 27 + 64}{10} = 10 \end{aligned}$$

(xii) Given $$A = \begin{vmatrix} 2 & -6 & 0\\ 0 & 9 & -4\\ 5 & -8 & 0 \end{vmatrix}$$

$$\therefore \quad \text{Minor of } a_{32} = \begin{vmatrix} 2 & 0\\ 0 & -4 \end{vmatrix}$$

$$= 2(-4) - 0$$

$$= -8$$

(xiii) Given that, aRb if $a \geq b$

$\Rightarrow aRa$

$\Rightarrow a \geq a$ which is true.

R is reflexive

Let aRb, $a \geq b$,

then $b \geq a$ which is not true so, R is not symmetric.

Now, let aRb and bRc

$\Rightarrow a \geq b$ and $b \geq c$

$\Rightarrow a \geq c$

Hence, R is transitive.

Here given relation is reflexive, and transitive but not symmetric, hence given relation is not an equivalence relation.

(xiv) A relation is a function if each input x is associated with exactly one output. This means that for each value of x, there must be only one corresponding value of y. So Graph A is a function but Graph B is not a function because at x = 0 then we get three value of y.

(xv) Probability that a chocolate has been physically examined $= P(A) = \frac{30}{990} = \frac{1}{33}$

Probability that a chocolate has been chemically examined $= P\left(\frac{B}{A}\right) = \frac{1}{30}$

Probability that a chocolate has been physically examined but not chemically examined $= P(A$ but not $B) = P(A \cap \overline{B}) = P(A) - P(A \cap B)$

$$= \frac{1}{33} - P(A) \times P\left(\frac{B}{A}\right) = \frac{1}{33} - \left(\frac{1}{33} \times \frac{1}{30}\right)$$

$$= \frac{1}{33} - \frac{1}{990}$$

$$= \frac{30-1}{990}$$

$$= \frac{29}{990}$$

[ISC Marking Scheme 2024]

2. **(i)** $x = a\left(\frac{1-t^2}{1+t^2}\right)$ and $y = \frac{2at}{1+t^2}$.

On differentiating with respect to t, respectively, we get

$$\frac{dx}{dt} = a\left(\frac{(1+t^2)(0-2t)-(1-t^2)(0+2t)}{(1+t^2)^2}\right)$$

$$= -\frac{4at}{(1+t^2)^2}$$

and $$\frac{dy}{dt} = \left(\frac{(1+t^2)(2a)-(2at)(2t)}{(1+t^2)^2}\right)$$

$$= \frac{2a(1-t^2)}{(1+t^2)^2}$$

$$\therefore \quad \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\frac{dx}{dt}} = \frac{1-t^2}{-2t}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-(1-t^2)}{2t} = -\frac{x}{y}$$

Now, $$\frac{d^2y}{dx^2} = -\left[\frac{y.1 - x\left(\frac{dy}{dx}\right)}{y^2}\right]$$

$$= -\left[\frac{y.1 - x\left(\frac{-x}{y}\right)}{y^2}\right]$$

$$= -\left[\frac{y^2 - x^2}{y^3}\right]$$

Hence, Proved

OR

(ii) $y^2 = 8x + 3$ ...(i) (given)

$\therefore \quad 2y\frac{dy}{dt} = 8\frac{dx}{dt}$ ...(ii)

$\frac{dy}{dt} = 8\frac{dx}{dt}$ ...(iii) (given)

$\therefore \quad 2y\ .8\frac{dx}{dt} = 8\frac{dx}{dt}$ [using (ii) & (iii)]

$\Rightarrow \quad y = \frac{8}{16} = \frac{1}{2}$

For $y = \frac{1}{2}$

From (i), $\left(\frac{1}{2}\right)^2 = 8x + 3$

$\Rightarrow \quad \frac{1}{4} - 3 = 8x$

$\Rightarrow \quad x = \frac{1-12}{4} \times \frac{1}{8} = -\frac{11}{32}$

Hence, required point is $\left(-\frac{11}{32}, \frac{1}{2}\right)$.

3. $$I = \int_0^{2\pi} \frac{1}{1+e^{\sin x}}dx \quad \text{...(i)}$$

$$I = \int_0^{2\pi} \frac{1}{1+e^{\sin(2\pi - x)}}dx$$

$$I = \int_0^{2\pi} \frac{1}{1+e^{-\sin x}}dx$$

$$I = \int_0^{2\pi} \frac{e^{\sin x}}{1+e^{\sin x}}dx \quad \text{...(ii)}$$

From (i) & (ii)

$$2I = \int_0^{2\pi} \frac{1+e^{\sin x}}{1+e^{\sin x}}dx$$

$$= \int_0^{2\pi} dx = [x]_0^{2\pi}$$

$$2I = 2\pi - 0$$

$$\therefore \quad I = \pi$$

4. $$y = (x-2)^2$$

$$\frac{dy}{dx} = 2(x-2)$$

$$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 2(x_1 - 2)$$

$$\text{Slope of chord} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{4-2} = 2$$

Slope of tangent = slope of chord

$$2(x_1 - 2) = 2$$

$$\therefore \quad x_1 = 3$$

Required point (3, 1)

5. **(i)** Here, $\frac{d}{dx}(6 - x^3 + 5x) = -3x^2 + 5 = 5 - 3x^2$

Since, the numerator is the derivative of denominator

$$\Rightarrow \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

$$\Rightarrow \int \frac{5-3x^2}{6-x^3+5x}\,dx = \ln|6 - x^3 + 5x| + C$$

[Marking Scheme ISC Specimen paper, 2023-24]

OR

(ii)
$$|\sin x| = \begin{cases} \sin x, \text{ if } \sin x \geq 0 \\ -\sin x, \text{ if } \sin x < 0 \end{cases}$$

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} |\sin x|\,dx = \int_{\frac{-\pi}{2}}^{0} -\sin x\,dx + \int_0^{\frac{\pi}{2}} \sin x\,dx$$

$$= [\cos x]_{-\frac{\pi}{2}}^{0} + (-\cos x)_0^{\frac{\pi}{2}}$$

$$= \cos 0 - \cos\left(\frac{-\pi}{2}\right) + \left[-\cos\frac{\pi}{2} - (-\cos 0)\right]$$

$$= 1 - 0 + [-0 + 1] = 2$$

[Marking Scheme ISC 2023-24]

Examiner's Comment

- Some students are unable to split the modulus of a trigonometric function within the given limits (interval) and make mistakes.

Answering Tip

- Revise all graphs of trigonometric functions so that student is able to decide whether the function attain positive or negative value within the interval.

6. To show, $\cos[\tan^{-1}\{\cot(\sin^{-1} x)\}] = x$

Solving LHS, $\cos[\tan^{-1}\{\cot(\sin^{-1} x)\}]$ ...(i)

Let $\sin^{-1} x = \theta$

$\Rightarrow \quad \sin\theta = x$

$\Rightarrow \quad \cot\theta = \frac{\sqrt{1-x^2}}{x}$

$\Rightarrow \quad \theta = \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

Thus, $\cos\left[\tan^{-1}\cot\left\{\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right\}\right]$

[Putting value of θ in (i)]

$\Rightarrow \cos\left[\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right]$ **...(ii)**

Now, let $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = t$

$\Rightarrow \quad \tan t = \left(\frac{\sqrt{1-x^2}}{x}\right)$

$\Rightarrow \quad \cos t = x$

$\Rightarrow \quad t = \cos^{-1} x$

Thus, $\cos t = \cos(\cos^{-1} x)$

[Putting value in (ii)]

$\Rightarrow \quad \cos t = x = \text{RHS}$ **Hence proved**

7.
$$\sin^{-1}\left(\frac{x}{2}\right) + \cos^{-1} x = \frac{\pi}{6}$$

$$\sin^{-1}\left(\frac{x}{2}\right) + \frac{\pi}{2} - \sin^{-1} x = \frac{\pi}{6}$$

$$\sin^{-1}\left(\frac{x}{2}\right) - \sin^{-1} x = \frac{\pi}{6} - \frac{\pi}{2}$$

$$\sin^{-1}\left(\frac{x}{2}\right) = -\frac{\pi}{3} + \sin^{-1} x$$

$$\left(\frac{x}{2}\right) = \sin\left(\sin^{-1} x - \frac{\pi}{3}\right)$$

$$\left(\frac{x}{2}\right) = \sin(\sin^{-1} x)\cos\frac{\pi}{3} - \cos\sin^{-1} x \sin\frac{\pi}{3}$$

$$\frac{x}{2} = x \times \frac{1}{2} - \sqrt{1-x^2}\,\frac{\sqrt{3}}{2}$$

$$x = x - \sqrt{3}\sqrt{1-x^2}$$

$$\sqrt{3}\sqrt{1-x^2} = 0$$

$$3(1 - x^2) = 0$$

$$x^2 = 1$$

$$\Rightarrow \quad x = \pm 1$$

$x = -1$ is not satisfied the equation

$$\therefore \quad x = 1$$

8. $\int x^2 \cos x\,dx$

$$= x^2 \int \cos x\,dx - \int\left[\frac{d}{dx}x^2 \int \cos x\,dx\right]dx$$

$$= x^2 \sin x - \int 2x \sin x\,dx$$

$$= x^2 \sin x - 2\left[-x\cos x - \int 1(-\cos x)\,dx\right]$$

$$= x^2 \sin x + 2x \cos x - \sin x + c$$

9. **(i)** Let $r = 3$ cm be the radius of the disc

Given, $\frac{dr}{dt} = 0.05$ cm/s

Let area, $A = \pi r^2$

$$\therefore \quad \frac{dA}{dt} = \frac{d}{dr}(\pi r^2)\frac{dr}{dt}$$

$$= 2\pi r(0.05)$$

$$= \frac{22}{7} \times r \times (0.1)$$

Now, $\left(\frac{dA}{dt}\right)_{r\,=\,3.2\text{ cm}} = \frac{22}{7} \times 3.2 \times 0.1$

$$= \frac{22}{7} \times 0.32$$

$$= 1.005 \text{ cm}^2/\text{s}$$

∴ Area increases at the rate of 1.005 cm^2/s

OR

(ii)

Topper Answer, 2019

$$\frac{dy}{dx} = \frac{x+y+2}{2(x+y)-1} \Rightarrow \frac{dy}{dx} = \frac{(x+y)+2}{2(x+y)-1}$$

Let $x+y = v$

$$1+\frac{dy}{dx} = \frac{dv}{dx} \Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\frac{dv}{dx} - 1 = \frac{v+2}{2v-1}$$

$$\frac{dv}{dx} = \frac{v+2}{2v-1} + 1$$

$$\frac{dv}{dx} = \frac{v+2+2v-1}{2v-1}$$

$$\frac{dv}{dx} = \frac{3v+1}{2v-1}$$

$$\frac{2v-1}{3v+1}\,dv = dx$$

Integrations on both LHS & RHS

$$\int\frac{2v-1}{3v+1}\,dv = \int dx$$

$$\frac{2}{3}\int\frac{3v-3/2}{3v+1}\,dv = x + c$$

$$\frac{2}{3}\int\frac{3v+1-(3/2+1)}{3v+1}\,dv = x + c$$

$$\frac{2}{3}\int\frac{3v+1}{3v+1}\,dv - \frac{5\times 2}{2\;\;3}\int\frac{dv}{3v+1} = x + c$$

$$\frac{2}{3}v - \frac{5}{3}\frac{\log|3v+1|}{3} = x + c$$

$$\frac{2}{3}v - \frac{5}{9}\log|3v+1| = x + c$$

$$\frac{2}{3}(x+y) - \frac{5}{9}\log|3(x+y)+1| = x + c \quad \underline{\underline{\text{Ans}}}$$

Examiner's Comment

- Most of the candidates identified the given differential equation as a homogeneous equation whereas, it was the type of equation reducible to a simple separable variable.

Answering Tip

- Understand all types of differential equations and applications and do adequate practice of various types of problems. Clarify the significance of constant in the solution of a differential equation.

10. (i) Finds required probability as:

(a) $P(5 \text{ from spinner } A) \cap P(8 \text{ from spinner } B)$

$$= \frac{1}{4}\times\frac{1}{8}$$

$$= \frac{1}{32}$$

(b) Use the conditional probability and find the required probability as follows:

$$P(\text{Even/Multiple of 3}) = P(\text{Even} \cap \text{Multiple of 3}) \div P(\text{Multiple of 3})$$

$$= \frac{\frac{1}{8}}{\frac{2}{8}} = \frac{1}{2}$$

(c) Find the probability of getting 2 from spinner A and getting 1 from spinner B as:

$$P_1 = \frac{1}{2}\times\frac{1}{8} = \frac{1}{16}$$

Find the probability of getting 5 from spinner A and getting either 1, 2, 3 or 4 from spinner B as:

$$P_2 = \frac{1}{4}\times\left[\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right]$$

$$= \frac{1}{4}\times\frac{4}{8} = \frac{1}{8}$$

Write that P_1 and P_2 are mutually exclusive and hence, find the probability that she wins a photo frame as:

$$P_1 + P_2 = \frac{1}{16}+\frac{1}{4}$$

$$= \frac{5}{16}$$

OR

(ii) (a) Let E_1 is the event that a student is regular

E_2 is the event that a student is irregular

A is the event that a student attains grade A

$$P(E_1) = \frac{30}{100},\ P(E_2) = \frac{70}{100}$$

$$P\left(\frac{A}{E_1}\right) = \frac{80}{100},\ P\left(\frac{A}{E_2}\right) = \frac{10}{100}$$

(a) Required probability $= P\left(\frac{A}{E_2}\right) = \frac{10}{100} = \frac{1}{10}$

(b) Required probability $= P(A)$

$$= P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{30}{100}\cdot\frac{80}{100}+\frac{70}{100}\cdot\frac{10}{100} = \frac{31}{100}$$

(c) Required probability $= P\left(\frac{E_1}{A}\right)$

$$= \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{30}{100}\times\frac{80}{100}}{\frac{30}{100}\times\frac{80}{100}+\frac{70}{100}\times\frac{10}{100}} = \frac{24}{31}$$

11. (i) According to the questions

$$[x\ y]\begin{bmatrix}\frac{10}{100}\\ \frac{12}{100}\end{bmatrix} = [2800]$$

i.e., $$\frac{10x}{100}+\frac{12y}{100}=2800$$

$\Rightarrow$ $$10x+12y=280000$$

$\Rightarrow$ $$5x+6y=140000 \quad \text{...(i)}$$

Also, $$[y \quad x]\begin{bmatrix}\frac{10}{100}\\ \frac{12}{100}\end{bmatrix}=[2700]$$

i.e., $$\frac{10y}{100}+\frac{12x}{100}=2700$$

$\Rightarrow$ $$10y+12x=270000$$

$\Rightarrow$ $$5y+6x=135000 \quad \text{...(ii)}$$

(ii) By matrix method,

$$\begin{bmatrix}5 & 6\\ 6 & 5\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}140000\\ 135000\end{bmatrix}$$

i.e., $AX = B$

where, $A=\begin{bmatrix}5 & 6\\ 6 & 5\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$, $B=\begin{bmatrix}140000\\ 135000\end{bmatrix}$

It is of the form $AX = B$

$\Rightarrow$ $$X = A^{-1}B$$

Now, $$|A|=\begin{vmatrix}5 & 6\\ 6 & 5\end{vmatrix}$$
$$= 25 - 36$$
$$= -11 \neq 0$$

Then, A^{-1} exists.

co-factors of $|A|$ are :

$$A_{11} = 5, A_{12} = -6,$$
$$A_{21} = -6, A_{22} = 5$$

$$adj\ A=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}^T=\begin{bmatrix}5 & -6\\ -6 & 5\end{bmatrix}$$

$\therefore$ $$A^{-1}=\frac{1}{|A|}(adj\,A)$$

$$X = A^{-1}\,B$$

$\Rightarrow$ $$X=\frac{1}{|A|}(adj\ A)B$$

$$=\frac{1}{|A|}\begin{bmatrix}5 & -6\\ -6 & 5\end{bmatrix}\begin{bmatrix}140000\\ 135000\end{bmatrix}$$

$$=\frac{1}{-11}\begin{bmatrix}700000-810000\\ -840000+675000\end{bmatrix}$$

$$=\frac{-1}{11}\begin{bmatrix}-110000\\ -165000\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}10000\\ 15000\end{bmatrix}$$

$\therefore$ $x = 10000$ and $y = 15000$

(iii) $\therefore$ Amount invested in type one bond ₹ x =₹ 10,000 and in type two bond ₹ y =₹ 15,000

Total amount invested is

₹ x+ ₹ y = ₹ 10,000 + ₹ 15,000
= ₹ 25,000

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12. **(i)** Given, $$e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)+\left(1+e^{\frac{x}{y}}\right)\frac{dx}{dy}=0$$

Substitute $x=vy \Rightarrow \frac{dx}{dy}=v+y\frac{dv}{dy}$

$$e^v-ve^v+v+ve^v+(1+e^v)y\frac{dv}{dy}=0$$

$\Rightarrow$ $$(v+e^v)+(1+e^v)y\frac{dv}{dy}=0$$

$\Rightarrow$ $$\left(\frac{1+e^v}{v+e^v}\right)dv+\frac{dy}{y}=0$$

Integrating, on both sides,

$$\int\left(\frac{1+e^v}{v+e^v}\right)dv=-\int\frac{dy}{y}$$

$\Rightarrow$ $$\log|v+e^v|=-\log|y|+\log c$$

$\Rightarrow$ $$\log|v+e^v|+\log|y|=\log c$$

$\Rightarrow$ $$\log|\{(v+e^v)y\}|=\log c$$

$\Rightarrow$ $$\log\left|\left\{\left(\frac{x}{y}+e^{\frac{x}{y}}\right)y\right\}\right|=\log c$$

$\Rightarrow$ $$x+e^{\frac{x}{y}}y=c$$

When $x = 0$ and $y = 1 \Rightarrow c = 1$

$$x+e^{\frac{x}{y}}y=1$$

[ISC Marking Scheme, 2015]

Examiner's Comment

- Many candidates substituted $y = vx$ and proceeded further to solve the given equation using the rules of homogeneous equation, which was an incorrect approach. The subsequent integrals were not correctly understood by some. A few candidates did not find the value of c (constant) under the given condition i.e., $x= 0$, $y = 1$.

Answering Tip

- All forms of integration need rigorous practice. The constant of integration should not be ignored.

OR

(ii) Let, $$I=\int\left(\frac{x^2}{x^4+1}\right)dx$$

$$=\frac{1}{2}\int\left(\frac{2x^2}{x^4+1}\right)dx$$

$$= \frac{1}{2}\int\left[\frac{(x^2+1)+(x^2-1)}{x^4+1}\right]dx$$

$$= \frac{1}{2}\int\frac{(x^2+1)}{x^4+1}dx + \frac{1}{2}\int\frac{(x^2-1)}{x^4+1}dx$$

$$= \frac{1}{2}\int\frac{\left(1+\frac{1}{x^2}\right)}{x^2+\frac{1}{x^2}}dx + \frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)}{x^2+\frac{1}{x^2}}dx$$

$$= \frac{1}{2}\int\frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+2}dx + \frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2-2}dx$$

Let $u = x - \frac{1}{x}$ and $v = x + \frac{1}{x}$

$\Rightarrow du = \left(1+\frac{1}{x^2}\right)dx$ and $dv = \left(1-\frac{1}{x^2}\right)dx$

$$I = \frac{1}{2}\int\frac{du}{(u)^2+2}dx + \frac{1}{2}\int\frac{dv}{(v)^2-2}dx$$

$$I = \frac{1}{2}\times\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + \frac{1}{2}\times\frac{1}{2\sqrt{2}}\log\left|\frac{v-\sqrt{2}}{v+\sqrt{2}}\right| + c$$

$$I = \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + \frac{1}{4\sqrt{2}}\log\left|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right| + c$$

Examiner's Comment

- Students find difficulty to solve such questions. Some of them left in mid-way and few students made wrong substitution ultimately unable to solve the problem.

Answering Tip

- Do ample practice of the problems involving variety of substitutions.

13. **(i)** Assumes the width of photo frame to be x cm and its length to be $\frac{80}{x}$ cm.

Subtracts the specified margins from the width and length to find the area (A) available to stick the photo as:

$$A = (x-2)\left(\frac{80}{x}-\frac{5}{2}\right)$$

$$\Rightarrow \quad A = 85 - \frac{5}{2}x - \frac{160}{x}$$

Differentiates area with respect to x as:

$$\frac{dA}{dx} = -\frac{5}{2} + \frac{160}{x^2}$$

Equates the above derivative to zero and finds the critical point as:

$$-\frac{5}{2} + \frac{160}{x^2} = 0$$

$$\Rightarrow \quad x^2 = 64$$

$\Rightarrow x = 8$ cm (as x being a length cannot be negative)

Finds $\frac{d^2A}{dx^2}$ at $x = 8$ as:

$$\frac{d^2A}{dx^2}(\text{at } x = 8) = -\frac{320}{x^3}(\text{at } x = 8) = -\frac{5}{8} < 0$$

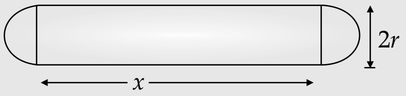
Concludes that by second derivative test, the area is maximum at $x = 8$ cm.

Finds the length of the frame as $\frac{80}{8} = 10$ cm.

Concludes that the required dimensions of the photo frame are 8 cm and 10 cm .

OR

(ii) Here, area of rectangular portion $= 2rx$



and perimeter of track $= 2x + 2\pi r$

Given perimeter of track $= 440$ m

$\therefore \quad 2x + 2\pi r = 440 \quad \ldots$(i)

So, area of rectangular portion, $A = r(440 - 2\pi r)$ [from (i)]

$$A = 440r - 2\pi r^2$$

Differentiating A w.r.t. r, we get

$$\Rightarrow \quad \frac{dA}{dr} = 440 - 4\pi r$$

Area to be maximum, $\frac{dA}{dr} = 0$

$\therefore \quad 440 - 4\pi r = 0$

$\Rightarrow \quad r = \frac{110}{\pi}$

Now, $\quad \frac{d^2A}{dr^2} = -4\pi < 0$

From (i), we get

$$\Rightarrow \quad 2x = 440 - 2\pi r$$

$$= 440 - 2\left(\pi\frac{110}{\pi}\right)$$

[Since, $r = \frac{110}{\pi}$]

$$= 440 - 220$$

$$= 220$$

$\therefore \quad x = 110$ m

Hence, sides of rectangle are $2r = \frac{220}{\pi}$ m and $x = 110$ m.

Area of football field = Area of rectangle + 2(area of semicircle)

$$= x \times 2r + 2\left(\frac{\pi r^2}{2}\right)$$

[r = radius of semi-circle]

$$= 110 \times \frac{220}{\pi} + \pi\left(\frac{110}{\pi}\right)^2$$

$$= \frac{24200}{\pi} + \frac{12100}{\pi} = \frac{36300}{\pi}$$

$$= \frac{36300}{22} \times 7$$

$$= 1650 \times 7 = 11{,}550 \text{ m}^2$$

[ISC Marking Scheme 2023]

14. Let A: event that students opted Mathematics
B: event that students opted Biology

$$\therefore\ P(A) = \frac{30}{60},\ P(B) = \frac{32}{60},\ P(A\cap B) = \frac{24}{60}$$

(i)
$$\begin{aligned} P(A\cup B) &= P(A) + P(B) - P(A\cap B)\\ &= \frac{30}{60}+\frac{32}{60}-\frac{24}{60}\\ &= \frac{38}{60}=\frac{19}{30} \end{aligned}$$

Hence, probability that student opted Mathematics or Biology is $\frac{19}{30}$.

(ii)
$$\begin{aligned} P(A'\cap B') &= P(A\cup B)' = 1 - P(A\cup B)\\ &= 1-\frac{19}{30}=\frac{11}{30} \end{aligned}$$

Hence, probability that student has neither opted for Mathematics nor Biology is $\frac{11}{30}$.

(iii) Probability of student has opted Mathematics but not Biology
$$\begin{aligned} &= P(A) - P(A\cap B)\\ &= \frac{30}{60}-\frac{24}{60}=\frac{6}{60}=\frac{1}{10} \end{aligned}$$

[ISC Marking Scheme 2017]

Section-B

15. (i) Option (a) is correct.

Explanation: Statement 1 is correct as the vectors having the same direction but different magnitude are known as parallel or like vectors.

Statement 2 is incorrect as vector quantity has magnitude as well as specific direction.

(ii) Option (a) is correct.

Explanation:

$$\vec{r}(2\hat{i}-4\hat{j}+3\hat{k})=12$$
$$\frac{x}{6}+\frac{y}{(-3)}+\frac{z}{4}=1$$

Intercepts = 6, –3, 4

$\Rightarrow a = 6,\ b = -3$ and $c = 4$

(iii) Option (a) is correct.

Explanation:

D.C′ of z-axis = <0, 0, 1>

(iv) Let $\vec{a}=\hat{i}-2\hat{j}+2\hat{k}$

Any vector in the direction of a vector $\vec{a}$ is given by

$$\frac{\vec{a}}{|\vec{a}|}=\frac{\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{1^2+2^2+2^2}}=\frac{\hat{i}-2\hat{j}+2\hat{k}}{3}$$

$\therefore$ Vector in the direction of $\vec{a}$ with magnitude 9

$$= 9\ \frac{\hat{i}-2\hat{j}+2\hat{k}}{3}\ = 3(\hat{i}-2\hat{j}+2\hat{k})$$

(v) The equation is $\frac{x}{3}+\frac{y}{-4}+\frac{z}{2}=1$ or $4x - 3y + 6z = 12$

The vector equation is

$$\vec{r}.(4\hat{i}-3\hat{j}+6\hat{k})=12$$

16. (i) We have, A(1, 2, –3) and B(–1, –2, 1)

$$\begin{aligned} \vec{AB} &= (-1-1)\hat{i}+(-2-2)\hat{j}+(1+3)\hat{k}\\ \therefore\quad \vec{AB} &= -2\hat{i}-4\hat{j}+4\hat{k} \end{aligned}$$

Unit vector in direction of $\vec{AB}$

$$\begin{aligned} &= \frac{-2\hat{i}-4\hat{j}+4\hat{k}}{\sqrt{(-2)^2+(-4)^2+(4)^2}}\\ &= \frac{-2(\hat{i}+2\hat{j}-2\hat{k})}{\sqrt{4+16+16}}\\ &= \frac{-2(\hat{i}+2\hat{j}-2\hat{k})}{6}\\ &= -\frac{1}{3}(\hat{i}+2\hat{j}-2\hat{k}) \end{aligned}$$

[ISC Marking Scheme 2023]

Examiner's Comment

- Some students find other vector instead of unit vector.

Answering Tip

- Read the question carefully and practice more problems involving unit vectors.

OR

(ii) Given,

$$\begin{aligned} (2\vec{x}-3\hat{a}).(2\vec{x}+3\hat{a}) &= 91\\ \Rightarrow 4|\vec{x}|^2+6\vec{x}.\hat{a}-6\hat{a}.\vec{x}-9|\hat{a}|^2 &= 91\\ \Rightarrow\quad 4|\vec{x}|^2+6.\vec{x}.1-6.1.\vec{x}-9.1 &= 91 \qquad [\because \hat{a}=1]\\ \Rightarrow\quad 4|\vec{x}|^2 &= 100\\ \Rightarrow\quad |\vec{x}|^2 &= 25\\ \Rightarrow\quad |\vec{x}| &= 5 \end{aligned}$$

[ISC Marking Scheme 2023]

17. (i) We have
$$\begin{aligned} a_1 &= 3\hat{i}+2\hat{j}-4\hat{k}\\ b_1 &= \hat{i}+2\hat{j}+2\hat{k}\\ a_2 &= 5\hat{i}-2\hat{j}\\ b_2 &= 3\hat{i}+2\hat{j}+6\hat{k}\\ \vec{a_2}-\vec{a_1} &= 2\hat{i}-4\hat{j}+4\hat{k} \end{aligned}$$

$$\vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 2\\ 3 & 2 & 6\end{vmatrix}=\hat{i}(12-4)-\hat{j}(6-6)+\hat{k}(2-6)$$

$$\vec{b_1}\times\vec{b_2}=8\hat{i}+0\hat{j}-4\hat{k}=8\hat{i}-4\hat{k}$$

$$\because\ \left(\vec{b_1}\times\vec{b_2}\right).\left(\vec{a_2}-\vec{a_1}\right) = 16 - 16 = 0$$

∴ The lines are intersecting and the shortest distance between the lines is 0.

Now for point of intersection

$$\vec{r} = 3\hat{i}+2\hat{j}-4\hat{k}+\lambda\left(\hat{i}+2\hat{j}+2\hat{k}\right)$$

$$\vec{r} = 5\hat{i}-2\hat{j}+\mu\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$$

$\Rightarrow$ $3+\lambda = 5+3\mu$ ...(i)

$2+2\lambda = -2+2\mu$ ...(ii)

$-4+2\lambda = 6\mu$ ...(iii)

Solving (i) and (ii) we get $\mu = -2$ and $\lambda = -4$

Substituting in equation of line we get

$$\vec{r} = 5i-2j+(-2)\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$$

$$= -\hat{i}-6\hat{j}-12\hat{k}$$

Point of intersection is (–1, –6, –12).

Examiner's Comment

- Some students write the wrong values of a_1, a_2 and b_1, b_2 that is why, the answer of shortest distance is wrongly calculated.

Answering Tip

- First student should write the question in standard form $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$, $\vec{r}=\vec{a_2}+\mu_1\vec{b_2}$ then solve for the shortest distance.

OR

(ii) Given point is P (–2, 3, 2) and plane is $3x-4y+2z = 11$

Equation of line $= \dfrac{x+2}{1} = \dfrac{y-3}{1} = \dfrac{z-2}{1} = k$

$Q = (k-2, k+3, k+2)$ lie on $3x-4y+2z = 11$

$\Rightarrow 3(k-2)-4(k+3)+2(k+2) = 11$

$\Rightarrow$ $3k-6-4k-12+2k+4 = 11$

$\Rightarrow$ $k-14 = 11$

$\Rightarrow$ $k = 25$

$Q = (23, 28, 27)$

$$PQ = \sqrt{(23-(-2))^2+(28-3)^2+(27-2)^2}$$

$$PQ = \sqrt{(25)^2+(25)^2+(25)^2}$$

$$PQ = \sqrt{625+625+625}$$

$PQ = 25\sqrt{3}$ **[ISC Marking Scheme 2024]**

18. (i) Given curve $x^2 = 4y$ ... (i)

represents an upward parabola with vertex (0, 0) and axis along y-axis

Given equation of line is $x = 4y-2$ ...(ii)

On solving eqs (i) & (ii), we get

$x^2 = x+2$

$\Rightarrow$ $x^2-x-2 = 0$

$\Rightarrow$ $(x-2)(x+1) = 0$

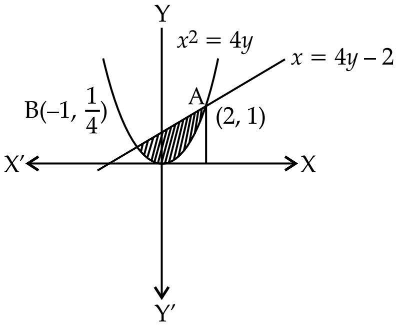
$\Rightarrow$ $x = 2, -1$

when $x = 2, y = 1$

and $x = -1, y = \dfrac{1}{4}$

Thus, line meets the parabola at the points

A(2, 1) and B$\left(-1, \dfrac{1}{4}\right)$.



(ii) Required area = (Area under the line $x = 4y-2$) – (Area under the parabola $x^2 = 4y$)

$$= \int_{-1}^{2}\left(\frac{x+2}{4}\right)dx - \int_{-1}^{2}\left(\frac{x^2}{4}\right)dx$$

[From eq. (ii), $y = \dfrac{x+2}{4}$ and from eq. (i), $y = \dfrac{x^2}{4}$]

$$= \frac{1}{4}\left[\frac{x^2}{2}+2x\right]_{-1}^{2} - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{4}\left[\left(\frac{2^2}{2}+2\times 2\right)-\left(\frac{(-1)^2}{2}+2\times(-1)\right)\right] - \frac{1}{4}\left[\frac{2^3}{3}-\frac{(-1)^3}{3}\right]$$

$$= \frac{1}{4}\left[6+\frac{3}{2}\right]-\frac{1}{12}\times 9$$

$= \dfrac{15}{8}-\dfrac{9}{12} = \dfrac{9}{8}$ sq. unit **[ISC Marking Scheme 2023]**

Examiner's Comment

- Mostly students fail to formulate the correct expression for required area and limits.

Answering Tip

- Practice more problems in integration.

Section-C

19. (i) Option (d) is correct.

Explanation:

$MC(201) = C(201)-C(200)$

[ISC Marking Scheme 2024]

(ii) Option (a) is correct.

Explanation: The slope of the regression line of X on Y is also called the Regression coefficient of X on Y.

(iii)

$$\begin{aligned} r &= \sqrt{0.8 \times 0.2} \\ &= \sqrt{0.16} \\ &= \sqrt{(0.4)^2} \\ &= 0.4 \end{aligned}$$

Here, the correlation coefficient will be positive because both the coefficients are positive.

[ISC Marking Scheme 2023]

(iv)

$$\begin{aligned} R(x) &= 4x^3 + \frac{1}{5}x^2 + x \\ \text{Price} &= \frac{R(x)}{x} = 4x^2 + \frac{1}{5}x + 1 \\ \therefore \quad P &= 4x^2 + \frac{1}{5}\ x + 1 \end{aligned}$$

Price for demand of 100 laptops

$$\begin{aligned} \therefore \quad x &= 100 \\ P &= 4(100)^2 + \frac{1}{5}\ (100) + 1 \\ P &= 40{,}000 + 20 + 1 \\ &= ₹\ 40{,}021 \end{aligned}$$

[ISC Marking Scheme 2024]

(v)

$$\begin{aligned} \text{Total cost} &= \text{fixed cost} + \text{variable cost} \\ C(x) &= ₹\ 30{,}000 + ₹\ 800x \end{aligned}$$

where, x = total unit

Also, revenue function, $R(x) = p.x$

$$\begin{aligned} &= (4{,}500 - 100x)x \\ &\qquad [\because \text{ Given } p = 4{,}500 - 100x] \\ &= 4{,}500x - 100x^2 \end{aligned}$$

Profit function $P(x)$ $= R(x) - C(x)$

$$\begin{aligned} &= 4{,}500x - 100x^2 - 30{,}000 - 800x \\ &= -100\ x^2 + 3{,}700x - 30{,}000 \end{aligned}$$

At break even point,

$$P(x) = 0$$

$$\therefore -100x^2 + 3700x - 30{,}000 = 0$$

or, $x^2 - 37x + 300 = 0$

$$\begin{aligned} x &= \frac{+37 \pm \sqrt{(-37)^2 - 4(1)(300)}}{2 \times 1} \\ &= \frac{37 \pm \sqrt{1369 - 1200}}{2} \\ &= \frac{37 \pm 13}{2} \\ &= \frac{50}{2} \text{ and } \frac{24}{2} \\ &= 25 \text{ and } 12 \end{aligned}$$

So, break even values are 25 and 12.

[ISC Marking Scheme 2023]

Examiner's Comment

- Some candidate put $C(x) = 0$, to find break even point, which is incorrect.

Answering Tip

- Always remember for break-even point put profit function, $P(x) = 0$.

20. (i)

$$\begin{aligned} C(x) &= \frac{99}{2}x + 525 \\ \text{Price} &= ₹50/\text{units} \\ \therefore \quad R(x) &= P \times x = 50x \\ \text{Profit} &= R(x) - C(x) \\ P(x) &= 50x - \frac{99}{2}x - 525 = \frac{1}{2}x - 525 \\ P(x) &\geq 0 \text{ then } x = 1050 \end{aligned}$$

Company sell 50 more units to ensure no loss.

[ISC Marking Scheme 2024]

OR

(ii)

$$\begin{aligned} P(x) &= -8x^3 + 29{,}400x - 60{,}000 \\ P'(x) &= -24x^2 + 29{,}400 - 0 \end{aligned}$$

For Maximum /Minimum

$$\begin{aligned} P'(x) &= 0 \\ -24x^2 + 29{,}400 &= 0 \\ x^2 &= 1{,}225 \\ x &= \pm 35 \qquad x \neq -35 \\ P''(x) &= -48x + 0 \\ P''(35) &= -48 \times 35 < 0 \end{aligned}$$

Hence, Profit is maximum at the price ₹35.

[ISC Marking Scheme 2024]

21. (i)

Temp·(0°F) (X)	Cones Sold (Y)	X^2	XY
65	180	4,225	11,700
75	250	5,625	18,750
60	160	3,600	9,600
70	210	4,900	14,700
80	300	6,400	24,000
70	220	4,900	15,400
420	**1,320**	**29,650**	**94,150**

$$\bar{x} = \frac{\sum x}{n} = \frac{420}{6} = 70, \bar{y} = \frac{\sum y}{n} = \frac{1{,}320}{6} = 220$$

$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2} = \frac{6 \times 94{,}150 - 420 \times 1{,}320}{6 \times 29{,}650 - (420)^2}$$

$$b_{yx} = \frac{5{,}64{,}900 - 5{,}54{,}400}{1{,}77{,}900 - 1{,}76{,}400}$$

$$b_{yx} = \frac{10{,}500}{1{,}500} = 7$$

$\therefore$ Regression line of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$\begin{aligned} y - 200 &= 7(x - 70) \\ y &= 7x - 490 + 220 \\ y &= 7x - 270 \end{aligned}$$

[ISC Marking Scheme 2024]

OR

(ii) (a) Given, $\Sigma x = 125$, $\Sigma y = 100$, $\Sigma x^2 = 1650$, $\Sigma y^2 = 1500$, $\Sigma xy = 50$, $n = 25$.

$$\bar{x} = \frac{\Sigma x}{n} = \frac{125}{25} = 5$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{100}{25} = 4$$

$$b_{xy} = \frac{n\Sigma xy - \Sigma x\,\Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

$$= \frac{25(50) - 125(100)}{25(1500) - (100)^2}$$

$$= \frac{-11250}{27500} = \frac{-9}{22}$$

∴ Regression line of y on x

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$\Rightarrow \quad x - 5 = \frac{-9}{22}(y - 4)$$

$\Rightarrow 22x + 9y - 146 = 0$

(b) When $y = 5$, $22x + 9 \times 5 - 146 = 0$

$\Rightarrow \quad 22x = 101$

$$\Rightarrow \quad x = \frac{101}{22} = 4.591$$

∴ Estimated value of $x = 4.591$ Approx.

22. **(a)** Let, x kg of product A be mined with y kg of product B.

∴ $\quad z = 20x + 40y$

Subject to the constraints

$36x + 6y \geq 108 \Rightarrow 6x + y \geq 18$

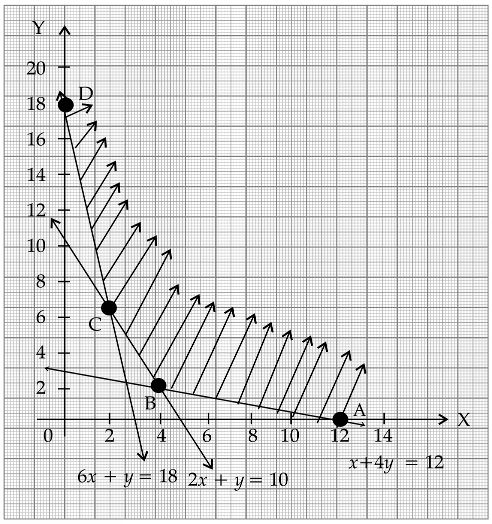
$3x + 12y \geq 36 \Rightarrow x + 4y \geq 12$

$20x + 10y \geq 100 \Rightarrow 2x + y \geq 10$

$x \geq 0$

$y \geq 0$

(b) Now considering these inequations in terms of equations to plot these on graph paper.



∴ The corner points are A(12, 0), B(4, 2), C(2,6) and D(0, 18).

∴ At A(12, 0); $z = 20 \times 12 = 240$

At B(4, 2); $z = 20 \times 4 + 40 \times 2 = 160$

At C(2, 6); $z = 20 \times 2 + 40 \times 6 = 280$

At D(0, 18); $z = 40 \times 18 = 720$

∴ 4 kg of product A and 2 kg of product B be mixed together so that the cost will be minimum and minimum cost = ₹160

❑❑

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